CKY Parsing

Ling 571
Deep Processing Techniques for NLP
January 15, 2014

Roadmap

- Motivation:
  - Parsing (In) efficiency
- Dynamic Programming
- Cocke-Kasami-Younger Parsing Algorithm
  - Chomsky Normal Form
    - Conversion
  - CKY Algorithm
    - Parsing by tabulation

Repeated Work

- Top-down and bottom-up parsing both lead to repeated substructures
  - Globally bad parses can construct good subtrees
    - But overall parse will fail
    - Require reconstruction on other branch
  - No static backtracking strategy can avoid
- Efficient parsing techniques require storage of shared substructure
  - Typically with dynamic programming
- Example: *a flight from Indianapolis to Houston on TWA*

Parsing w/Dynamic Programming

- Avoids repeated work
- Allows implementation of (relatively) efficient parsing algorithms
  - Polynomial time in input length
    - Typically cubic (n^3) or less
- Several different implementations
  - Cocke-Kasami-Younger (CKY) algorithm
  - Earley algorithm
  - Chart parsing

Dynamic Programming in CKY

- Key idea:
  - For a parse spanning substring [i,j] , there exists some k such there are parses spanning [i,k] and [k,j]
    - We can construct parses for whole sentence by building up from these stored partial parses

Dynamic Programming in CKY

- Key idea:
  - For a parse spanning substring [i,j] , there exists some k such there are parses spanning [i,k] and [k,j]
    - We can construct parses for whole sentence by building up from these stored partial parses
- So,
  - To have a rule A -> B C in [i,j],
    - We must have B in [i,k] and C in [k,j], for some i<k<j
      - CNF grammar forces this for all j>i+1

CKY

- Given an input string S of length n,
  - Build table (n+1) x (n+1)
  - Indexes correspond to inter-word positions
    - E.g., 0 Book 1 That 2 Flight 3

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  - Indexes correspond to inter-word positions
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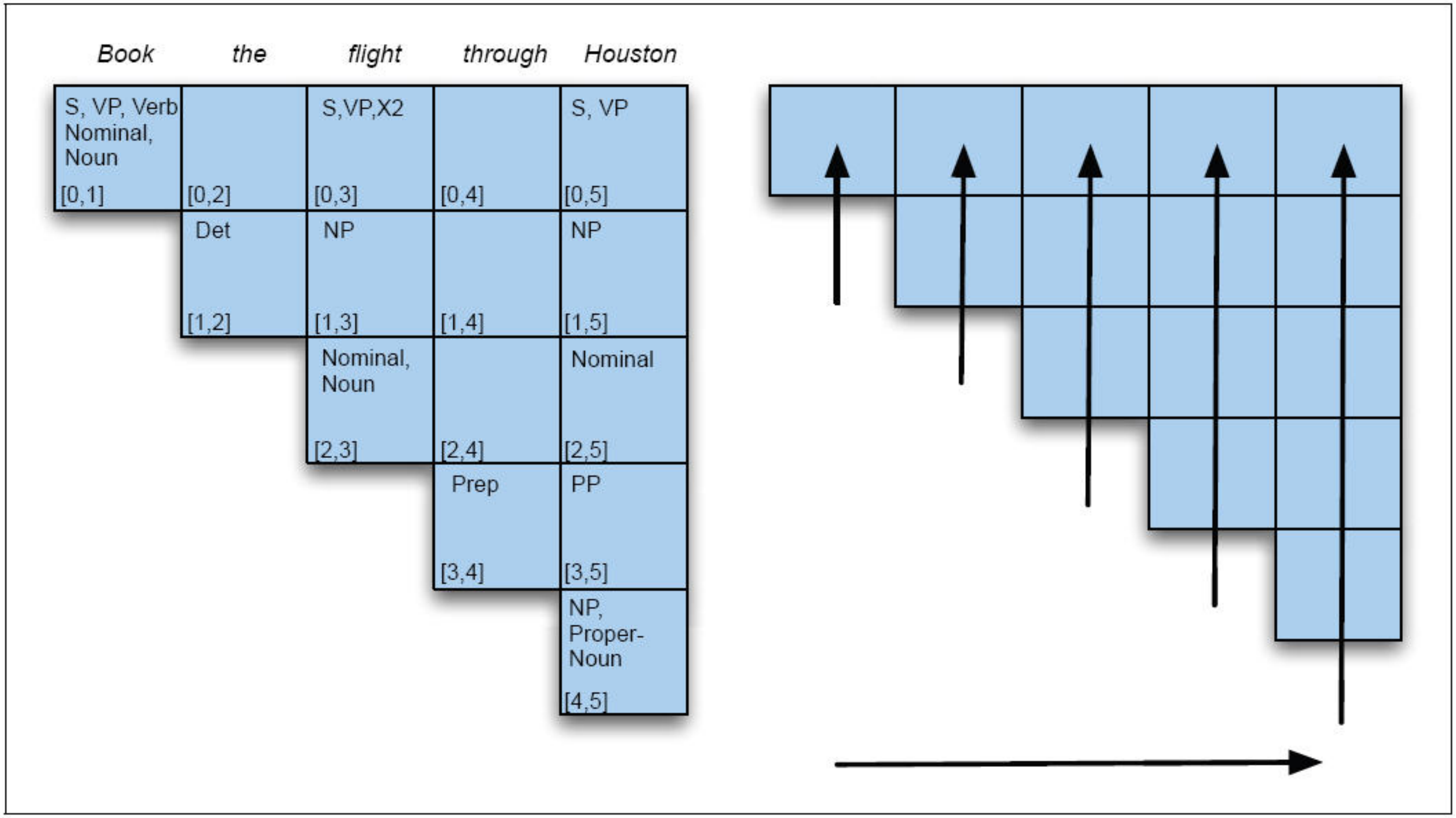
- Cells [i,j] contain sets of non-terminals of ALL constituents spanning i,j
  - [j-1,j] contains pre-terminals
  - If [0,n] contains Start, the input is recognized

