Conditional random field

LING 572
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Highlights

- CRF is a form of undirected graphical model
- Proposed by Lafferty, McCallum and Pereira in 2001
- Used in many NLP tasks: e.g., Named-entity detection
- Types:
  – Linear-chain CRF
  – Skip-chain CRF
  – General CRF

Outline

- Graphical models
- Linear-chain CRF
- Skip-chain CRF

Graphical models

Graphical model

- A graphical model is a probabilistic model for which a graph denotes the conditional independence structure between random variables:
  - Nodes: random variables
  - Edges: dependency relation between random variables

- Types of graphical models:
  - Bayesian network: directed acyclic graph (DAG)
  - Markov random fields: undirected graph

Bayesian network

Bayesian network

- Graph: directed acyclic graph (DAG)
  - Nodes: random variables
  - Edges: conditional dependencies
  - Each node X is associated with a probability function P(X | parents(X))

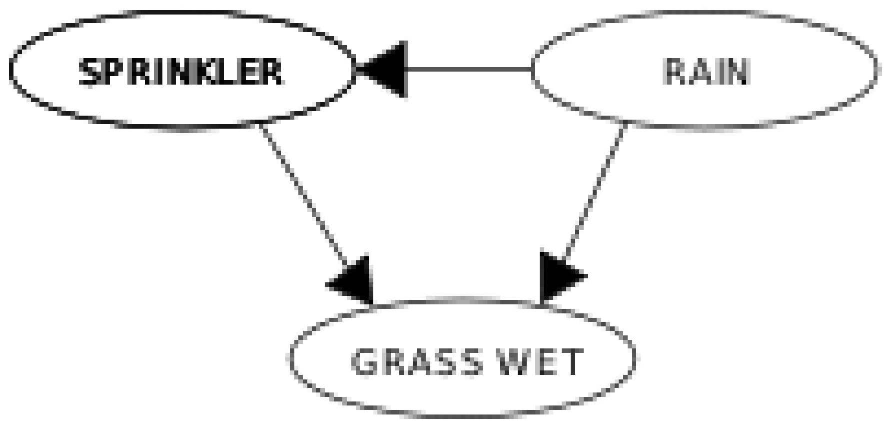
- Learning and inference: efficient algorithms exist.

An example

(from http://en.wikipedia.org/wiki/Bayesian_network)

P(sprinker | rain)

