# Parsing: Earley & PCFGs

Ling 571 Deep Processing Techniques for NLP January 19, 2011

### Roadmap

- Motivation I: CKY limitations
- The Earley algorithm

- Motivation II: Ambiguity
- Probabilistic Context-free Grammars (PCFGs)

# CKY Algorithm

- Dynamic programming approach
  - Yields parsing in cubic time in length of input string
  - Fairly efficient
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# CKY Algorithm

- Dynamic programming approach
  - Yields parsing in cubic time in length of input string
  - Fairly efficient
- Issues:
  - Requires CNF conversion
    - Limits expressiveness
  - Maintains ambiguity

# Earley Parsing

- Avoid repeated work/recursion problem
  - Dynamic programming
    - Store partial parses in "chart"
      - Compactly encodes ambiguity

•  $O(N^3)$ 

- Chart entries:
  - Subtree for a single grammar rule
  - Progress in completing subtree
  - Position of subtree wrt input

# Earley Algorithm

- Uses dynamic programming to do parallel top-down search in (worst case) O(N<sup>3</sup>) time
- First, left-to-right pass fills out a chart with N +1 states
  - Think of chart entries as sitting between words in the input string keeping track of states of the parse at these positions
  - For each word position, chart contains set of states representing all partial parse trees generated to date. E.g. chart[0] contains all partial parse trees generated at the beginning of the sentence

### Chart Entries

#### Represent three types of constituents:

predicted constituents

in-progress constituents

completed constituents

# Parse Progress

- Represented by Dotted Rules
- Position of indicates type of constituent
- Book 1 that 2 flight 3
  - $S \rightarrow \bullet VP$ , [0,0] (predicted)
  - NP  $\rightarrow$  Det Nom, [1,2] (in progress)
  - VP →V NP •, [0,3] (completed)
- [x,y] tells us what portion of the input is spanned so far by this rule
- Each State s<sub>i</sub>: <dotted rule>, [<back pointer>,<current position>]

# 0 Book 1 that 2 flight 3

 $S \rightarrow \bullet VP$ , [0,0]

- First 0 means S constituent begins at the start of input
- Second 0 means the dot here too
- So, this is a top-down prediction

 $NP \rightarrow Det \cdot Nom, [1,2]$ 

- the NP begins at position 1
- the dot is at position 2
- so, Det has been successfully parsed
- Nom predicted next

# O Book 1 that 2 flight 3 (continued)

#### $VP \rightarrow V NP \bullet, [0,3]$

Successful VP parse of entire input



### Successful Parse

- Final answer found by looking at last entry in chart
- If entry resembles S  $\rightarrow \alpha \cdot [0,N]$  then input parsed successfully
- Chart will also contain record of all possible parses of input string, given the grammar

# Parsing Procedure for the Earley Algorithm

- Move through each set of states in order, applying one of three operators to each state:
  - **predictor:** add predictions to the chart
  - **scanner:** read input and add corresponding state to chart
  - **completer:** move dot to right when new constituent found
- Results (new states) added to current or next set of states in chart
  - No backtracking and no states removed: keep complete history of parse

### States and State Sets

- Dotted Rule s<sub>i</sub> represented as <dotted rule>, [<back pointer>, <current position>]
- State Set S<sub>j</sub> to be a collection of states s<sub>i</sub> with the same <current position>.

# Earley Algorithm from Book

#### function EARLEY-PARSE(words, grammar) returns chart

ENQUEUE(( $\gamma \rightarrow \bullet S, [0,0]$ ), *chart[0]*) for  $i \leftarrow$  from 0 to LENGTH(words) do for each state in chart[i] do if INCOMPLETE?(state) and NEXT-CAT(*state*) is not a part of speech **then** PREDICTOR(*state*) elseif INCOMPLETE?(state) and NEXT-CAT(*state*) is a part of speech **then** SCANNER(*state*) else COMPLETER(*state*) end end return(chart)

Earley Algorithm from Book **procedure** PREDICTOR( $(A \rightarrow \alpha \bullet B \beta, [i, j])$ ) for each  $(B \rightarrow \gamma)$  in GRAMMAR-RULES-FOR(B, grammar) do ENQUEUE( $(B \rightarrow \bullet \gamma, [j, j]), chart[j]$ ) end **procedure** SCANNER( $(A \rightarrow \alpha \bullet B \beta, [i, j])$ ) if  $B \subset PARTS-OF-SPEECH(word[j])$  then ENQUEUE( $(B \rightarrow word[j], [j, j+1]), chart[j+1]$ ) **procedure** COMPLETER( $(B \rightarrow \gamma \bullet, [j,k])$ ) for each  $(A \rightarrow \alpha \bullet B \beta, [i, j])$  in *chart*[j] do ENQUEUE(( $A \rightarrow \alpha B \bullet \beta, [i,k]$ ), chart[k]) end

