

Parsing: Earley & PCFGs

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Deep Processing Techniques for NLP

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Roadmap

- Earley wrap-up:
 - Scanner
 - Completer
- Probabilistic Context-free Grammars (PCFGs)
 - Motivation: Ambiguity
 - Approach:
 - Definition
 - Disambiguation
 - Parsing
 - Evaluation
 - Enhancements

3 Main Sub-Routines of Earley Algorithm

- **Predictor:** Adds predictions into the chart.
- **Completer:** Moves the dot to the right when new constituents are found.
- **Scanner:** Reads the input words and enters states representing those words into the chart.

Predictor

- Intuition: create new state for top-down prediction of new phrase.
- Applied when non part-of-speech non-terminals are to the right of a dot: **S** → •
VP [0,0]
- Adds new states to *current* chart
 - One new state for each expansion of the non-terminal in the grammar
VP → • **V** [0,0]
VP → • **V NP** [0,0]
- Formally:
$$S_i: A \rightarrow \alpha \cdot B \beta, [i,j]$$
$$S_j: B \rightarrow \cdot \gamma, [j,j]$$

Chart[0]

S0	$\gamma \rightarrow \bullet S$	[0,0]	Dummy start state
S1	$S \rightarrow \bullet NP VP$	[0,0]	Predictor
S2	$S \rightarrow \bullet Aux NP VP$	[0,0]	Predictor
S3	$S \rightarrow \bullet VP$	[0,0]	Predictor
S4	$NP \rightarrow \bullet Pronoun$	[0,0]	Predictor
S5	$NP \rightarrow \bullet Proper-Noun$	[0,0]	Predictor
S6	$NP \rightarrow \bullet Det Nominal$	[0,0]	Predictor
S7	$VP \rightarrow \bullet Verb$	[0,0]	Predictor
S8	$VP \rightarrow \bullet Verb NP$	[0,0]	Predictor
S9	$VP \rightarrow \bullet Verb NP PP$	[0,0]	Predictor
S10	$VP \rightarrow \bullet Verb PP$	[0,0]	Predictor
S11	$VP \rightarrow \bullet VP PP$	[0,0]	Predictor

Note that given a grammar, these entries are the same for all inputs; they can be pre-loaded.

Scanner

- Intuition: Create new states for rules matching part of speech of next word.
- Applicable when part of speech is to the right of a dot: $VP \rightarrow \cdot V NP [0,0]$ ‘Book...’
- Looks at current word in input
- If match, adds state(s) to *next* chart
 $V \rightarrow \text{Book} \cdot [0,1]$
- Formally:
 $S_j: A \rightarrow \alpha \cdot B \beta, [i,j], B \text{ in } \text{POS}(\text{words}[j])$
- $S_{j+1}: B \rightarrow \text{words}[j] \cdot, [j,j+1]$

Completer

- Intuition: parser has finished a new phrase, so must find and advance all states that were waiting for this
- Applied when dot has reached right end of rule
 $NP \rightarrow \text{Det Nom} \cdot [1,3]$
- Find all states w/dot at 1 and expecting an NP:
 - $VP \rightarrow V \cdot NP [0,1]$
- Adds new (completed) state(s) to *current* chart :
 - $VP \rightarrow V NP \cdot [0,3]$
- Formally: $S_k: B \rightarrow \delta \cdot, [j,k]$
 $S_k: A \rightarrow \alpha B \cdot \beta, [i,k],$
 where: $S_j: A \rightarrow \alpha \cdot B \beta, [i,j].$

Chart[1]

S12	$Verb \rightarrow book \bullet$	[0,1]	Scanner
S13	$VP \rightarrow Verb \bullet$	[0,1]	Completer
S14	$VP \rightarrow Verb \bullet NP$	[0,1]	Completer
S15	$VP \rightarrow Verb \bullet NP PP$	[0,1]	Completer
S16	$VP \rightarrow Verb \bullet PP$	[0,1]	Completer
S17	$S \rightarrow VP \bullet$	[0,1]	Completer
S18	$VP \rightarrow VP \bullet PP$	[0,1]	Completer
S19	$NP \rightarrow \bullet Pronoun$	[1,1]	Predictor
S20	$NP \rightarrow \bullet Proper-Noun$	[1,1]	Predictor
S21	$NP \rightarrow \bullet Det Nominal$	[1,1]	Predictor
S22	$PP \rightarrow \bullet Prep NP$	[1,1]	Predictor