	SPRINKLER	
RAIN	T	F
F	0.4	0.6
T	0.01	0.99



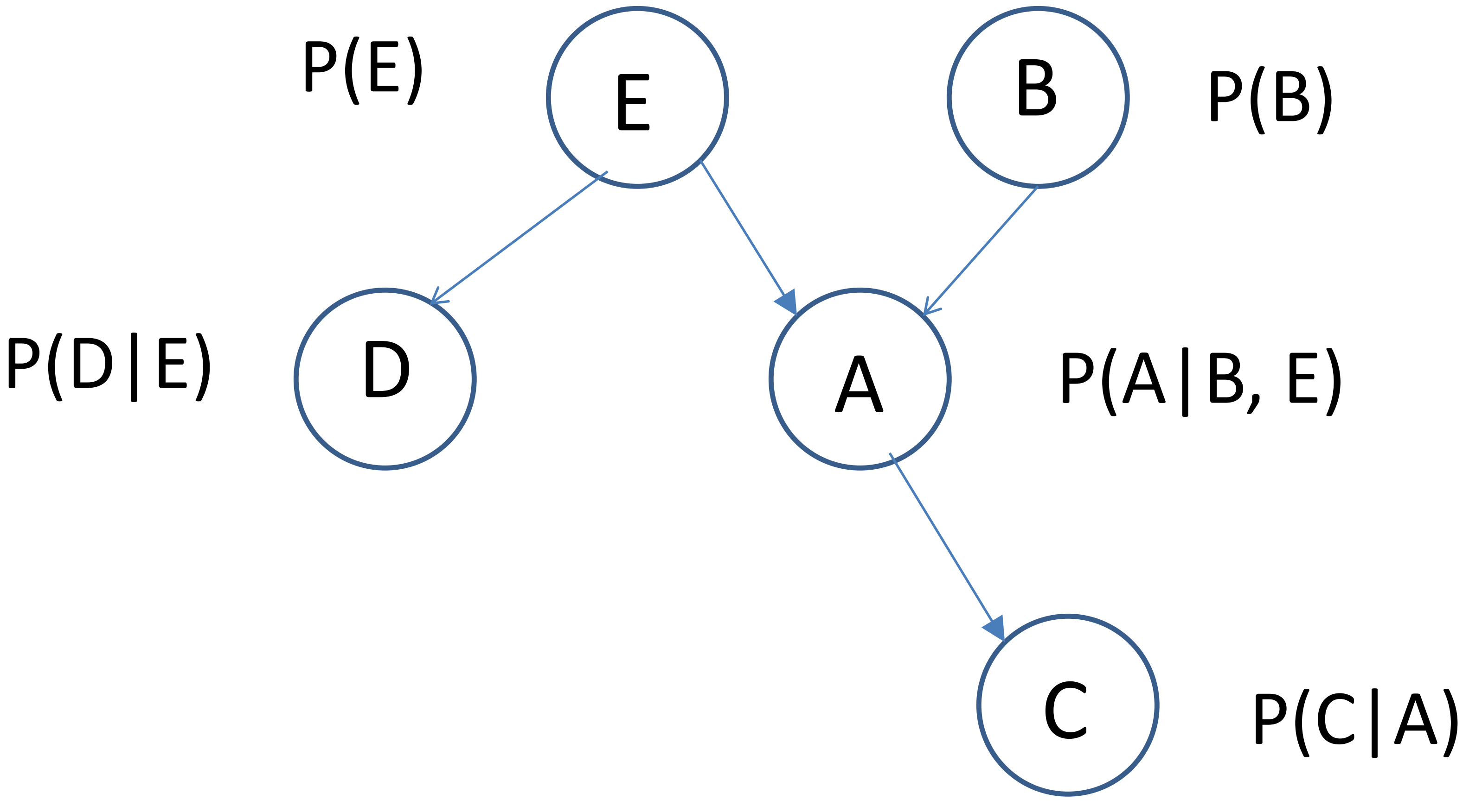
P(rain)

RAIN	
T	F
0.2	0.8

P(grassWet | sprinkler, rain)