# Earley Algorithm (simpler!)

- 1. Add Start  $\rightarrow$  · S, [0,0] to state set 0 Let i=1
- 2. **Predict** all states you can, adding new predictions to state set 0
- 3. Scan input word i—add all matched states to state set S<sub>i</sub>. Add all new states produced by Complete to state set S<sub>i</sub> Add all new states produced by Predict to state set S<sub>i</sub> Let i = i + 1 Unless i=n, repeat step 3.

4. At the end, see if state set *n* contains Start  $\rightarrow$  S  $\cdot$ , [0,n]

3 Main Sub-Routines of Earley Algorithm

- **Predictor**: Adds predictions into the chart.
- **Completer**: Moves the dot to the right when new constituents are found.
- Scanner: Reads the input words and enters states representing those words into the chart.

### Predictor

- Intuition: create new state for top-down prediction of new phrase.
- Applied when non part-of-speech non-terminals are to the right of a dot:  $S \rightarrow \cdot VP[0,0]$
- Adds new states to *current* chart
  - One new state for each expansion of the nonterminal in the grammar
     VP → • V [0,0]
     VP → • V NP [0,0]

• Formally:  $S_j: A \rightarrow \alpha \cdot B \beta, [i,j]$  $S_j: B \rightarrow \cdot \gamma, [j,j]$ 

### Scanner

- Intuition: Create new states for rules matching part of speech of next word.
- Applicable when part of speech is to the right of a dot: VP → • V NP [0,0] 'Book...'
- Looks at current word in input
- If match, adds state(s) to *next* chart VP  $\rightarrow$  V NP [0,1]

• Formally:  $S_j: A \rightarrow \alpha \cdot B \beta, [i,j]$  $S_{j+1}: A \rightarrow \alpha B \cdot \beta, [i,j+1]$ 

# Completer

- Intuition: parser has finished a new phrase, so must find and advance states all that were waiting for this
- Applied when dot has reached right end of rule NP → Det Nom • [1,3]
- Find all states w/dot at 1 and expecting an NP:  $VP \rightarrow V \cdot NP [0,1]$
- Adds new (completed) state(s) to *current* chart :  $VP \rightarrow V NP \cdot [0,3]$ 
  - Formally:  $S_k : B \to \delta \cdot , [j,k]$   $S_k : A \to \alpha \ B \cdot \beta , [i,k],$ where:  $S_i : A \to \alpha \ \cdot B \ \beta , [i,j].$

# Chart[0]

S0	$\gamma \rightarrow \bullet S$	[0,0]	Dummy start state
S1	$S \rightarrow \bullet NP VP$	[0,0]	Predictor
S2	$S \rightarrow \bullet Aux NP VP$	[0,0]	Predictor
S3	$S \rightarrow \bullet VP$	[0,0]	Predictor
S4	$NP \rightarrow \bullet Pronoun$	[0,0]	Predictor
S5	$NP \rightarrow \bullet Proper-Noun$	[0,0]	Predictor
S6	$NP \rightarrow \bullet Det Nominal$	[0,0]	Predictor
S7	$VP \rightarrow \bullet Verb$	[0,0]	Predictor
S8	$VP \rightarrow \bullet Verb NP$	[0,0]	Predictor
S9	$VP \rightarrow \bullet Verb NP PP$	[0,0]	Predictor
S10	$VP \rightarrow \bullet Verb PP$	[0,0]	Predictor
S11	$VP \rightarrow \bullet VP PP$	[0,0]	Predictor

Note that given a grammar, these entries are the same for all inputs; they can be pre-loaded.