Charts[2] and [3]

S23	<i>Det</i> → <i>that</i> •	[1,2]	Scanner
S24	<i>NP</i> → <i>Det</i> • <i>Nominal</i>	[1,2]	Completer
S25	<i>Nominal</i> → • <i>Noun</i>	[2,2]	Predictor
S26	<i>Nominal</i> → • <i>Nominal Noun</i>	[2,2]	Predictor
S27	<i>Nominal</i> → • <i>Nominal PP</i>	[2,2]	Predictor
S28	<i>Noun</i> → <i>flight</i> •	[2,3]	Scanner
S29	<i>Nominal</i> → <i>Noun</i> •	[2,3]	Completer
S30	<i>NP</i> → <i>Det Nominal</i> •	[1,3]	Completer
S31	<i>Nominal</i> → <i>Nominal</i> • <i>Noun</i>	[2,3]	Completer
S32	<i>Nominal</i> → <i>Nominal</i> • <i>PP</i>	[2,3]	Completer
S33	<i>VP</i> → <i>Verb NP</i> •	[0,3]	Completer
S34	<i>VP</i> → <i>Verb NP</i> • <i>PP</i>	[0,3]	Completer
S35	<i>PP</i> → • <i>Prep NP</i>	[3,3]	Predictor
S36	<i>S</i> → <i>VP</i> •	[0,3]	Completer

How do we retrieve the parses at the end?

- Augment the Completer to add pointers to prior states it advances as a field in the current state
 - i.e. what state did we advance here?
 - Read the pointers back from the final state

- What about ambiguity?
- CKY/Earley can represent it
- Can't resolve it

Probabilistic Parsing

- Provides strategy for solving disambiguation problem
 - Compute the probability of all analyses
 - Select the most probable
- Employed in language modeling for speech recognition
 - N-gram grammars predict words, constrain search
 - Also, constrain generation, translation

PCFGs

- Probabilistic Context-free Grammars
 - Augmentation of CFGs

N a set of **non-terminal symbols** (or **variables**)

Σ a set of **terminal symbols** (disjoint from N)

R a set of **rules** or productions, each of the form $A \rightarrow \beta$ [p],

where A is a non-terminal,

β is a string of symbols from the infinite set of strings $(\Sigma \cup N)^*$,

and p is a number between 0 and 1 expressing $P(\beta|A)$

S a designated **start symbol**

PCFGs

- Augment each production with probability that LHS will be expanded as RHS
 - $P(A \rightarrow B)$ or $P(A \rightarrow B | A)$, $P(\text{RHS} | \text{LHS})$
 - Sum over all possible expansions is 1

$$\sum_{\beta} P(A \rightarrow \beta) = 1$$

- A PCFG is consistent if sum of probabilities of all sentences in language is 1.
 - Recursive rules often yield inconsistent grammars