CKY Table

- Book the flight through Houston



CKY Algorithm

function CKY-PARSE(*words*, *grammar*) **returns** *table*

for $j \leftarrow$ **from** 1 **to** LENGTH(*words*) **do**
  $table[j-1, j] \leftarrow \{A \mid A \rightarrow words[j] \in grammar\}$
  for $i \leftarrow$ **from** $j-2$ **downto** 0 **do**
    for $k \leftarrow i+1$ **to** $j-1$ **do**
      $table[i,j] \leftarrow table[i,j] \cup$
        $\{A \mid A \rightarrow BC \in grammar,$
          $B \in table[i,k],$
          $C \in table[k,j]\}$

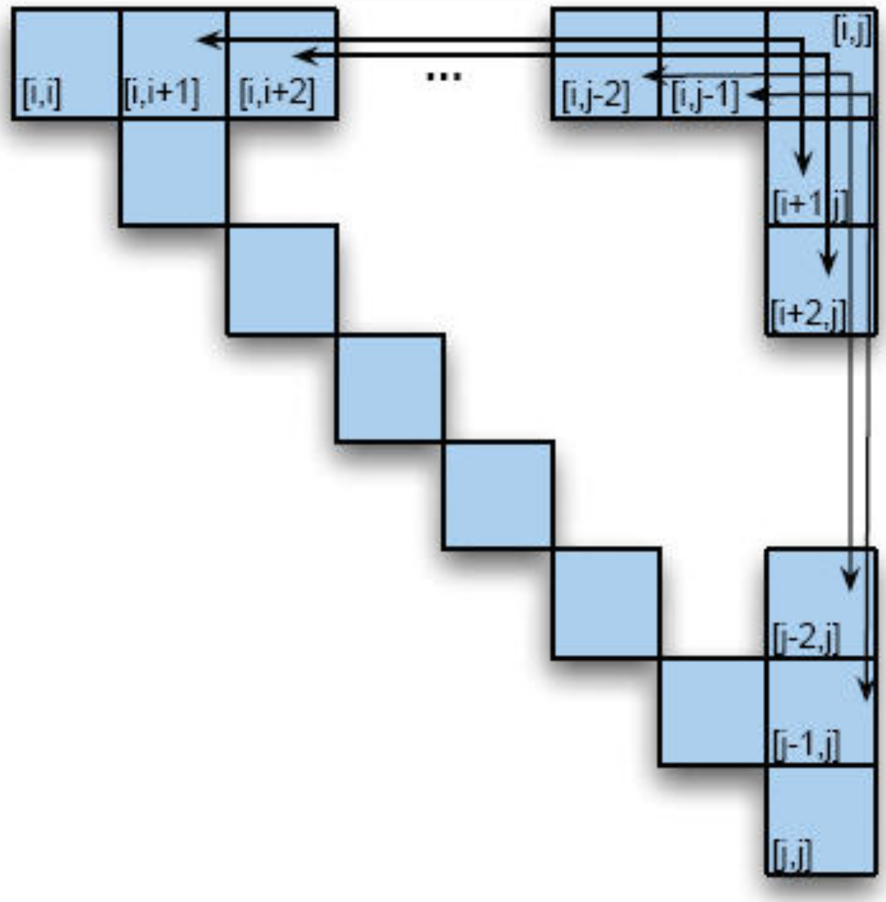
CKY Parsing

- Table fills:
  - Column-by-column
  - Left-to-right
  - Bottom-to-top
- Why?

CKY Parsing

- Table fills:
  - Column-by-column
  - Left-to-right
  - Bottom-to-top
- Why?
  - Necessary info available (below and left)
  - Allows online sentence analysis
    - Works across input string as it arrives

Filling a CKY cell



$\mathscr{L}_1$ **Grammar**	$\mathscr{L}_1$ **in CNF**
$S \rightarrow NP\ VP$	$S \rightarrow NP\ VP$
$S \rightarrow Aux\ NP\ VP$	$S \rightarrow X1\ VP$
	$X1 \rightarrow Aux\ NP$
$S \rightarrow VP$	$S \rightarrow book \mid include \mid prefer$
	$S \rightarrow Verb\ NP$
	$S \rightarrow X2\ PP$
	$S \rightarrow Verb\ PP$
	$S \rightarrow VP\ PP$
$NP \rightarrow Pronoun$	$NP \rightarrow I \mid she \mid me$
$NP \rightarrow Proper\text{-}Noun$	$NP \rightarrow TWA \mid Houston$
$NP \rightarrow Det\ Nominal$	$NP \rightarrow Det\ Nominal$
$Nominal \rightarrow Noun$	$Nominal \rightarrow book \mid flight \mid meal \mid money$
$Nominal \rightarrow Nominal\ Noun$	$Nominal \rightarrow Nominal\ Noun$
$Nominal \rightarrow Nominal\ PP$	$Nominal \rightarrow Nominal\ PP$
$VP \rightarrow Verb$	$VP \rightarrow book \mid include \mid prefer$
$VP \rightarrow Verb\ NP$	$VP \rightarrow Verb\ NP$
$VP \rightarrow Verb\ NP\ PP$	$VP \rightarrow X2\ PP$
	$X2 \rightarrow Verb\ NP$
$VP \rightarrow Verb\ PP$	$VP \rightarrow Verb\ PP$
$VP \rightarrow VP\ PP$	$VP \rightarrow VP\ PP$
$PP \rightarrow Preposition\ NP$	$PP \rightarrow Preposition\ NP$