Speech and Language Froces

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1/17/11

# Chart[1]

S12	$Verb \rightarrow book \bullet$	[0,1]	Scanner
S13	$VP \rightarrow Verb \bullet$	[0,1]	Completer
S14	$VP \rightarrow Verb \bullet NP$	[0,1]	Completer
S15	$VP \rightarrow Verb \bullet NP PP$	[0,1]	Completer
S16	$VP \rightarrow Verb \bullet PP$	[0,1]	Completer
S17	$S \rightarrow VP \bullet$	[0,1]	Completer
S18	$VP \rightarrow VP \bullet PP$	[0,1]	Completer
S19	$NP \rightarrow \bullet Pronoun$	[1,1]	Predictor
S20	$NP \rightarrow \bullet Proper-Noun$	[1,1]	Predictor
S21	$NP \rightarrow \bullet Det Nominal$	[1,1]	Predictor
S22	$PP \rightarrow \bullet Prep NP$	[1,1]	Predictor

# Charts[2] and [3]

S23	$Det \rightarrow that \bullet$	[1,2]	Scanner
S24	$NP \rightarrow Det \bullet Nominal$	[1,2]	Completer
S25	$Nominal \rightarrow \bullet Noun$	[2,2]	Predictor
S26	$Nominal \rightarrow \bullet Nominal Noun$	[2,2]	Predictor
S27	Nominal $\rightarrow \bullet$ Nominal PP	[2,2]	Predictor
S28	Noun $\rightarrow$ flight $\bullet$	[2,3]	Scanner
S29	$Nominal \rightarrow Noun \bullet$	[2,3]	Completer
S30	$NP \rightarrow Det Nominal ullet$	[1,3]	Completer
S31	$Nominal \rightarrow Nominal \bullet Noun$	[2,3]	Completer
S32	Nominal $\rightarrow$ Nominal $\bullet$ PP	[2,3]	Completer
S33	$VP \rightarrow Verb NP \bullet$	[0,3]	Completer
S34	$VP \rightarrow Verb NP \bullet PP$	[0,3]	Completer
S35	$PP \rightarrow \bullet Prep NP$	[3,3]	Predictor
S36	$S \rightarrow VP \bullet$	[0,3]	Completer
S37	$VP \rightarrow VP \bullet PP$	[0,3]	Completer

Jurafsky and Martin

# How do we retrieve the parses at the end?

- Augment the Completer to add pointers to prior states it advances as a field in the current state
  - i.e. what state did we advance here?
  - Read the pointers back from the final state

#### • What about ambiguity?

• What about ambiguity?

• CKY/Earley can represent it

• What about ambiguity?

• CKY/Earley can represent it

Can't resolve it

# **Probabilistic Parsing**

- Provides strategy for solving disambiguation problem
  - Compute the probability of all analyses
  - Select the most probable
- Employed in language modeling for speech recognition
  - N-gram grammars predict words, constrain search
  - Also, constrain generation, translation

# PCFGs

- Probabilistic Context-free Grammars
  - Augmentation of CFGs
  - N a set of **non-terminal symbols** (or **variables**)
  - $\Sigma$  a set of **terminal symbols** (disjoint from *N*)
  - *R* a set of **rules** or productions, each of the form  $A \rightarrow \beta$  [*p*], where *A* is a non-terminal,

 $\beta$  is a string of symbols from the infinite set of strings  $(\Sigma \cup N)*$ , and *p* is a number between 0 and 1 expressing  $P(\beta | A)$ 

*S* a designated **start symbol** 

# PCFGs

- Augments each production with probability that LHS will be expanded as RHS
  - $P(A \rightarrow B)$  or  $P(A \rightarrow B|A)$ , p(RHS|LHS)
  - Sum over all possible expansions is 1

$$\sum_{\beta} P(A \to \beta) = 1$$

- A PCFG is consistent if sum of probabilities of all sentences in language is 1.
  - Recursive rules often yield inconsistent grammars

# Disambiguation

- A PCFG assigns probability to each parse tree T for input S.
  - Probability of T: product of all rules to derive T

$$P(T,S) = \prod_{i=1}^{n} P(RHS_i \mid LHS_i)$$
$$P(T,S) = P(T)P(S \mid T) = P(T)$$

S   VP		
Verb NP		
		Verb NP NP
Book Det Nominal		
the Nominal Noun		Book Det Nominal Nominal
		the Noun Noun
Noun flight		
dinner		dinner flight
Bulas	D	Dulas D
Rules	P 05	Kules P
$S \rightarrow VP$	.05	$S \rightarrow VP .05$
$VP \longrightarrow Verb NP$	.20	$VP \rightarrow Verb NP NP .10$
NP $\rightarrow$ Det Nominal	.20	NP $\rightarrow$ Det Nominal .20
Nominal $\rightarrow$ Nominal Noum	n .20	NP $\rightarrow$ Nominal .15
Nominal $\rightarrow$ Noun	.75	Nominal $\rightarrow$ Noun .75
		Nominal $\rightarrow$ Noun .75
Verb $\rightarrow$ book	.30	Verb $\rightarrow$ book .30
Det $\rightarrow$ the	.60	Det $\rightarrow$ the .60
Noun $\rightarrow$ dinner	.10	Noun $\rightarrow$ dinner .10
Noun $\rightarrow$ flights	.40	Noun $\rightarrow$ flights .40