Example PCFG

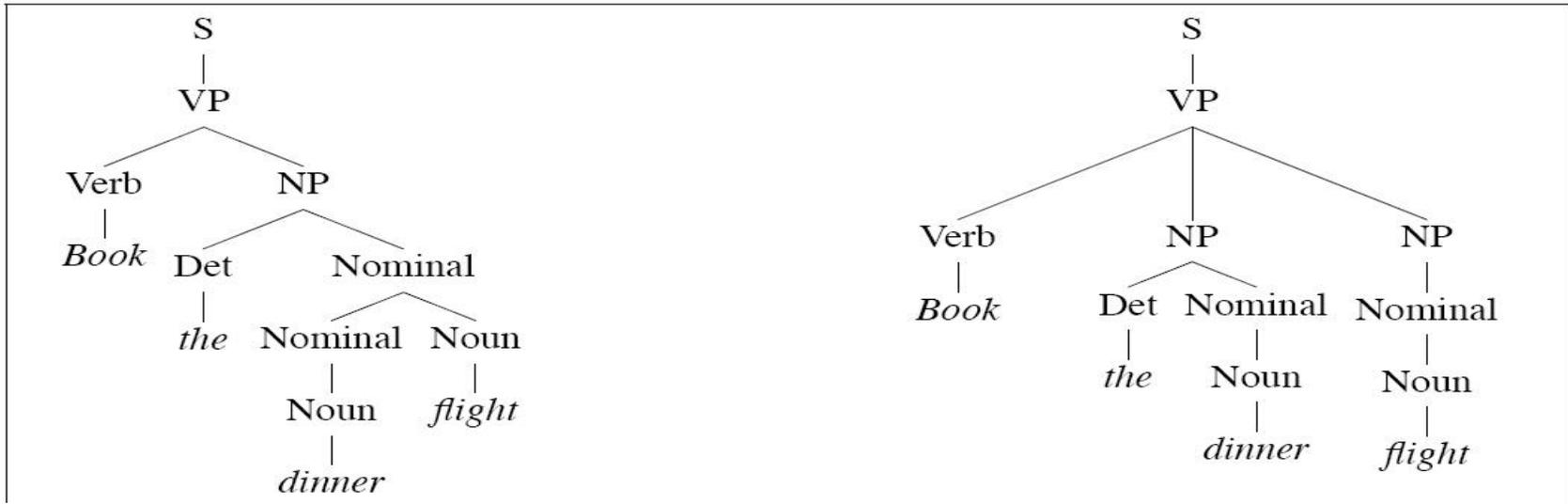
Grammar		Lexicon
$S \rightarrow NP VP$	[.80]	$Det \rightarrow that [.10] \mid a [.30] \mid the [.60]$
$S \rightarrow Aux NP VP$	[.15]	$Noun \rightarrow book [.10] \mid flight [.30]$
$S \rightarrow VP$	[.05]	$\mid meal [.15] \mid money [.05]$
$NP \rightarrow Pronoun$	[.35]	$\mid flights [.40] \mid dinner [.10]$
$NP \rightarrow Proper-Noun$	[.30]	$Verb \rightarrow book [.30] \mid include [.30]$
$NP \rightarrow Det Nominal$	[.20]	$\mid prefer; [.40]$
$NP \rightarrow Nominal$	[.15]	$Pronoun \rightarrow I [.40] \mid she [.05]$
$Nominal \rightarrow Noun$	[.75]	$\mid me [.15] \mid you [.40]$
$Nominal \rightarrow Nominal Noun$	[.20]	$Proper-Noun \rightarrow Houston [.60]$
$Nominal \rightarrow Nominal PP$	[.05]	$\mid NWA [.40]$
$VP \rightarrow Verb$	[.35]	$Aux \rightarrow does [.60] \mid can [.40]$
$VP \rightarrow Verb NP$	[.20]	$Preposition \rightarrow from [.30] \mid to [.30]$
$VP \rightarrow Verb NP PP$	[.10]	$\mid on [.20] \mid near [.15]$
$VP \rightarrow Verb PP$	[.15]	$\mid through [.05]$
$VP \rightarrow Verb NP NP$	[.05]	
$VP \rightarrow VP PP$	[.15]	
$PP \rightarrow Preposition NP$	[1.0]	

Disambiguation

- A PCFG assigns probability to each parse tree T for input S .
 - Probability of T : product of all rules to derive T

$$P(T, S) = \prod_{i=1}^n P(RHS_i | LHS_i)$$

$$P(T, S) = P(T)P(S | T) = P(T)$$



	Rules	P		Rules	P
S	→ VP	.05	S	→ VP	.05
VP	→ Verb NP	.20	VP	→ Verb NP NP	.10
NP	→ Det Nominal	.20	NP	→ Det Nominal	.20
Nominal	→ Nominal Noun	.20	NP	→ Nominal	.15
Nominal	→ Noun	.75	Nominal	→ Noun	.75
Verb	→ book	.30	Nominal	→ Noun	.75
Det	→ the	.60	Verb	→ book	.30
Noun	→ dinner	.10	Det	→ the	.60
Noun	→ flights	.40	Noun	→ dinner	.10
			Noun	→ flights	.40

$$P(T,S)=0.05*0.2*0.2*0.2*0.75*0.3*0.6*0.1*0.4=2.2 \times 10^{-6}$$

$$P(T,S)=0.05*0.1*0.2*0.15*0.75*0.75*0.3*0.6*0.1*0.4=6.1 \times 10^{-7}$$

Parsing Problem for PCFGs

- Select T such that:

$$\hat{T}(S) = \underset{T \text{ s.t. } S = \text{yield}(T)}{\operatorname{argmax}} P(T)$$