0 Book 1 the 2 flight 3 through 4 Houston 5

Book	the	Flight	through	Houston
NN, VB, Nominal, VP, S [0,1]				

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Book	the	Flight	through	Houston
NN, VB, Nominal, VP, S [0,1]				
	Det [1,2]			

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NN, VB, Nominal, VP, S [0,1]	[0,2]			
	Det [1,2]			

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Book	the	Flight	through	Houston
NN, VB, Nominal, VP, S [0,1]	[0,2]			
	Det [1,2]			
		NN, Nominal [2,3]		

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Book	the	Flight	through	Houston
NN, VB, Nominal, VP, S [0,1]	[0,2]			
	Det [1,2]	NP [1,3]		
		NN, Nominal [2,3]		

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Book	the	Flight	Through	Houston
NN, VB, Nominal, VP, S [0,1]	[0,2]	S, VP, X2 [0,3]		
	Det [1,2]	NP [1,3]		
		NN, Nominal [2,3]		

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NN, VB, Nominal, VP, S [0,1]	[0,2]	S, VP, X2 [0,3]		
	Det [1,2]	NP [1,3]		
		NN, Nominal [2,3]		
			Prep [3,4]	

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Book	the	Flight	Through	Houston
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	Det [1,2]	NP [1,3]	[1,4]	
		NN, Nominal [2,3]	[2,4]	
			Prep [3,4]	

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NN, VB, Nominal, VP, S [0,1]	[0,2]	S, VP, X2 [0,3]	[0,4]	
	Det [1,2]	NP [1,3]	[1,4]	
		NN, Nominal [2,3]	[2,4]	
			Prep [3,4]	
				NNP, NP [4,5]

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		NN, Nominal [2,3]	[2,4]	Nominal [2,5]
			Prep [3,4]	PP [3,5]
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Book	the	Flight	Through	Houston
NN, VB, Nominal, VP, S [0,1]	[0,2]	S, VP, X2 [0,3]	[0,4]	S, VP, X2 [0,5]
	Det [1,2]	NP [1,3]	[1,4]	NP [1,5]
		NN, Nominal [2,3]	[2,4]	Nominal [2,5]
			Prep [3,4]	PP [3,5]
				NNP, NP [4,5]

Is this a parser?

From Recognition to Parsing

- Limitations of current recognition algorithm:

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- Parsing solution:
  - All repeated versions of non-terminals
  - Pair each non-terminal with pointers to cells
    - Backpointers
  - Last step: construct trees from back-pointers in [0,n]

Filling column 5

Book	the	flight	through	Houston
S, VP, Verb, Nominal, Noun [0,1]	[0,2]	S,VP,X2 [0,3]	[0,4]	[0,5]
	Det [1,2]	NP [1,3]	[1,4]	[1,5]
		Nominal, Noun [2,3]	[2,4]	Nominal [2,5]
			Prep [3,4]	[3,5]
				NP, Proper-Noun [4,5]