		GRASS WET	
SPRINKLER	RAIN	T	F
F	F	0.0	1.0
F	T	0.8	0.2
T	F	0.9	0.1
T	T	0.99	0.01

Another example

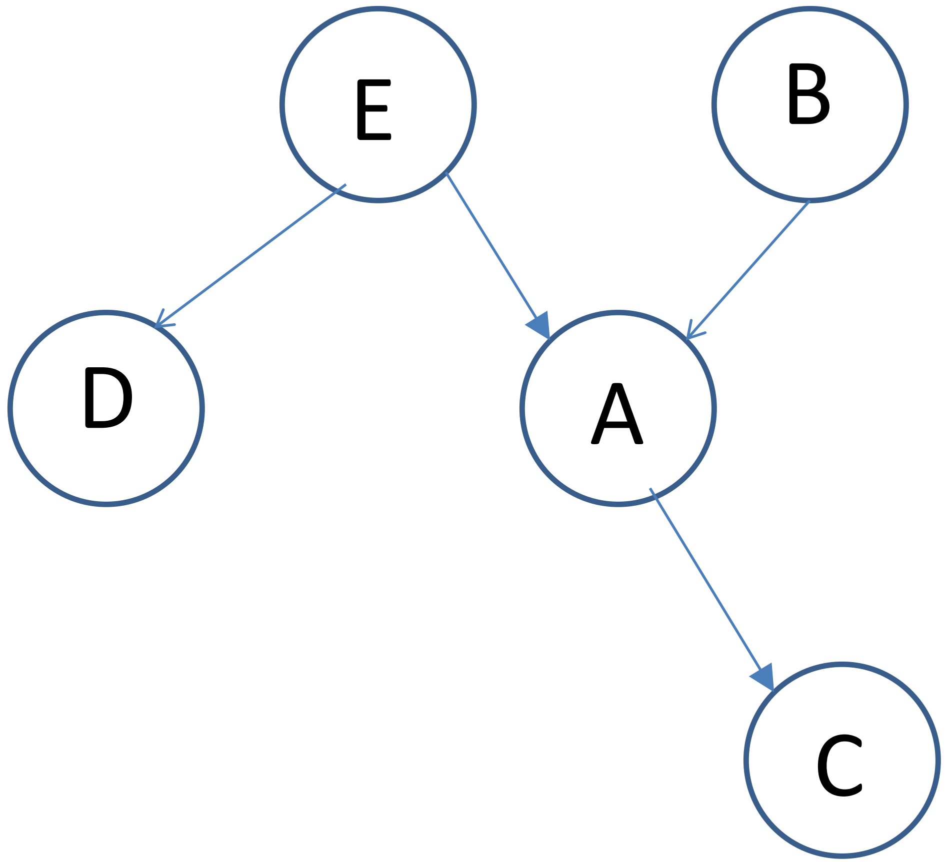


Bayesian network: properties

Local Markov property: each variable X_i is conditionally independent of its nondecendants given its parents variables.

$$P(X_1, ..., X_n)$$
$$= \prod_{i=1}^{n} P(X_i | X_1, ..., X_{i-1}))$$

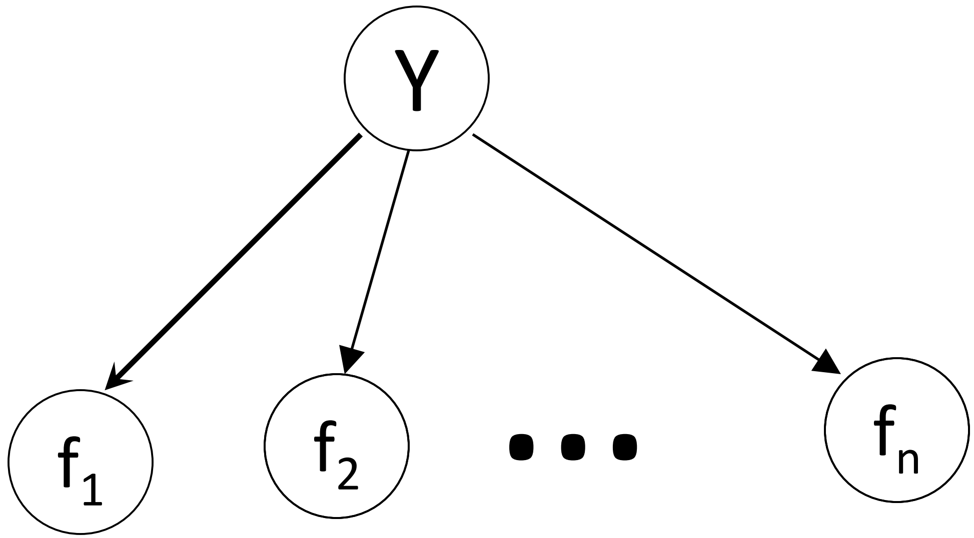
$$= \prod_{i=1}^{n} P(X_i | parents(X_i))$$



$$P(A,B,C,D,E) = P(B)P(E)P(A|B,E)P(C|A)P(D|E)$$

$$P(B,E|C,D) = \frac{P(B,E,C,D)}{P(C,D)} = \frac{\sum_A P(A,B,C,D,E)}{\sum_A \sum_B \sum_E P(A,B,C,D,E)}$$

