Hyperplane

Point and vector

- A point in n-dimensional space is given by an n-tuple, e.g., P = (p_i)
  - A point denotes an absolute position in space
- A vector represents magnitude and direction in space, and is given by an n-tuple.
  - Vectors do not have a fixed position in place, but can be located at any initial base point P.
- The vector from point P to Q is given by v = Q – P = (q_i – p_i)
- Vector addition: v +/- w = (v_i +/- w_i)
- The length of a vector v: |v| = $\sqrt{\sum_{i=1}^{n} {v_i}^2}$
- http://geomalgorithms.com/points_and_vectors.html

Normal vector

- A normal vector is a vector perpendicular to another object (e.g., a plane).

- A unit normal vector is a normal vector of length one.

  If N is a normal vector, the unit normal vector is

  $N/|N|$, where $|N|$ is the length of N.

The equation for a hyperplane

- A 3-D plane determined by normal vector N=(A, B, C) and
  point Q=(x0, y0, z0) is:
  $$A(x - x0) + B(y - y0) + C(z - z0) = 0$$

  which can be written as
  $$Ax + By + Cz + D = 0, \quad \text{where } D = - A\,x0 - B\,y0 - c\,z0$$

- Hyperplane: $w\,x + d = 0$,
  where w is a normal vector, x is a point on the hyperplane
  - It separates the space into two half-spaces:
    $wx + d > 0$ and $wx + d < 0$

The distance from a point to a plane

- Given a plane Ax + By + Cz + D = 0, and a point P=(x1, y1, z1), the distance from P to the plane is:

$$d = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

See http://mathinsight.org/distance_point_plane

- Distance from a point x to a hyperplane wx + d = 0 is:

|w x + d |/||w||

Distance between two parallel planes

- Two planes $A_1x + B_1y + C_1z + D_1=0$ and $A_2 x + B_2y + C_2z + D_2=0$ are parallel if $A_1=k A_2$, $B_1=k B_2$ and $C_1=k C_2$

- The distance between Ax + By + Cz + D1 = 0 and Ax + By + Cz + D2 = 0 is equal to the distance from a point (x1, y1, z1) on the first plane to the second plane:

$$\frac{|Ax_1+By_1+Cz_1+D2|}{\sqrt{A^2+B^2+C^2}} = \frac{|D2-D1|}{\sqrt{A^2+B^2+C^2}}$$

- The distance between two hyperplanes wx + d1 = 0 and wx + d2 = 0 is

$$\frac{|d2 - d1|}{||w||}$$