P(T,S)=0.05

					S   VP	
Verb NP Book Det Non	ninal		Verb	1	NP N	Р
the Nominal Noun	Noun   flight		Book	Det   the	Nominal Nom     Noun No     dinner flig	inal un <i>ht</i>
Ru	les	Р		Rı	ıles	P
$\overline{S} \rightarrow T$	VP	.05	S	$\rightarrow$	VP	.05
$VP \rightarrow V$	Verb NP	.20	VP	$\rightarrow$	Verb NP NP	.10
$NP \rightarrow 1$	Det Nominal	.20	NP	$\rightarrow$	Det Nominal	.20
Nominal $\rightarrow$ 1	Nominal Noun	.20	NP	$\rightarrow$	Nominal	.15
Nominal $\rightarrow$ 1	Noun	.75	Nominal	$\rightarrow$	Noun	.75
			Nominal	$\rightarrow$	Noun	.75
Verb $\rightarrow$ 1	book	.30	Verb	$\rightarrow$	book	.30
Det $\rightarrow$ t	the	.60	Det	$\rightarrow$	the	.60
Noun $\rightarrow$ o	dinner	.10	Noun	$\rightarrow$	dinner	.10
Noun $\rightarrow$ f	flights	.40	Noun	$\rightarrow$	flights	.40

P(T,S)=0.05\*0.2

S VP Verb NP				_	S VP	
			Verb	]	NP N	P
Book Det No.	minal		Ĩ	/	$\sim$	
the Nomine	Noun		Book	Det	Nominal Nom	inal
				the	Noun No	un
Noun	flight					un
dinner					dinner flig	ht
ainner						
R	ules	Р		Rı	iles	P
$S \rightarrow$	VP	.05	S	$\rightarrow$	VP	.05
$VP \longrightarrow$	Verb NP	.20	VP	$\rightarrow$	Verb NP NP	.10
$NP \longrightarrow$	Det Nominal	.20	NP	$\rightarrow$	Det Nominal	.20
Nominal $\rightarrow$	Nominal Noun	.20	NP	$\rightarrow$	Nominal	.15
Nominal $\rightarrow$	Noun	.75	Nominal	$\rightarrow$	Noun	.75
			Nominal	$\rightarrow$	Noun	.75
Verb $\rightarrow$	book	.30	Verb	$\rightarrow$	book	.30
Det $\rightarrow$	the	.60	Det	$\rightarrow$	the	.60
Noun $\rightarrow$	dinner	.10	Noun	$\rightarrow$	dinner	.10
Noun $\rightarrow$	flights	.40	Noun	$\rightarrow$	flights	.40

P(T,S)=0.05\*0.2\*0.2

S VP Verb NP				_	S VP	
	<		Verb	]	NP N	P
Book Det No.	minal		Ĩ	/	$\sim$	
the Nomine	Noun		Book	Det	Nominal Nom	inal
				the	Noun No	un
Noun	flight					un
dinner					dinner flig	ht
ainner						
R	ules	Р		Rı	iles	P
$S \rightarrow$	VP	.05	S	$\rightarrow$	VP	.05
$VP \longrightarrow$	Verb NP	.20	VP	$\rightarrow$	Verb NP NP	.10
$NP \longrightarrow$	Det Nominal	.20	NP	$\rightarrow$	Det Nominal	.20
Nominal $\rightarrow$	Nominal Noun	.20	NP	$\rightarrow$	Nominal	.15
Nominal $\rightarrow$	Noun	.75	Nominal	$\rightarrow$	Noun	.75
			Nominal	$\rightarrow$	Noun	.75
Verb $\rightarrow$	book	.30	Verb	$\rightarrow$	book	.30
Det $\rightarrow$	the	.60	Det	$\rightarrow$	the	.60
Noun $\rightarrow$	dinner	.10	Noun	$\rightarrow$	dinner	.10
Noun $\rightarrow$	flights	.40	Noun	$\rightarrow$	flights	.40