Naïve Bayes Model

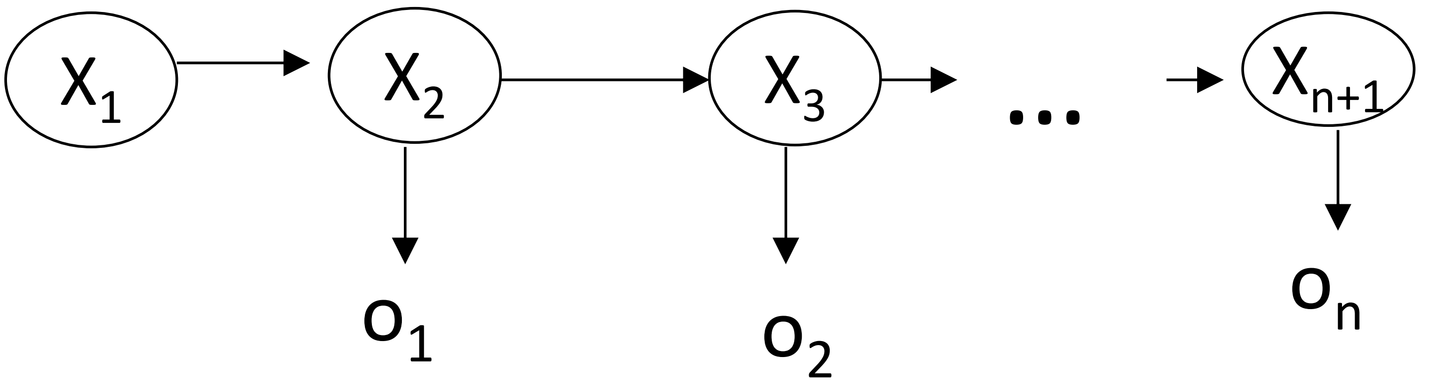


$$P(X,Y) = P(f1, f2, ..., f_n, Y)$$

$$= P(Y)P(f_1|Y)...P(f_n|Y)$$

$$= P(Y)\prod_{k=1}^{n} P(f_k|Y)$$

HMM



- State sequence: $X_{1,n+1}$
- Output sequence: $O_{1,n}$

$$P(O_{1,n}, X_{1,n+1}) = \pi(X_1)\prod_{i=1}^{n}(P(X_{i+1} \mid X_i)P(O_i \mid X_{i+1}))$$

Generative model

- It is a directed graphical model in which the output (i.e., what to be predicted) topologically precede the input (i.e., what is given as observation).

- Naïve Bayes and HMM are generative models.

Markov random field

Markov random field

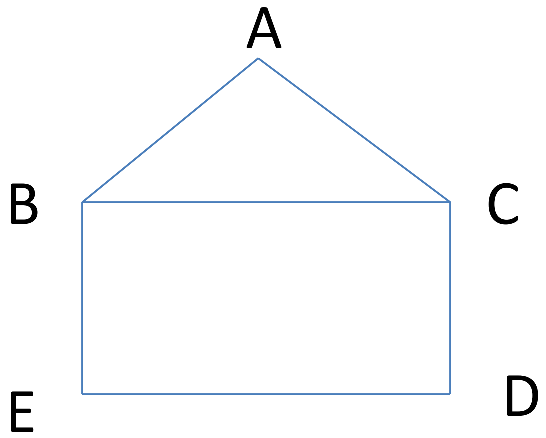
- Also called "Markov network"

- It is a graphical mode in which a set of random variables have a Markov property:
  - Local Markov property: A variable is conditionally independent of all other variables given its neighbors.

$$P(X_i|X_j, ne(X_i)) = P(X_i|ne(X_i))$$

Cliques

- A **clique** in an undirected graph is a subset of its vertices such that every two vertices in the subset are connected by an edge.
- A **maximal clique** is a clique that cannot be extended by adding one more vertex.
- A **maximum clique** is a clique of the largest possible size in a given graph.



clique:

maximal clique:

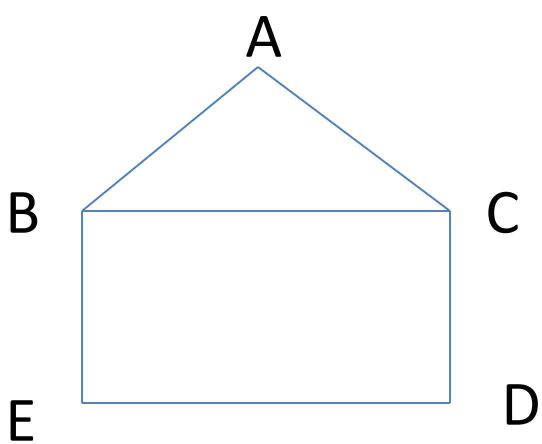
maximum clique:

Clique factorization

$G = (V, E)$ be an undirected graph.

$cl(G)$ be the set of cliques of G.

$$P(X) = \frac{1}{Z} \prod_{C \in cl(G)} \phi_C(X_C)$$



$$P(A, B, C, D, E)$$

$$= \frac{1}{Z} \phi_{ABC}(A, B, C) \phi_{BE}(B, E) \phi_{DE}(D, E) \phi_{C,D}(C, D)$$

Conditional Random Field

Definition. *Let $G = (V, E)$ be a graph such that $\mathbf{Y} = (\mathbf{Y}_v)_{v \in V}$, so that $\mathbf{Y}$ is indexed by the vertices of G. Then $(\mathbf{X}, \mathbf{Y})$ is a conditional random field in case, when conditioned on $\mathbf{X}$, the random variables $\mathbf{Y}_v$ obey the Markov property with respect to the graph: $p(\mathbf{Y}_v \mid \mathbf{X}, \mathbf{Y}_w, w \neq v) = p(\mathbf{Y}_v \mid \mathbf{X}, \mathbf{Y}_w, w \sim v)$, where $w \sim v$ means that w and v are neighbors in G.*

A CRF is a random field globally conditioned on the observation X.

$$p(\mathbf{y}|\mathbf{x}) = \frac{1}{Z(\mathbf{x})} \prod_{\Psi_A \in G} \exp\left\{ \sum_{k=1}^{K(A)} \lambda_{Ak} f_{Ak}(\mathbf{y}_A, \mathbf{x}_A) \right\}$$

