# Parsing: PCFGs 

Ling 571
Deep Processing Techniques for NLP January 21, 2015

## Roadmap

- Motivation: Ambiguity
- Approach:
- Probabilistic Context-free Grammars (PCFGs)
- Definition
- Disambiguation
- Parsing
- Evaluation
- Enhancements


## Probabilistic Parsing

- Provides strategy for solving disambiguation problem
- Compute the probability of all analyses
- Select the most probable


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- Provides strategy for solving disambiguation problem
- Compute the probability of all analyses
- Select the most probable
- Employed in language modeling for speech recognition
- N-gram grammars predict words, constrain search
- Also, constrain generation, translation


## PCFGs

- Probabilistic Context-free Grammars
- Augmentation of CFGs
$N$ a set of non-terminal symbols (or variables)
$\Sigma$ a set of terminal symbols (disjoint from $N$ )
$R$ a set of rules or productions, each of the form $A \rightarrow \beta[p]$, where $A$ is a non-terminal,
$\beta$ is a string of symbols from the infinite set of strings $(\Sigma \cup N) *$, and $p$ is a number between 0 and 1 expressing $P(\beta \mid A)$
$S$ a designated start symbol


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- A PCFG is consistent if sum of probabilities of all sentences in language is 1 .
- Recursive rules often yield inconsistent grammars


## Example PCFG

| Grammar | Lexicon |  |
| :--- | :---: | :---: |
| $S \rightarrow N P V P$ | $[.80]$ | Det $\rightarrow$ that $[.10]\|a\|[.30] \mid$ the $[.60]$ |
| $S \rightarrow$ Aux $N P V P$ | $[.15]$ | Noun $\rightarrow$ book $[.10] \mid$ flight $[.30]$ |
| $S \rightarrow V P$ | $[.05]$ | $\mid$ meal $[.15] \mid$ money $[.05]$ |
| $N P \rightarrow$ Pronoun | $[.35]$ | $\mid$ flights $[.40] \mid$ dinner $[.10]$ |
| $N P \rightarrow$ Proper-Noun | $[.30]$ | Verb $\rightarrow$ book $[.30] \mid$ include $[.30]$ |
| $N P \rightarrow$ Det Nominal | $[.20]$ | $\mid$ prefer $;[.40]$ |
| $N P \rightarrow$ Nominal | $[.15]$ | Pronoun $\rightarrow I[.40] \mid$ she $[.05]$ |
| Nominal $\rightarrow$ Noun | $[.75]$ | $\mid$ me $[.15] \mid$ you $[.40]$ |
| Nominal $\rightarrow$ Nominal Noun $[.20]$ | Proper-Noun $\rightarrow$ Houston $[.60]$ |  |
| Nominal $\rightarrow$ Nominal PP | $[.05]$ | $\mid$ NWA $[.40]$ |
| $V P \rightarrow$ Verb | $[.35]$ | Aux $\rightarrow$ does $[.60] \mid$ can $[40]$ |
| $V P \rightarrow$ Verb NP | $[.20]$ | Preposition $\rightarrow$ from $[.30] \mid$ to $[.30]$ |
| $V P \rightarrow$ Verb NP $P P$ | $[.10]$ | $\mid$ on $[.20] \mid$ near $[.15]$ |
| $V P \rightarrow$ Verb PP | $[.15]$ | $\mid$ through $[.05]$ |
| $V P \rightarrow$ Verb NP NP | $[.05]$ |  |
| $V P \rightarrow$ VP PP | $[.15]$ |  |
| $P P \rightarrow$ Preposition NP | $[1.0]$ |  |

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## Disambiguation

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$$
\begin{aligned}
& P(T, S)=\prod_{i=1}^{n} P\left(R H S_{i} \mid L H S_{i}\right) \\
& P(T, S)=P(T) P(S \mid T)=P(T)
\end{aligned}
$$


$P(T, S)=0.05$

$P(T, S)=0.05 * 0.2$

$P(T, S)=0.05 * 0.2 * 0.2$

$P(T, S)=0.05^{*} 0.2^{*} 0.2^{*} 0.2$

$P(T, S)=0.05^{*} 0.2 * 0.2 * 0.2^{*} 0.75$

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0.3

$P(T, S)=0.05 * 0.2 * 0.2 * 0.2 * 0.75^{*}$
$0.3 * 0.6$

$P(T, S)=0.05^{*} 0.2^{*} 0.2 * 0.2 * 0.75^{*}$
$0.3 * 0.6 * 0.1$

$P(T, S)=0.05^{*} 0.2 * 0.2 * 0.2 * 0.75^{*}$
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$P(T, S)=0.05 * 0.2 * 0.2 * 0.2 * 0.75^{*} \quad P(T, S)=0.05^{*} 0.1 * 0.15$
$0.3^{*} 0.6 * 0.1 * 0.4=2.2 \times 10^{\wedge} .6$

$P(T, S)=0.05 * 0.2 * 0.2 * 0.2 * 0.75^{*} \quad \mathrm{P}(\mathrm{T}, \mathrm{S})=0.05^{*} 0.1 * 0.15^{*} 0.75$
$0.3^{*} 0.6 * 0.1 * 0.4=2.2 \times 10^{\wedge} .6$