#### P(T,S)=0.05\*0.2\*0.2\*0.2

S   VP		S   VP
Verb NP		Verb NP NP
<i>the</i> Nominal Noun		Book Det Nominal Nominal         the Noun Noun     dinner flight
dinner		unner jugni
Rules	Р	Rules P
$S \longrightarrow VP$	.05	$S \longrightarrow VP$ .05
$VP \longrightarrow Verb NP$	.20	VP $\rightarrow$ Verb NP NP .10
NP $\rightarrow$ Det Nominal	.20	NP $\rightarrow$ Det Nominal .20
Nominal $\rightarrow$ Nominal Noun	.20	NP $\rightarrow$ Nominal .15
Nominal $\rightarrow$ Noun	.75	Nominal $\rightarrow$ Noun .75
		Nominal $\rightarrow$ Noun .75
Verb $\rightarrow$ book	.30	Verb $\rightarrow$ book .30
Det $\rightarrow$ the	.60	Det $\rightarrow$ the .60
Noun $\rightarrow$ dinner	.10	Noun $\rightarrow$ dinner .10
Noun $\rightarrow$ flights	.40	Noun $\rightarrow$ flights .40

P(T,S)=0.05\*0.2\*0.2\*0.2\*0.75

				, /	S ↓ VP	
Verb NP						
Book Det Nom	ninal		Verb	1		P
			Book	Det	Nominal Nom	inal
the Nominal	Noun				<u> </u>	
Noun	flight			the	Noun No	un
	J. 8.				dinner flig	ht
dinner						
Rul	les	Р		Rı	iles	Р
$\overline{S} \rightarrow \overline{S}$	VP	.05	S	$\rightarrow$	VP	.05
$VP \rightarrow V$	Verb NP	.20	VP	$\rightarrow$	Verb NP NP	.10
$NP \rightarrow I$	Det Nominal	.20	NP	$\rightarrow$	Det Nominal	.20
Nominal $\rightarrow$ 1	Nominal Noun	.20	NP	$\rightarrow$	Nominal	.15
Nominal $\rightarrow$ 1	Noun	.75	Nominal	$\rightarrow$	Noun	.75
			Nominal	$\rightarrow$	Noun	.75
Verb $\rightarrow$ t	book	.30	Verb	$\rightarrow$	book	.30
Det $\rightarrow$ t	the	.60	Det	$\rightarrow$	the	.60
Noun $\rightarrow$ c	dinner	.10	Noun	$\rightarrow$	dinner	.10
Noun $\rightarrow$ f	flights	.40	Noun	$\rightarrow$	flights	.40

P(T,S)=0.05\*0.2\*0.2\*0.2\*0.75\* 0.3

			S VP	
Verb NP				
Book Dia	1	Verb	NP NP	
Deer Det Nomin		Past Det	Nominal Naminal	
the Nominal	Noun	BOOK Det		
		the	Noun Noun	
Noun	flight		dinner di L	
dinner			ainner jught	
Rule	es P	R	iles P	
$S \rightarrow V$	P .05	$S \longrightarrow$	VP .05	
$VP \rightarrow Ve$	erb NP .20	$VP \longrightarrow$	Verb NP NP .10	
$NP \rightarrow De$	et Nominal .20	$NP \longrightarrow$	Det Nominal .20	
Nominal $\rightarrow$ No	ominal Noun .20	$NP \longrightarrow$	Nominal .15	
Nominal $\rightarrow$ No	oun .75	Nominal $\rightarrow$	Noun .75	
		Nominal $\rightarrow$	Noun .75	
Verb $\rightarrow$ bc	ook .30	Verb $\rightarrow$	book .30	
Det $\rightarrow$ the	.60	Det $\rightarrow$	the .60	
Noun $\rightarrow$ di	inner .10	Noun $\rightarrow$	dinner .10	
Noun $\rightarrow$ fli	ights .40	Noun $\rightarrow$	flights .40	