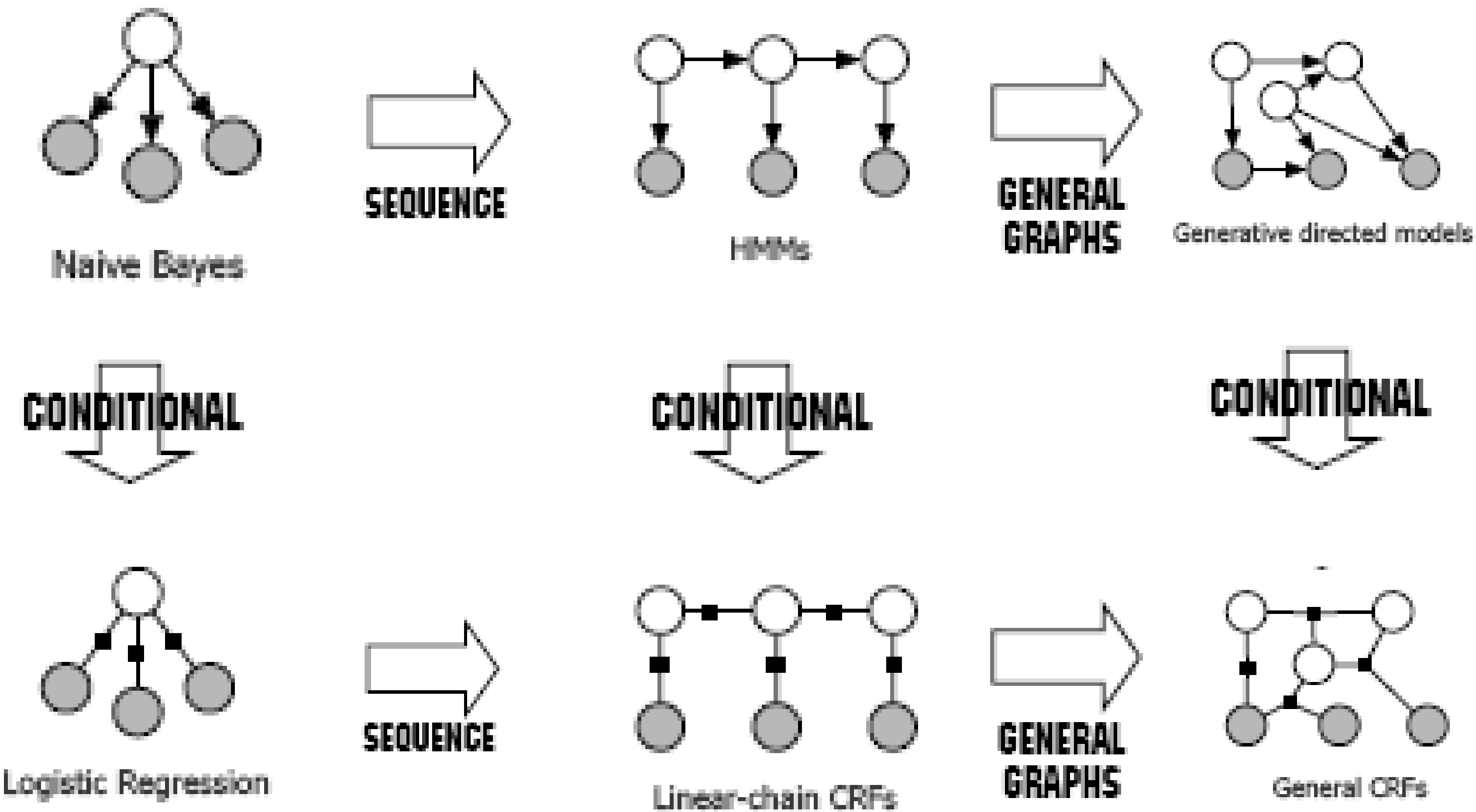
Linear-chain CRF

Motivation

- Sequence labeling problem: e.g., POS tagging
  - HMM: Find best sequence, but cannot use rich features
  - MaxEnt: Use rich features, but may not find the best sequence

- Linear-chain CRF: HMM + MaxEnt

Relations between NB, MaxEnt, HMM, and CRF



Linear-chain CRF

$$f_j(y_{t-1}, y_t, x, t) = \begin{cases} 1 & (y_{t-1} = IN) \wedge (y_t = NNP) \wedge (x_t = Sept) \\ 0 & otherwise \end{cases}$$

$$F_j(y, x) = \sum_{t=1}^{T} f_j(y_{t-1}, y_t, x, t)$$

$$\begin{aligned} P(y|x) &= \frac{1}{Z(x)} exp(\sum_j \lambda_j F_j(y, x)) \\ &= \frac{1}{Z(x)} exp(\sum_j (\lambda_j \sum_{t=1}^{T} f_j(y_t, y_{t-1}, x, t))) \\ &= \frac{1}{Z(x)} exp(\sum_j \sum_{t=1}^{T} (\lambda_j f_j(y_t, y_{t-1}, x, t))) \\ &= \frac{1}{Z(x)} exp(\sum_{t=1}^{T} \sum_j (\lambda_j f_j(y_t, y_{t-1}, x, t))) \\ &= \frac{1}{Z(x)} \prod_{t=1}^{T} \boxed{exp(\sum_j (\lambda_j f_j(y_t, y_{t-1}, x, t)))} \\ &= \frac{1}{Z(x)} \prod_{t=1}^{T} \phi_t(y_t, y_{t-1}, x) \end{aligned}$$