Book	the	flight	through	Houston
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		Nominal, Noun [2,3]	[2,4]	[2,5]
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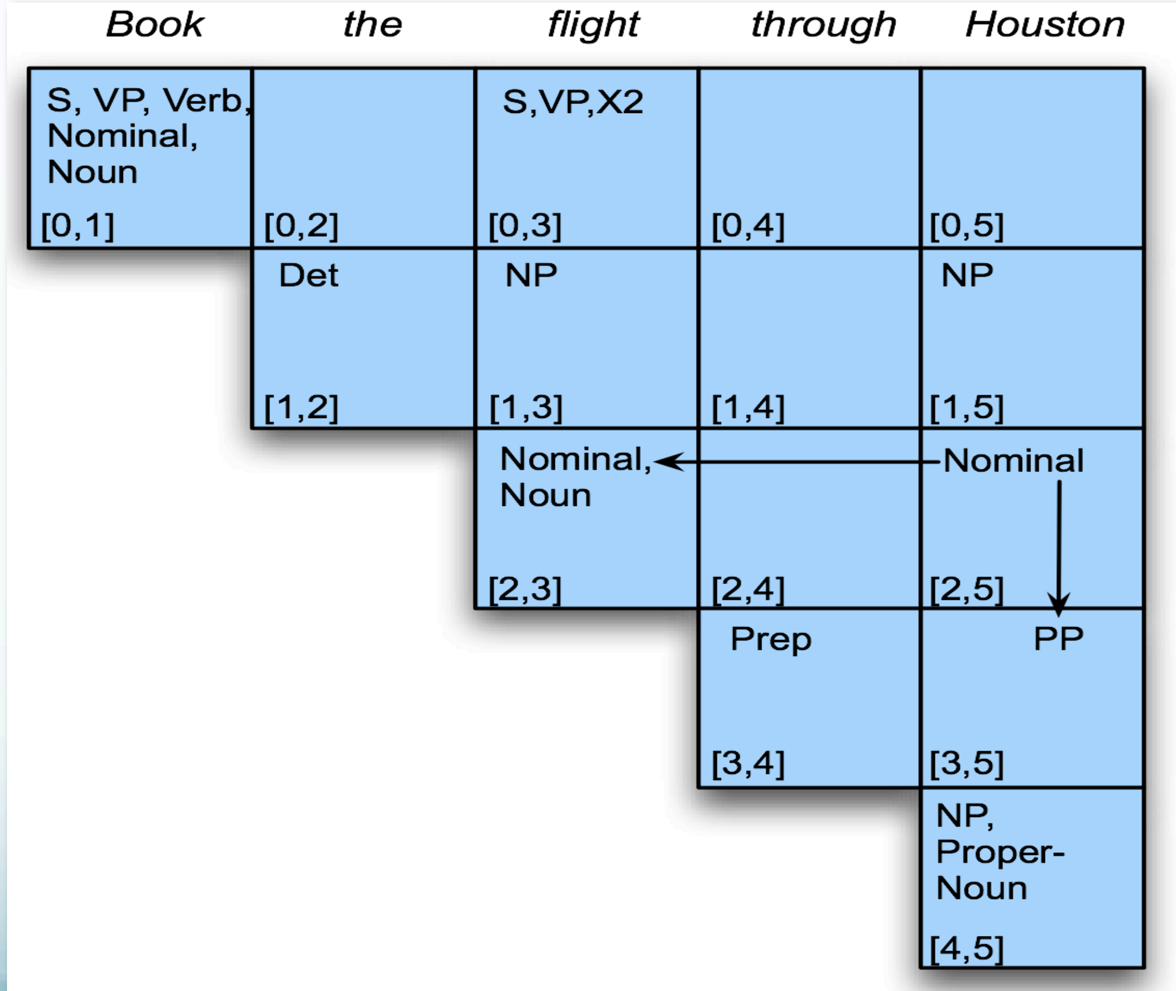
Book
the
flight
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Houston
S, VP, Verb, Nominal, Noun
[0,1]
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S,VP,X2
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[0,4]
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Det
[1,2]
NP
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[1,4]
NP
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Nominal, Noun
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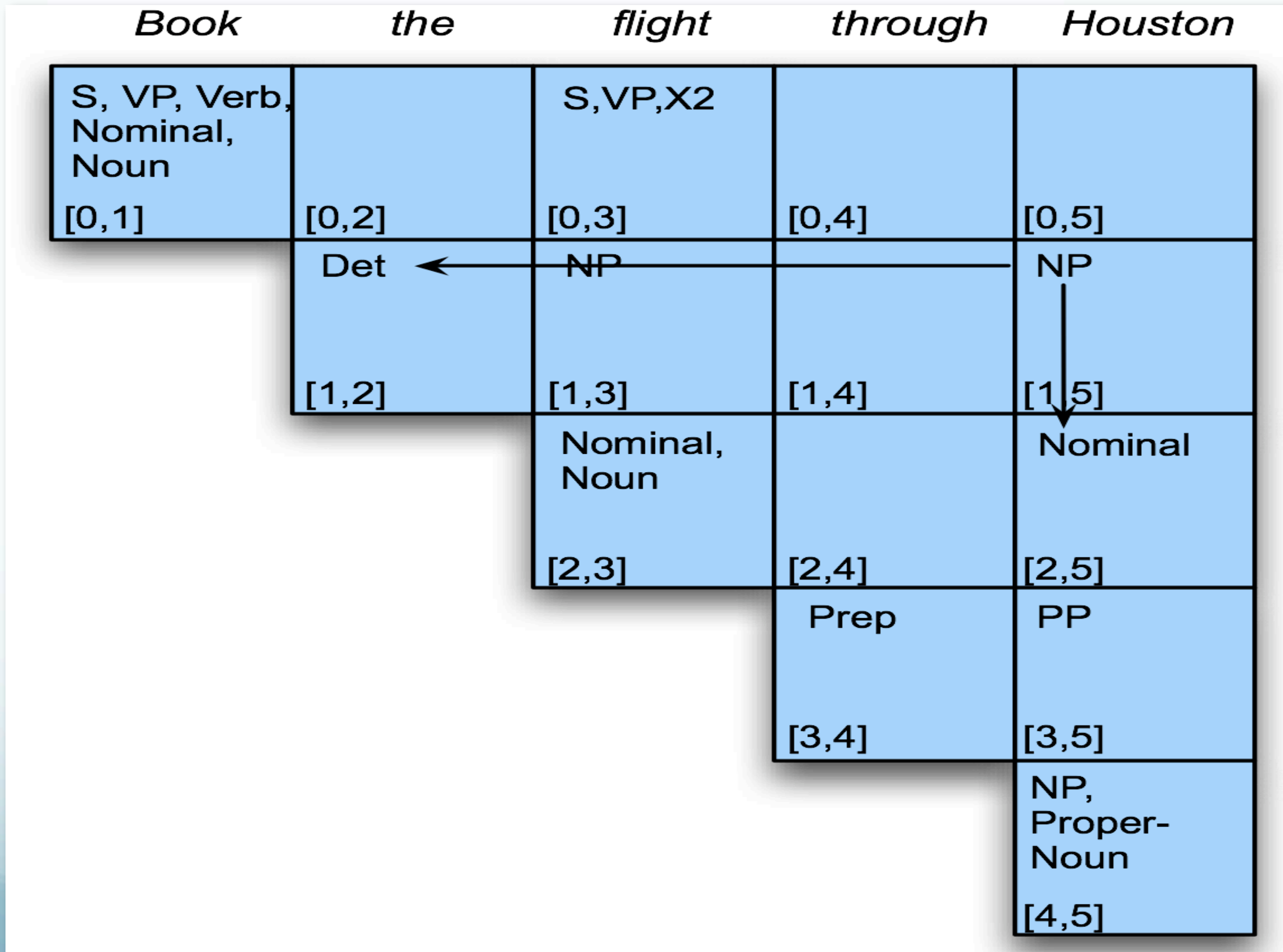
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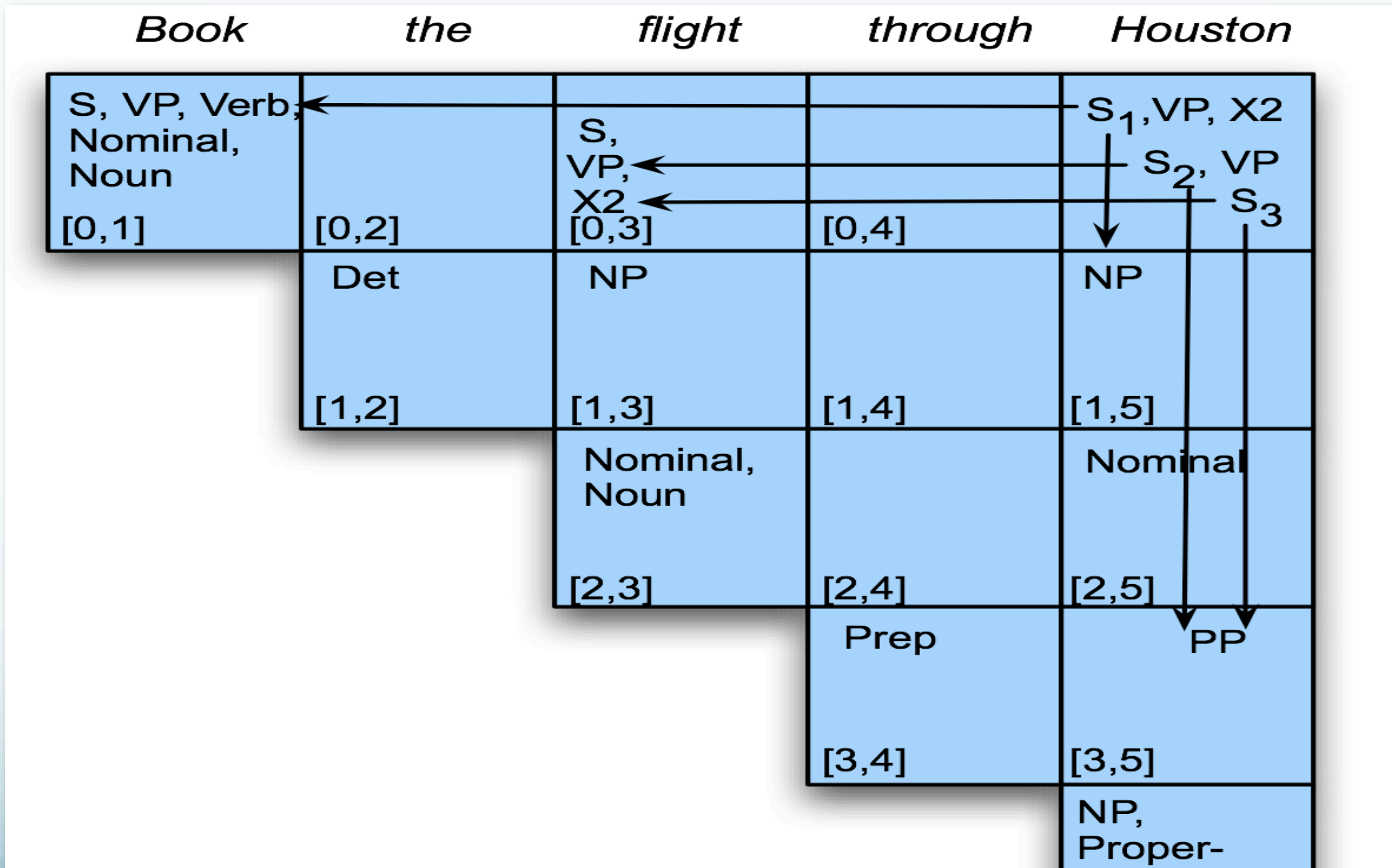