P(T,S)=0.05\*0.2\*0.2\*0.2\*0.75\* 0.3\*0.6

					S   VP	
Verb NP Book Det Not	minal		Verb			Р
the Nomina	l Noun   flight		Book	Det   the	Nominal Nom     Noun No     dinner flig	inal un tht
R	ıles	Р		Rı	ıles	Р
$S \longrightarrow$	VP	.05	S	$\rightarrow$	VP	.05
$VP \rightarrow$	Verb NP	.20	VP	$\rightarrow$	Verb NP NP	.10
$NP \rightarrow$	Det Nominal	.20	NP	$\rightarrow$	Det Nominal	.20
Nominal $\rightarrow$	Nominal Noun	.20	NP	$\rightarrow$	Nominal	.15
Nominal $\rightarrow$	Noun	.75	Nominal	$\rightarrow$	Noun	.75
			Nominal	$\rightarrow$	Noun	.75
Verb $\rightarrow$	book	.30	Verb	$\rightarrow$	book	.30
Det $\rightarrow$	the	.60	Det	$\rightarrow$	the	.60
Noun $\rightarrow$	dinner	.10	Noun	$\rightarrow$	dinner	.10
Noun $\rightarrow$	flights	.40	Noun	$\rightarrow$	flights	.40

P(T,S)=0.05\*0.2\*0.2\*0.2\*0.75\* 0.3\*0.6\*0.1

S   VP		S   VP
Verb NP Book Det Nominal		Verb NP NP
the Nominal Noun     Noun flight   dinner		Book Det Nominal Nominal         the Noun Noun     dinner flight
Rules	Р	Rules P
$S \rightarrow VP$	.05	$S \rightarrow VP$ .05
$VP \longrightarrow Verb NP$	.20	VP $\rightarrow$ Verb NP NP .10
NP $\rightarrow$ Det Nominal	.20	NP $\rightarrow$ Det Nominal .20
Nominal $\rightarrow$ Nominal Noun	.20	NP $\rightarrow$ Nominal .15
Nominal $\rightarrow$ Noun	.75	Nominal $\rightarrow$ Noun .75
		Nominal $\rightarrow$ Noun .75
Verb $\rightarrow$ book	.30	Verb $\rightarrow$ book .30
Det $\rightarrow$ the	.60	Det $\rightarrow$ the .60
Noun $\rightarrow$ dinner	.10	Noun $\rightarrow$ dinner .10
Noun $\rightarrow$ flights	.40	Noun $\rightarrow$ flights .40

S $VP$ $Verb$ $Verb$ $NP$ $Book$ $Det$ $Nomin$ $He$ $Nominal$ $Noun$ $f$ $dinner$	al Noun   <i>light</i>	Verb   Book De   th	S VP NP N et Nominal Nom e Noun No dinner flig	P ninal oun ght
Rules	Р	2 8	Rules	Р
$S \rightarrow VF$	.05	S –	$\rightarrow$ VP	.05
$VP \rightarrow Vei$	rb NP .20	VP -	$\rightarrow$ Verb NP NP	.10
$NP \rightarrow De$	t Nominal .20	NP -	→ Det Nominal	.20
Nominal $\rightarrow$ No	minal Noun .20	NP –	$\rightarrow$ Nominal	.15
Nominal $\rightarrow$ No	oun .75	Nominal -	→ Noun	.75
		Nominal -	→ Noun	.75
Verb $\rightarrow$ boo	ok .30	Verb -	→ book	.30
Det $\rightarrow$ the	.60	Det -	$\rightarrow$ the	.60
Noun $\rightarrow$ dim	nner .10	Noun -	→ dinner	.10
Noun $\rightarrow$ flig	ghts .40	Noun -	$\rightarrow$ flights	.40

P(T,S)=0.05

S VP Verb NP		Verb	, 		P
Book Det Nomina	1		/	$\sim$	
the Nominal N	oun	Book I	Det	Nominal Nom	inal
		1	the	Noun No	un
Noun fli	ight			dinner di	1
dinner				anner jug	ht
Rules	Р		Ru	ıles	Р
$S \longrightarrow VP$	.05	S	$\rightarrow$	VP	.05
$VP \rightarrow Ver$	b NP .20	VP	$\rightarrow$	Verb NP NP	.10
$NP \rightarrow Det$	Nominal .20	NP	$\rightarrow$	Det Nominal	.20
Nominal $\rightarrow$ Nor	ninal Noun .20	NP	$\rightarrow$	Nominal	.15
Nominal $\rightarrow$ Not	ın .75	Nominal	$\rightarrow$	Noun	.75
		Nominal	$\rightarrow$	Noun	.75
Verb $\rightarrow$ boo	k .30	Verb	$\rightarrow$	book	.30
Det $\rightarrow$ the	.60	Det	$\rightarrow$	the	.60
Noun $\rightarrow$ dim	ner .10	Noun	$\rightarrow$	dinner	.10
Noun $\rightarrow$ flight	hts .40	Noun	$\rightarrow$	flights	.40