$P(T, S)=0.05^{*} 0.2 * 0.2 * 0.2 * 0.75^{*}$
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$P(T, S)=0.05^{*} 0.2 * 0.2 * 0.2 * 0.75^{*}$
$P(T, S)=0.05 * 0.1 * 0.15 * 0.75^{*} 0.75^{*}$
$0.3 * 0.6 * 0.1 * 0.4=2.2 \times 10^{\wedge}-6$ $0.3^{*} 0.6 * 0.1 * 0.4=6.1 \times 10^{\wedge}-7$

## Formalizing Disambiguation

- Select T such that:

$$
\hat{T}(S)=\underset{T s . t, S=\text { yield }(T)}{\operatorname{argmax}} P(T)
$$

- String of words S is yield of parse tree over S
- Select tree that maximizes probability of parse


## Parsing Problem for PCFGs

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- String of words $S$ is yield of parse tree over $S$
- Select tree that maximizes probability of parse
- Extend existing algorithms: CKY \& Earley
- Most modern PCFG parsers based on CKY
- Augmented with probabilities


## Probabilistic CKY

- Like regular CKY
- Assume grammar in Chomsky Normal Form (CNF)
- Productions:
- A -> B C or A -> w
- Represent input with indices b/t words
- E.g., o Book ${ }_{1}$ that ${ }_{2}$ flight ${ }_{3}$ through $_{4}$ Houston $_{5}$


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- E.g., o Book ${ }_{1}$ that ${ }_{2}$ flight ${ }_{3}$ through $_{4}$ Houston $_{5}$
- For input string length $n$ and non-terminals $V$
- Cell[i,j,A] in $(n+1) x(n+1) x V$ matrix contains
- Probability that constituent A spans [i,j]


## Probabilistic CKY Algorithm

function PROBABILISTIC-CKY(words,grammar) returns most probable parse and its probability
for $j \leftarrow$ from 1 to LENGTH(words) do
for all $\{A \mid A \rightarrow$ words $[j] \in$ grammar $\}$
table $[j-1, j, A] \leftarrow P(A \rightarrow$ words $[j])$
for $i \leftarrow$ from $j-2$ downto 0 do
for $k \leftarrow i+1$ to $j-1$ do
for all $\{A \mid A \rightarrow B C \in$ grammar,
and table $[i, k, B]>0$ and table $[k, j, C]>0\}$
if $($ table $[i, j, A]<P(A \rightarrow B C) \times$ table $[i, k, B] \times$ table $[k, j, C])$ then table $[i, j, A] \leftarrow P(A \rightarrow B C) \times$ table $[i, k, B] \times$ table $[k, j, C]$ back $[i, j, A] \leftarrow\{k, B, C\}$
return BUILD_TREE(back[1, LENGTH(words), S]), table[1, LENGTH(words), $S$ ]

## PCKY Grammar Segment

| $S$ | $\rightarrow N P V P$ | .80 | Det $\rightarrow$ the |
| ---: | :--- | ---: | :--- |

## PCKY Matrix: <br> The flight includes a meal

| Det: 0.4 |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| [0,1] |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

## PCKY Matrix: <br> The flight includes a meal

$\left.\begin{array}{|l|l|l|l|l|}\hline \begin{array}{l}\text { Det: } \mathbf{0 . 4} \\ \text { [0,1] }\end{array} & & & & \\ \hline & \text { N: 0.02 } \\ {[1,2]}\end{array}\right)$

## PCKY Matrix: <br> The flight includes a meal

| Det: 0.4 | NP: <br> $0.3^{* 0.4 * 0.02 ~}$ <br> [0,1] <br> $=.0024$ <br> $[0,2]$ |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  | N: 0.02 |  |  |  |
|  | $[1,2]$ |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

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| Det: 0.4 | NP: <br> $0.3 * 0.4 * 0.02$ |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| [0,1] | .0024 <br> $[0,2]$ |  |  |  |
|  | N: 0.02 |  |  |  |
|  | $[1,2]$ |  |  |  |
|  |  | V: 0.05 |  |  |
|  |  | $[2,3]$ |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

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$\left.\begin{array}{|l|l|l|l|l|}\hline \text { Det: } 0.4 & \begin{array}{l}\text { NP: } \\ 0.3 * 0.4 * 0.02\end{array} & & & \\ \text { [0,1] } & .0024 \\ {[0,2]}\end{array}\right)$

## PCKY Matrix: <br> The flight includes a meal

| Det: 0.4 | NP: <br> $0.3 * 0.4 * 0.02$ <br> [0,1] <br> $[0.224$ |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  | N: 0,3$]$ |  |  |  |
|  | $[1,2]$ | $[1,3]$ |  |  |
|  |  | V: 0.05 |  |  |
|  |  | $[2,3]$ |  |  |
|  |  |  |  |  |
|  |  |  |  |  |
|  |  |  |  |  |