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[0,4]
S_1,VP, X2
S_2, VP
S_3
Det
[1,2]
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CKY Discussion

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  $O(n^3)$

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- Expressiveness:
  - As implemented, requires CNF
    - Weakly equivalent to original grammar
    - Doesn't capture full original structure
      - Back-conversion?
        - Can do binarization, terminal conversion
        - Unit non-terminals require change in CKY

Parsing Efficiently

- With arbitrary grammars
  - Earley algorithm
    - Top-down search
    - Dynamic programming
      - Tabulated partial solutions
    - Some bottom-up constraints

Earley Parsing

- Avoid repeated work/recursion problem
  - Dynamic programming
    - Store partial parses in “chart”
      - Compactly encodes ambiguity
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Earley Parsing

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    - $O(N^3)$

- Chart entries:
  - Subtree for a single grammar rule
  - Progress in completing subtree
  - Position of subtree wrt input

Earley Algorithm

- First, left-to-right pass fills out a chart with N+1 states
  - Think of chart entries as sitting between words in the input string, keeping track of states of the parse at these positions
  - For each word position, chart contains set of states representing all partial parse trees generated to date. E.g. chart[0] contains all partial parse trees generated at the beginning of the sentence

Chart Entries

Represent three types of constituents:

- predicted constituents
- in-progress constituents
- completed constituents

Parse Progress

- Represented by Dotted Rules
- Position of • indicates type of constituent
- $_0$ Book $_1$ that $_2$ flight $_3$
  - S → • VP, [0,0] (predicted)

