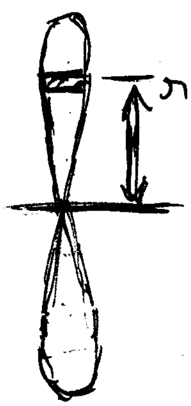
Lecture #17

Wind Turbine Performance

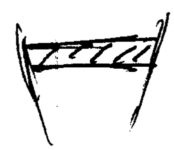
The tangential force pulling the wind turbine in its direction of rotation is:

$$F_T = L \sin\phi - D\cos\phi$$

The equation holds for one location along the blade.



Multiplication of F_T by r gives the torque produced by the small part of the blade in question:



$$T = \text{torque} = F_T\, r$$

F_T and T vary along the length of the blade, from hub to tip.

Integrating T along the whole blade gives the total torque produced:

$$\mathcal{T} = \text{total torque}$$

The power produced is:

$$P = \text{power} = T\,\omega$$

ω ← rotational speed of turbine as radians/sec

or

$$P = T\,2\pi N$$

N ← revolutions/sec

How does this compare to the Power of the wind?

The power of the wind is:

$$P_{wind} = \underbrace{\frac{1}{2}\rho u_o^2}_{\text{dynamic pressure}}\ \underbrace{A}_{\text{area of turbine}}\ u_o$$

u_o ← velocity magnitude of wind undisturbed by turbine

$$\boxed{P_{wind} = \frac{1}{2}\rho u_o^3 A}$$

This equation holds for a single wind speed.

Note: the power of the wind varies as the cube of the ambient wind speed. There is a big difference between a site with a mean wind speed of 10 mph and a site with 15 mph. The 15 mph site is 3.375 times as wind-powerful as the 10 mph site.

Also, note:

$$\overline{P}_{wind} \neq \frac{1}{2} \rho \overline{U}_o^{\,3} A$$

That is, the mean wind power is not equal to $\frac{1}{2}\rho A$ times the mean wind speed cubed.

This is because the high components in a distribution of wind speeds count much much more than the low components.

Statistical analysis of wind speeds shows a typical behavior of

$$\left(\overline{u_o^3}\right)^{1/3} \cong 1.24\,\overline{u}_o$$

(Note: not all winds obey this result.)

Thus:

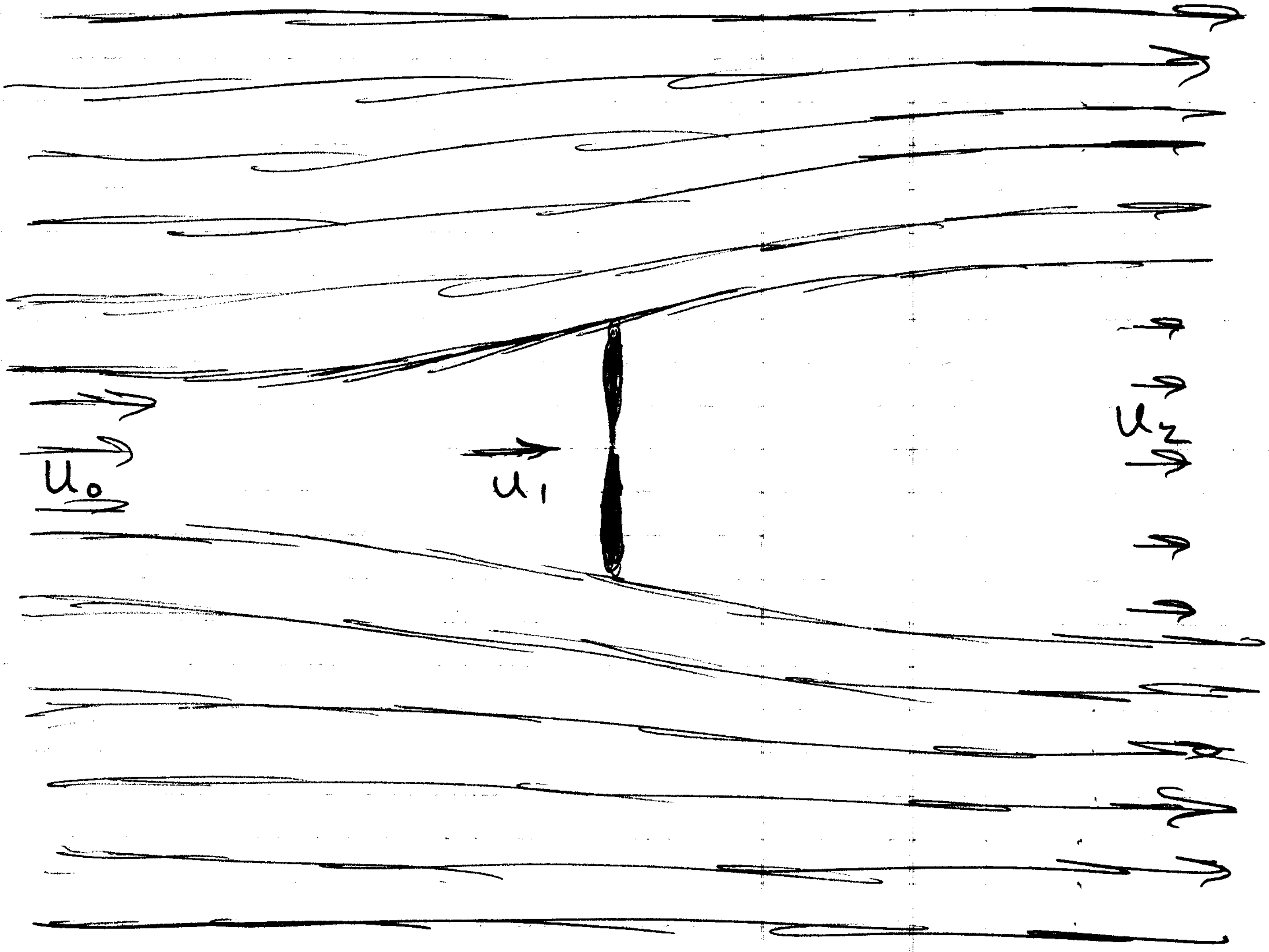
$$\overline{P}_{wind} \cong \frac{1}{2}\rho\left(1.24\,\overline{u}_o\right)^3 A$$

$$\overline{P}_{wind} \cong \frac{1.91}{2}\rho\,\overline{u}_o^{\,3} A$$

$$\boxed{\overline{P}_{wind} \cong \rho\,\overline{u}_o^{\,3} A}$$

Note also that u_o, the ambient wind speed is larger than the speed u_1, the "undisturbed" wind speed just ahead of the blade, used in our analysis of lift and drag forces on the blade.

This occurs because the wind diverages as it flows around and through the turbine. See picture on next page.



Because of the diveraging winds, $u_1 < u_0$. The wake has the lowest speed (u_2).

The efficiency of a wind turbine is a measure of how well the wind turbine extracts the power of the wind (P_{wind}), creating the mechanical power $P = T\,2\pi N$.

Performance as a function of tip-speed ratio ($\lambda = V_{Tip}/u_o$)

Wind turbines are most efficient when operated at speed, N, that is best matched to the wind speed, u_o.

The time required for a turbine blade to make one complete revolution is

$$t = \frac{1}{N} = \frac{2\pi}{\omega}$$

The time required for a blade (on a turbine) to move into the position previously occupied by the preceding blade is

$$t_b = 2\pi/\omega n$$

where n = number of blades on turbine.

The time of duration of the aerodynamic disturbance created by the preceding blade is

$$t_w = kR/u_0$$

where R = hub to tip radius of turbine and K is an empirical coefficient of value of about 1/2 to 1

(This is like aircraft waiting to take off -- waiting for the wake of the previous departure to die away.)

Best power extraction by the turbine occurs when

$$t_b \cong t_w$$

$$\frac{2\pi}{\omega n} \cong \frac{kR}{u_0}$$

or, rearranging: $\dfrac{2\pi}{nk} \cong \dfrac{\omega R}{u_0} = \dfrac{V_{tip}}{u_0}$

$$\frac{2\pi}{nk} \cong \frac{V_{tip}}{u_0} \quad \text{or} \quad \boxed{\frac{2\pi}{nK} = \lambda \cong \frac{V_{tip}}{u_0}}$$

tip speed ratio

This equation has very significant design and operating implications.