## PCKY Matrix: <br> The flight includes a meal

| Det: $\mathbf{0 . 4}$ | NP: <br> O.3*0.4*0.02 <br> [0,1] <br> $[0,024$ |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  | N: 0.02 |  |  |  |
|  | $[1,2]$ | $[1,3]$ |  |  |
|  |  | V: 0.05 |  |  |
|  |  | $[2,3]$ |  |  |
|  |  |  | Det: 0.4 |  |
|  |  |  | $[3,4]$ |  |
|  |  |  |  |  |

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| :--- | :--- | :--- | :--- | :--- |
|  | N: 0.02 |  |  |  |
|  | $[1,2]$ | $[1,3]$ |  |  |
|  |  | V: 0.05 |  |  |
|  |  | $[2,3]$ | $[2,4]$ |  |
|  |  |  | Det: 0.4 |  |
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|  |  |  |  |  |

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| Det: $\mathbf{0 . 4}$ | NP: <br> O.3*0.4*0.02 <br> [0,1] <br> $[0,024$ |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
|  | N: 0.02 |  |  |  |
| $[1,2]$ | $[1,3]$ | $[1,4]$ |  |  |
|  |  | V: 0.05 |  |  |
|  |  | $[2,3]$ | $[2,4]$ |  |
|  |  |  | Det: 0.4 |  |
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| Det: 0.4 $[0,1]$ | NP: $\begin{aligned} & 0.3 * 0.4 * 0.02 \\ & =.0024 \end{aligned}$ $[0,2]$ | [0,3] | [0,4] |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & \mathrm{N}: 0.02 \\ & {[1,2]} \end{aligned}$ | [1,3] | [1,4] |  |
|  |  | $\begin{aligned} & \mathrm{V}: 0.05 \\ & {[2,3]} \end{aligned}$ | [2,4] |  |
|  |  |  | Det: 0.4 <br> [3,4] |  |
|  |  |  |  |  |

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| Det: 0.4 $[0,1]$ | $\begin{aligned} & \text { NP: } \\ & 0.3^{*} 0.4^{*} 0.02 \\ & =.0024 \\ & {[0,2]} \end{aligned}$ | [0,3] | [0,4] |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & \mathrm{N}: 0.02 \\ & {[1,2]} \end{aligned}$ | [1,3] | [1,4] |  |
|  |  | $\begin{aligned} & \mathrm{V}: 0.05 \\ & {[2,3]} \end{aligned}$ | [2,4] |  |
|  |  |  | Det: 0.4 $[3,4]$ |  |
|  |  |  |  | $\begin{aligned} & \text { N: } 0.01 \\ & {[4,5]} \end{aligned}$ |

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| Det: 0.4 $[0,1]$ | $\begin{aligned} & \text { NP: } \\ & 0.3^{*} 0.4 * 0.02 \\ & =.0024 \\ & {[0,2]} \end{aligned}$ | [0,3] | [0,4] |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\begin{array}{\|l} \mathrm{N}: 0.02 \\ {[1,2]} \end{array}$ | [1,3] | [1,4] |  |
|  |  | $\begin{aligned} & \mathrm{V}: 0.05 \\ & {[2,3]} \end{aligned}$ | [2,4] |  |
|  |  |  | Det: 0.4 $[3,4]$ | $\begin{aligned} & \text { NP: } \\ & 0.3^{*} 0.4^{*} 0.01 \\ & =0.0012 \\ & {[3,5]} \end{aligned}$ |
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| :---: | :---: | :---: | :---: | :---: |
|  | $\begin{aligned} & \mathrm{N}: 0.02 \\ & {[1,2]} \end{aligned}$ | [1,3] | [1,4] |  |
|  |  | $\begin{aligned} & V: 0.05 \\ & {[2,3]} \end{aligned}$ | [2,4] | VP: <br> 0.2*0.05* <br> $0.0012=0.0$ $00012[2,5]$ <br> 00012 [2,5] |
|  |  |  | Det: 0.4 $[3,4]$ | $\begin{aligned} & \text { NP: } \\ & 0.3 * 0.4 * 0.01 \\ & =0.0012 \\ & {[3,5]} \end{aligned}$ |
|  |  |  |  | $\begin{aligned} & \mathrm{N}: 0.01 \\ & {[4,5]} \end{aligned}$ |

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| Det: $\mathbf{0 . 4}$ | NP: <br> 0.3*0.4*0.02 <br> [0,1] <br> =.0024 <br> $[0,2]$ |  | $[\mathbf{0 , 3 ]}$ | $[\mathbf{0 , 4 ]}$ |
| :--- | :--- | :--- | :--- | :--- |

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$$

- Alternative: Learn probabilities by re-estimating
- (Later)


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- Training:
- (Large) Set of sentences with associated parses (Treebank)
- E.g., Wall Street Journal section of Penn Treebank, sec 2-21
- 39,830 sentences
- Used to estimate rule probabilities


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- Used to estimate rule probabilities
- Development (dev):
- (Small) Set of sentences with associated parses (WSJ, 22)
- Used to tune/verify parser; check for overfitting, etc.
- Test:
- (Small-med) Set of sentences w/parses (WSJ, 23)
- 2416 sentences
- Held out, used for final evaluation