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Parse Progress

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- Position of • indicates type of constituent
- $_0$ Book $_1$ that $_2$ flight $_3$
  - S → • VP, [0,0] (predicted)
  - NP → Det • Nom, [1,2] (in progress)
  - VP →V NP •, [0,3] (completed)
- [x,y] tells us what portion of the input is spanned so far by this rule
- **Each State s_i:**
  <dotted rule>, [<back pointer>,<current position>]

$_0$ Book $_1$ that $_2$ flight $_3$

S → • VP, [0,0]

- First 0 means S constituent begins at the start of input
- Second 0 means the dot here, too
- So, this is a top-down prediction

$_0$ Book $_1$ that $_2$ flight $_3$

S → • VP, [0,0]

- First 0 means S constituent begins at the start of input
- Second 0 means the dot here too
- So, this is a top-down prediction

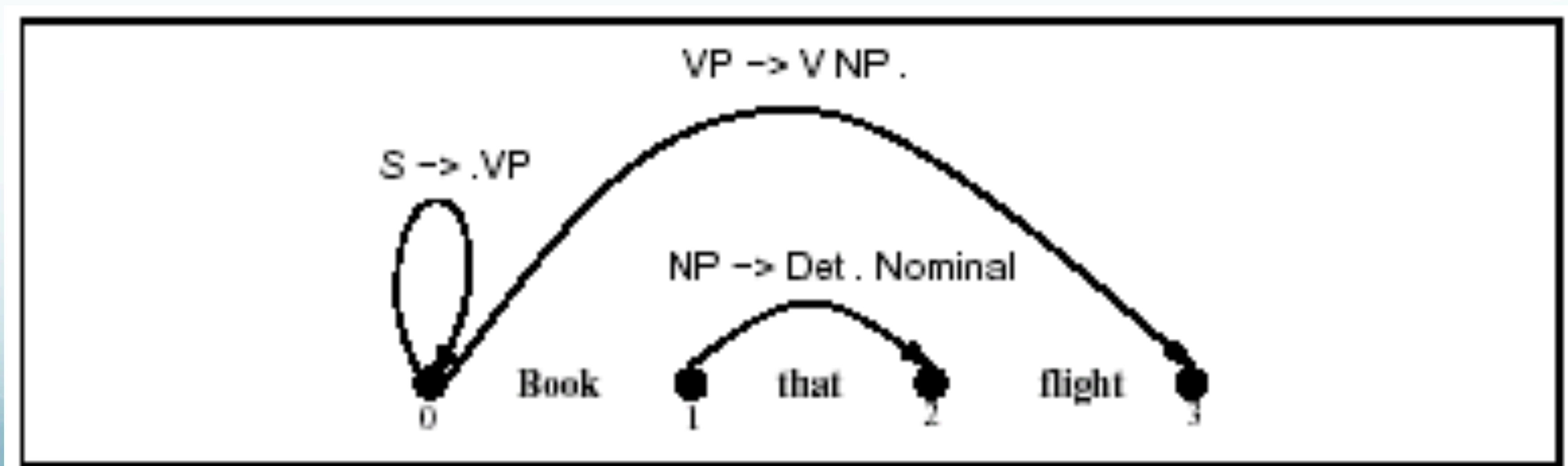
NP → Det • Nom, [1,2]

- the NP begins at position 1
- the dot is at position 2
- so, Det has been successfully parsed
- Nom predicted next

$_0$ Book $_1$ that $_2$ flight $_3$ (continued)

VP → V NP •, [0,3]

- Successful VP parse of entire input



Successful Parse

- Final answer found by looking at last entry in chart

Successful Parse

- Final answer found by looking at last entry in chart
- If entry resembles $S \rightarrow \alpha \bullet [0,N]$ then input parsed successfully
- Chart will also contain record of all possible parses of input string, given the grammar

Parsing Procedure for the Earley Algorithm

- Move through each set of states in order, applying one of three operators to each state:
  - **predictor:** add predictions to the chart
  - **scanner:** read input and add corresponding state to chart
  - **completer:** move dot to right when new constituent found

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- Results (new states) added to current or next set of states in chart
- No backtracking and no states removed: keep complete history of parse

States and State Sets

- **Dotted Rule s_i** represented as
  <dotted rule>, [<back pointer>, <current position>]

- **State Set S_j** to be a collection of states s_i with the same <current position>.

Earley Algorithm from Book

```
function EARLEY-PARSE(words, grammar) returns chart

  ENQUEUE((γ → • S, [0,0]), chart[0])
  for i ← from 0 to LENGTH(words) do
   for each state in chart[i] do
     if INCOMPLETE?(state) and
              NEXT-CAT(state) is not a part of speech then
        PREDICTOR(state)
     elseif INCOMPLETE?(state) and
              NEXT-CAT(state) is a part of speech then
        SCANNER(state)
     else
        COMPLETER(state)
   end
  end
  return(chart)
```

Earley Algorithm from Book

procedure PREDICTOR($(A \rightarrow \alpha \bullet B\,\beta,\ [i,j])$)
  for each $(B \rightarrow \gamma)$ **in** GRAMMAR-RULES-FOR(B, *grammar*) **do**
    ENQUEUE($(B \rightarrow \bullet\,\gamma,\ [j,j])$, *chart*[$j$])
  end

procedure SCANNER($(A \rightarrow \alpha \bullet B\,\beta,\ [i,j])$)
  if B $\subset$ PARTS-OF-SPEECH(*word[j]*) **then**
    ENQUEUE($(B \rightarrow$ *word*$[j],\ [j, j+1])$, *chart[j+1]*)

procedure COMPLETER($(B \rightarrow \gamma \bullet,\ [j,k])$)
  for each $(A \rightarrow \alpha \bullet B\,\beta,\ [i,j])$ **in** *chart*[j] **do**
    ENQUEUE($(A \rightarrow \alpha\, B \bullet \beta,\ [i,k])$, *chart[k]*)
  end

Earley Algorithm (simpler!)