P(T,S)=0.05\*0.1

S VP Verb NP		Verb	, 		P
Book Det Nomina	1		/	$\sim$	
the Nominal N	oun	Book I	Det	Nominal Nom	inal
		1	the	Noun No	un
Noun fli	ight			dinner di	1
dinner				anner jug	ht
Rules	Р		Ru	ıles	Р
$S \longrightarrow VP$	.05	S	$\rightarrow$	VP	.05
$VP \rightarrow Ver$	b NP .20	VP	$\rightarrow$	Verb NP NP	.10
$NP \rightarrow Det$	Nominal .20	NP	$\rightarrow$	Det Nominal	.20
Nominal $\rightarrow$ Nor	ninal Noun .20	NP	$\rightarrow$	Nominal	.15
Nominal $\rightarrow$ Not	ın .75	Nominal	$\rightarrow$	Noun	.75
		Nominal	$\rightarrow$	Noun	.75
Verb $\rightarrow$ boo	k .30	Verb	$\rightarrow$	book	.30
Det $\rightarrow$ the	.60	Det	$\rightarrow$	the	.60
Noun $\rightarrow$ dim	ner .10	Noun	$\rightarrow$	dinner	.10
Noun $\rightarrow$ flight	hts .40	Noun	$\rightarrow$	flights	.40

#### P(T,S)=0.05\*0.1\*0.15

S   VP		S   VP
Verb NP Book Det Nominal the Nominal Noun Noun flight		Verb NP NP   Book Det Nominal Nominal       the Noun Noun   dinner flight
dinner		Jughi
Rules	Р	Rules P
$S \longrightarrow VP$	.05	$S \rightarrow VP$ .05
$VP \longrightarrow Verb NP$	.20	VP $\rightarrow$ Verb NP NP .10
NP $\rightarrow$ Det Nominal	.20	NP $\rightarrow$ Det Nominal .20
Nominal $\rightarrow$ Nominal Not	m .20	NP $\rightarrow$ Nominal .15
Nominal $\rightarrow$ Noun	.75	Nominal $\rightarrow$ Noun .75
		Nominal $\rightarrow$ Noun .75
Verb $\rightarrow$ book	.30	Verb $\rightarrow$ book .30
Det $\rightarrow$ the	.60	Det $\rightarrow$ the .60
Noun $\rightarrow$ dinner	.10	Noun $\rightarrow$ dinner .10
Noun $\rightarrow$ flights	.40	Noun $\rightarrow$ flights .40

#### P(T,S)=0.05\*0.1\*0.15\*0.75

S VP Verb NP Book Det No the Nomina Noun dinner	minal al Noun <i>flight</i>		Verb   Book	Det   the	S VP NP N Nominal Nom Noun No <i>i</i> <i>dinner flig</i>	P iinal un eht
R	ules	Р		Rı	ıles	Р
$S \rightarrow$	VP	.05	S	$\rightarrow$	VP	.05
$VP \rightarrow$	Verb NP	.20	VP	$\rightarrow$	Verb NP NP	.10
$NP \rightarrow$	Det Nominal	.20	NP	$\rightarrow$	Det Nominal	.20
Nominal $\rightarrow$	Nominal Noun	.20	NP	$\rightarrow$	Nominal	.15
Nominal $\rightarrow$	Noun	.75	Nominal	$\rightarrow$	Noun	.75
			Nominal	$\rightarrow$	Noun	.75
Verb $\rightarrow$	book	.30	Verb	$\rightarrow$	book	.30
Det $\rightarrow$	the	.60	Det	$\rightarrow$	the	.60
Noun $\rightarrow$	dinner	.10	Noun	$\rightarrow$	dinner	.10
Noun $\rightarrow$	flights	.40	Noun	$\rightarrow$	flights	.40