- String of words S is *yield* of parse tree over S
 - Select tree that maximizes probability of parse
-
- Extend existing algorithms: CKY & Earley
 - Most modern PCFG parsers based on CKY
 - Augmented with probabilities

Probabilistic CKY

- Like regular CKY
 - Assume grammar in Chomsky Normal Form (CNF)
 - Productions:
 - $A \rightarrow B C$ or $A \rightarrow w$
 - Represent input with indices b/t words
 - E.g., $_0$ Book $_1$ that $_2$ flight $_3$ through $_4$ Houston $_5$
- For input string length n and non-terminals V
 - Cell $[i,j,A]$ in $(n+1) \times (n+1) \times V$ matrix contains
 - Probability that constituent A spans $[i,j]$

Probabilistic CKY Algorithm

```
function PROBABILISTIC-CKY(words,grammar) returns most probable parse
                                     and its probability

for  $j \leftarrow$  from 1 to LENGTH(words) do
  for all  $\{ A \mid A \rightarrow \text{words}[j] \in \text{grammar} \}$ 
     $\text{table}[j-1, j, A] \leftarrow P(A \rightarrow \text{words}[j])$ 
  for  $i \leftarrow$  from  $j-2$  downto 0 do
    for  $k \leftarrow i+1$  to  $j-1$  do
      for all  $\{ A \mid A \rightarrow BC \in \text{grammar},$ 
                and  $\text{table}[i, k, B] > 0$  and  $\text{table}[k, j, C] > 0 \}$ 
        if  $(\text{table}[i, j, A] < P(A \rightarrow BC) \times \text{table}[i, k, B] \times \text{table}[k, j, C])$  then
           $\text{table}[i, j, A] \leftarrow P(A \rightarrow BC) \times \text{table}[i, k, B] \times \text{table}[k, j, C]$ 
           $\text{back}[i, j, A] \leftarrow \{k, B, C\}$ 
    return BUILD_TREE( $\text{back}[1, \text{LENGTH}(\text{words}), S]$ ),  $\text{table}[1, \text{LENGTH}(\text{words}), S]$ 
```

PCKY Grammar Segment

- $S \rightarrow NP VP$ [0.80]
- $NP \rightarrow Det N$ [0.30]
- $VP \rightarrow V NP$ [0.20]
- $V \rightarrow includes$ [0.05]
- $Det \rightarrow the$ [0.40]
- $Det \rightarrow a$ [0.40]
- $N \rightarrow meal$ [0.01]
- $N \rightarrow flight$ [0.02]

PCKY Matrix:

The flight includes a meal

Det: 0.4 [0,1]	NP: $0.3*0.4*0.02$ =.0024 [0,2]	[0,3]	[0,4]	S: 0.8* 0.000012* 0.0024 [0,5]
	N: 0.02 [1,2]	[1,3]	[1,4]	[1,5]
		V: 0.05 [2,3]	[2,4]	VP: $0.2*0.05*$ 0.0012=0.0 00012 [2,5]
			Det: 0.4 [3,4]	NP: $0.3*0.4*0.01$ =0.0012 [3,5]
				N: 0.01 [4,5]

Learning Probabilities

- Simplest way:
 - Treebank of parsed sentences
 - To compute probability of a rule, count:
 - Number of times non-terminal is expanded
 - Number of times non-terminal is expanded by given rule

$$P(\alpha \rightarrow \beta | \alpha) = \frac{\text{Count}(\alpha \rightarrow \beta)}{\sum_{\gamma} \text{Count}(\alpha \rightarrow \gamma)} = \frac{\text{Count}(\alpha \rightarrow \beta)}{\text{Count}(\alpha)}$$

- Alternative: Learn probabilities by re-estimating
 - (Later)