1. Add Start → · S, [0,0] to state set 0
   Let i=1

2. **Predict** all states you can, adding new predictions to state set 0

3. **Scan** input word i—add all matched states to state set S_i.
   Add all new states produced by **Complete** to state set S_i
   Add all new states produced by **Predict** to state set S_i
   Let i = i + 1
   Unless i=*n*, repeat step 3.

4. At the end, see if state set *n* contains Start → S ·, [0,n]

3 Main Sub-Routines of Earley Algorithm

- **Predictor**: Adds predictions into the chart.
- **Completer**: Moves the dot to the right when new constituents are found.
- **Scanner**: Reads the input words and enters states representing those words into the chart.

Predictor

- Intuition: create new state for top-down prediction of new phrase.

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- Applied when non part-of-speech non-terminals are to the right of a dot: **S → • VP [0,0]**
- Adds new states to *current* chart
  - One new state for each expansion of the non-terminal in the grammar
    VP → • V [0,0]
    VP → • V NP [0,0]

Predictor

- Intuition: create new state for top-down prediction of new phrase.
- Applied when non part-of-speech non-terminals are to the right of a dot: **S → • VP [0,0]**
- Adds new states to *current* chart
  - One new state for each expansion of the non-terminal in the grammar
    VP → • V [0,0]
    VP → • V NP [0,0]
- Formally:
  S_j: $A \rightarrow \alpha \cdot B \beta$, [i,j]
  S_j: $B \rightarrow \cdot \gamma$, [j,j]

Chart[0]

S0	$\gamma \rightarrow \bullet S$	[0,0]	Dummy start state

Note that given a grammar, these entries are the same for all inputs; they can be pre-loaded.

Chart[0]

S0	$\gamma \rightarrow \bullet\, S$	[0,0]	Dummy start state
S1	$S \rightarrow \bullet\, NP\ VP$	[0,0]	Predictor
S2	$S \rightarrow \bullet\, Aux\ NP\ VP$	[0,0]	Predictor
S3	$S \rightarrow \bullet\, VP$	[0,0]	Predictor

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Chart[0]

S0	$\gamma \rightarrow \bullet\, S$	[0,0]	Dummy start state
S1	$S \rightarrow \bullet\, NP\ VP$	[0,0]	Predictor
S2	$S \rightarrow \bullet\, Aux\ NP\ VP$	[0,0]	Predictor
S3	$S \rightarrow \bullet\, VP$	[0,0]	Predictor
S4	$NP \rightarrow \bullet\, Pronoun$	[0,0]	Predictor
S5	$NP \rightarrow \bullet\, Proper\text{-}Noun$	[0,0]	Predictor
S6	$NP \rightarrow \bullet\, Det\ Nominal$	[0,0]	Predictor

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Chart[0]

S0	$\gamma \rightarrow \bullet S$	[0,0]	Dummy start state
S1	$S \rightarrow \bullet NP\ VP$	[0,0]	Predictor
S2	$S \rightarrow \bullet Aux\ NP\ VP$	[0,0]	Predictor
S3	$S \rightarrow \bullet VP$	[0,0]	Predictor
S4	$NP \rightarrow \bullet Pronoun$	[0,0]	Predictor
S5	$NP \rightarrow \bullet Proper\text{-}Noun$	[0,0]	Predictor
S6	$NP \rightarrow \bullet Det\ Nominal$	[0,0]	Predictor
S7	$VP \rightarrow \bullet Verb$	[0,0]	Predictor
S8	$VP \rightarrow \bullet Verb\ NP$	[0,0]	Predictor
S9	$VP \rightarrow \bullet Verb\ NP\ PP$	[0,0]	Predictor
S10	$VP \rightarrow \bullet Verb\ PP$	[0,0]	Predictor
S11	$VP \rightarrow \bullet VP\ PP$	[0,0]	Predictor

Note that given a grammar, these entries are the same for all inputs; they can be pre-loaded.

Scanner

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- Formally:
  S_j: $A \rightarrow \alpha \cdot B\ \beta$, $[i,j]$
  S_{j+1}: $A \rightarrow \alpha\ B \cdot \beta$, $[i,j+1]$

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  NP → Det Nom • [1,3]
- Find all states w/dot at 1 and expecting an NP:
  - VP → V • NP [0,1]
- Adds new (completed) state(s) to *current* chart :
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  NP → Det Nom • [1,3]
- Find all states w/dot at 1 and expecting an NP:
  - VP → V • NP [0,1]
- Adds new (completed) state(s) to *current* chart :
  - VP → V NP • [0,3]
- Formally: S_k: B → δ ·, [j,k]
  S_k: A → α B · β , [i,k],
  where: S_j: A → α · B β , [i,j].

Chart[0]

S0	$\gamma \rightarrow \bullet S$	[0,0]	Dummy start state
S1	$S \rightarrow \bullet NP\ VP$	[0,0]	Predictor
S2	$S \rightarrow \bullet Aux\ NP\ VP$	[0,0]	Predictor
S3	$S \rightarrow \bullet VP$	[0,0]	Predictor
S4	$NP \rightarrow \bullet Pronoun$	[0,0]	Predictor
S5	$NP \rightarrow \bullet Proper\text{-}Noun$	[0,0]	Predictor
S6	$NP \rightarrow \bullet Det\ Nominal$	[0,0]	Predictor
S7	$VP \rightarrow \bullet Verb$	[0,0]	Predictor
S8	$VP \rightarrow \bullet Verb\ NP$	[0,0]	Predictor
S9	$VP \rightarrow \bullet Verb\ NP\ PP$	[0,0]	Predictor
S10	$VP \rightarrow \bullet Verb\ PP$	[0,0]	Predictor
S11	$VP \rightarrow \bullet VP\ PP$	[0,0]	Predictor

Note that given a grammar, these entries are the same for all inputs; they can be pre-loaded.