#### P(T,S)=0.05\*0.1\*0.15\*0.75\*0.75\*

					S   VP	
Verb NP	~		Verb	1	NP N	P
<i>Book</i> Det Nor <i>the</i> Nominal	ninal 1 Noun		 Book	Det	Nominal Nom	inal
 Noun   dinner	flight			the	Noun No     dinner flig	un ht
Ru	iles	Р		Rı	iles	P
$\frac{1}{S} \rightarrow$	VP	.05	S	$\rightarrow$	VP	.05
$VP \rightarrow$	Verb NP	.20	VP	$\rightarrow$	Verb NP NP	.10
$NP \rightarrow$	Det Nominal	.20	NP	$\rightarrow$	Det Nominal	.20
Nominal $\rightarrow$	Nominal Noun	.20	NP	$\rightarrow$	Nominal	.15
Nominal $\rightarrow$	Noun	.75	Nominal	$\rightarrow$	Noun	.75
			Nominal	$\rightarrow$	Noun	.75
Verb $\rightarrow$	book	.30	Verb	$\rightarrow$	book	.30
Det $\rightarrow$	the	.60	Det	$\rightarrow$	the	.60
Noun $\rightarrow$	dinner	.10	Noun	$\rightarrow$	dinner	.10
Noun $\rightarrow$	flights	.40	Noun	$\rightarrow$	flights	.40

P(T,S)=0.05\*0.1\*0.15\*0.75\*0.75\* 0.3\*0.6\*0.1\*0.4=6.1x10^-7

# Formalizing Disambiguation

• Select T such that:

$$\hat{T}(S) = \underset{Ts.t, S=yield(T)}{\operatorname{argmax}} P(T)$$

- String of words S is *yield* of parse tree over S
- Select tree that maximizes probability of parse

# Learning Probabilities

- Simplest way:
  - Treebank of parsed sentences
  - To compute probability of a rule, count:
    - Number of times non-terminal is expanded
    - Number of times non-terminal is expanded by given rule

$$P(\alpha \to \beta \mid \alpha) = \frac{Count(\alpha \to \beta)}{\sum_{\gamma} Count(\alpha \to \gamma)} = \frac{Count(\alpha \to \beta)}{Count(\alpha)}$$

Alternative: Learn probabilities by re-estimating
(Later)

# Example PCFG

Grammar		Lexicon
$S \rightarrow NP VP$	[.80]	$Det \rightarrow that [.10] \mid a [.30] \mid the [.60]$
$S \rightarrow Aux NP VP$	[.15]	<i>Noun</i> $\rightarrow$ <i>book</i> [.10]   <i>flight</i> [.30]
$S \rightarrow VP$	[.05]	meal [.15]   money [.05]
$NP \rightarrow Pronoun$	[.35]	<i>flights</i> [.40]   <i>dinner</i> [.10]
$NP \rightarrow Proper-Noun$	[.30]	$Verb \rightarrow book [.30] \mid include [.30]$
$NP \rightarrow Det Nominal$	[.20]	<i>prefer</i> ; [.40]
$NP \rightarrow Nominal$	[.15]	$Pronoun \rightarrow I[.40] \mid she [.05]$
$Nominal \rightarrow Noun$	[.75]	<i>me</i> [.15]   <i>you</i> [.40]
$Nominal \rightarrow Nominal Noun$	[.20]	<i>Proper-Noun</i> $\rightarrow$ <i>Houston</i> [.60]
Nominal $\rightarrow$ Nominal PP	[.05]	NWA [.40]
$VP \rightarrow Verb$	[.35]	$Aux \rightarrow does [.60] \mid can [40]$
$VP \rightarrow Verb NP$	[.20]	Preposition $\rightarrow$ from [.30]   to [.30]
$VP \rightarrow Verb NP PP$	[.10]	<i>on</i> [.20]   <i>near</i> [.15]
$VP \rightarrow Verb PP$	[.15]	<i>through</i> [.05]
$VP \rightarrow Verb NP NP$	[.05]	
$VP \rightarrow VP PP$	[.15]	
$PP \rightarrow Preposition NP$	[1.0]	

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  - Rule expansion is context-independent
    - Allows us to multiply probabilities
  - Is this valid?

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#### • Is this valid?

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Subject	91%	9%
Object		

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- How can we handle this?

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Subject	91%	9%
Object	34%	66%

- In Treebank: roughly equi-probable
- How can we handle this?
  - Condition on Subj/Obj with parent annotation

- Insufficient lexical conditioning
  - Present in pre-terminal rules
- Are there cases where other rules should be conditioned on words?

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Different verbs & prepositions have different attachment preferences

# PCFGs

Augment parsers to handle probabilities

- Adapt PCFGs to handle
  - Structural dependencies
    - By splitting nodes
  - Lexical dependencies
    - By lexicalizing PCFGs