Training and decoding

$$P(y|x) = \frac{1}{Z(x)} \prod_{t=1}^{T} \phi_t(y_t, y_{t-1}, x)$$

$$\phi_t(y_t, y_{t-1}, x) = exp(\sum_j (\lambda_j f_j(y_t, y_{t-1}, x, t)))$$

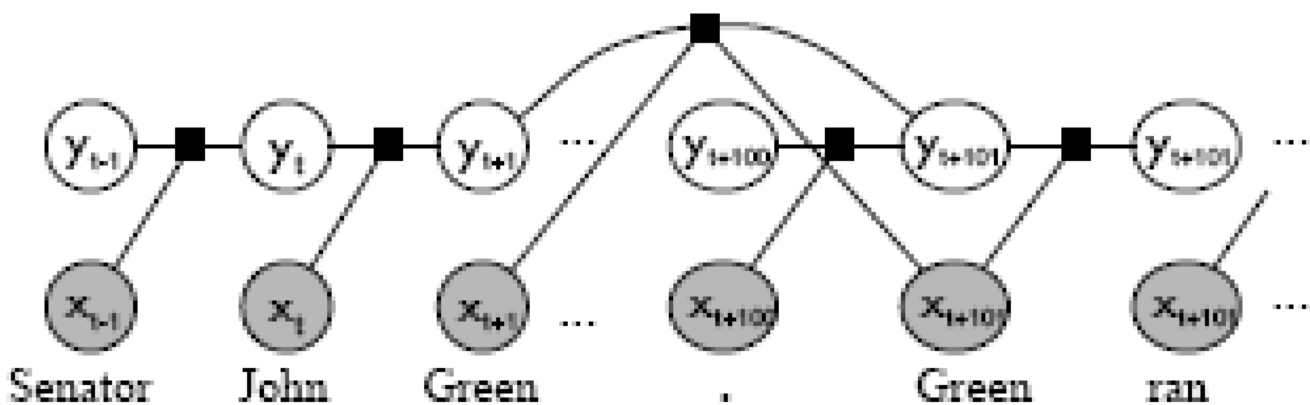
- Training: $\text{estimate } \lambda_j$
  - similar to the one used for MaxEnt
  - Ex: L-BFGS
- Decoding: find the best sequence y
  - similar to the one used for HMM
  - Viterbi algorithm

Skip-chain CRF

Motivation

- Sometimes, we need to handle long-distance dependency, which is not allowed by linear-chain CRF

- An example: NE detection
  - "Senator John Green … Green ran …"



Linear-chain CRF:

$$P(y|x) = \frac{1}{Z(x)} \prod_{t=1}^{T} \phi_t(y_t, y_{t-1}, x)$$

$$\phi_t(y_t, y_{t-1}, x) = exp(\sum_k (\lambda_k f_k(y_t, y_{t-1}, x, t)))$$

Skip-chain CRF:

$$P(y|x) = \frac{1}{Z(x)} \boxed{\prod_{t=1}^{T} \phi_t(y_t, y_{t-1}, x)} \boxed{\prod_{(u,v)\in D} \phi_{uv}(y_u, y_v, x)}$$

$$\phi_t(y_t, y_{t-1}, x) = exp(\sum_k (\lambda_k f_k(y_t, y_{t-1}, x, t)))$$

$$\phi_{uv}(y_u, y_v, x) = exp(\sum_k (\lambda_{2k} f_{2k}(y_u, y_v, x, u, v)))$$

Summary

- Graphical models:
  - Bayesian network (BN)
  - Markov random field (MRF)
- CRF is a variant of MRF:
  - Linear-chain CRF: HMM + MaxEnt
  - Skip-chain CRF: can handle long-distance dependency
  - General CRF
- Pros and cons of CRF:
  - Pros: higher accuracy than HMM and MaxEnt
  - Cons: training and inference can be very slow