Chart[1]

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S12	$Verb \rightarrow book \bullet$	[0,1]	Scanner

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S12	$Verb \rightarrow book \bullet$	[0,1]	Scanner
S13	$VP \rightarrow Verb \bullet$	[0,1]	Completer
S14	$VP \rightarrow Verb \bullet NP$	[0,1]	Completer
S15	$VP \rightarrow Verb \bullet NP\ PP$	[0,1]	Completer
S16	$VP \rightarrow Verb \bullet PP$	[0,1]	Completer

Chart[1]

S12	$Verb \rightarrow book \bullet$	[0,1]	Scanner
S13	$VP \rightarrow Verb \bullet$	[0,1]	Completer
S14	$VP \rightarrow Verb \bullet NP$	[0,1]	Completer
S15	$VP \rightarrow Verb \bullet NP\ PP$	[0,1]	Completer
S16	$VP \rightarrow Verb \bullet PP$	[0,1]	Completer
S17	$S \rightarrow VP \bullet$	[0,1]	Completer
S18	$VP \rightarrow VP \bullet PP$	[0,1]	Completer

Chart[1]

S12	$Verb \rightarrow book \bullet$	[0,1]	Scanner
S13	$VP \rightarrow Verb \bullet$	[0,1]	Completer
S14	$VP \rightarrow Verb \bullet NP$	[0,1]	Completer
S15	$VP \rightarrow Verb \bullet NP\ PP$	[0,1]	Completer
S16	$VP \rightarrow Verb \bullet PP$	[0,1]	Completer
S17	$S \rightarrow VP \bullet$	[0,1]	Completer
S18	$VP \rightarrow VP \bullet PP$	[0,1]	Completer
S19	$NP \rightarrow \bullet Pronoun$	[1,1]	Predictor
S20	$NP \rightarrow \bullet Proper\text{-}Noun$	[1,1]	Predictor
S21	$NP \rightarrow \bullet Det\ Nominal$	[1,1]	Predictor
S22	$PP \rightarrow \bullet Prep\ NP$	[1,1]	Predictor

Prediction of Next Rule

- When VP → V • is itself processed by the Completer, S → VP • is added to Chart[1] since VP is a left corner of S
- Last few rules in Chart[1] are added by **Predictor** when VP → V • NP is processed
- And so on....

Charts[2] and [3]

Charts[2] and [3]

S23	$Det \rightarrow that \bullet$	[1,2]	Scanner

Charts[2] and [3]

S23	$Det \rightarrow that \bullet$	[1,2]	Scanner
S24	$NP \rightarrow Det \bullet Nominal$	[1,2]	Completer

Charts[2] and [3]

S23	$Det \rightarrow that\,\bullet$	[1,2]	Scanner
S24	$NP \rightarrow Det \bullet Nominal$	[1,2]	Completer
S25	$Nominal \rightarrow \bullet\, Noun$	[2,2]	Predictor
S26	$Nominal \rightarrow \bullet\, Nominal\ Noun$	[2,2]	Predictor
S27	$Nominal \rightarrow \bullet\, Nominal\ PP$	[2,2]	Predictor

Charts[2] and [3]

S23	$Det \rightarrow that \bullet$	[1,2]	Scanner
S24	$NP \rightarrow Det \bullet Nominal$	[1,2]	Completer
S25	$Nominal \rightarrow \bullet Noun$	[2,2]	Predictor
S26	$Nominal \rightarrow \bullet Nominal\ Noun$	[2,2]	Predictor
S27	$Nominal \rightarrow \bullet Nominal\ PP$	[2,2]	Predictor
S28	$Noun \rightarrow flight \bullet$	[2,3]	Scanner

Charts[2] and [3]

S23	$Det \rightarrow that \bullet$	[1,2]	Scanner
S24	$NP \rightarrow Det \bullet Nominal$	[1,2]	Completer
S25	$Nominal \rightarrow \bullet Noun$	[2,2]	Predictor
S26	$Nominal \rightarrow \bullet Nominal\ Noun$	[2,2]	Predictor
S27	$Nominal \rightarrow \bullet Nominal\ PP$	[2,2]	Predictor
S28	$Noun \rightarrow flight \bullet$	[2,3]	Scanner
S29	$Nominal \rightarrow Noun \bullet$	[2,3]	Completer
S30	$NP \rightarrow Det\ Nominal \bullet$	[1,3]	Completer
S31	$Nominal \rightarrow Nominal \bullet Noun$	[2,3]	Completer
S32	$Nominal \rightarrow Nominal \bullet PP$	[2,3]	Completer
S33	$VP \rightarrow Verb\ NP \bullet$	[0,3]	Completer
S34	$VP \rightarrow Verb\ NP \bullet PP$	[0,3]	Completer
S35	$PP \rightarrow \bullet Prep\ NP$	[3,3]	Predictor
S36	$S \rightarrow VP \bullet$	[0,3]	Completer

How do we retrieve the parses at the end?

- Augment the Completer to add pointers to prior states it advances as a field in the current state
  - i.e. what state did we advance here?
  - Read the pointers back from the final state

- What about ambiguity?

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- Can’t resolve it