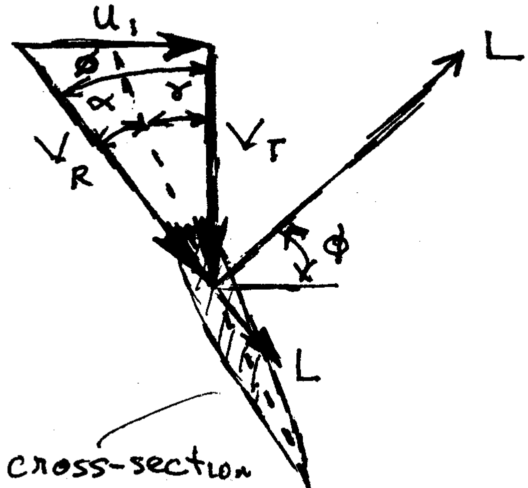
Lecture #18

So far we have looked in detail at the air flow around the turbine blade and the forces on the blades it produces.



The tangential force pulling the turbine in its direction of rotation is:

$$F_T = L \sin\phi - D \cos\phi$$

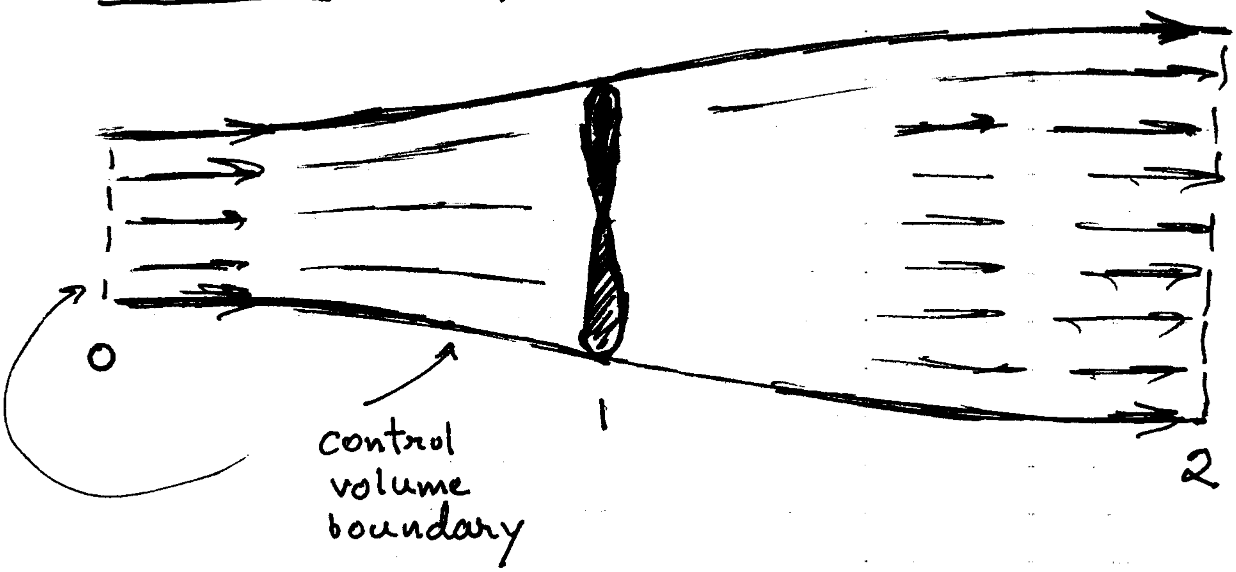
or

$$\frac{F_T}{\frac{1}{2}\rho V_R^2 A} = \frac{L}{\frac{1}{2}\rho V_R^2 A}\sin\phi - \frac{D}{\frac{1}{2}\rho V_R^2 A}\cos\phi$$

$$C_{F_T} = C_L \sin\phi - C_D \cos\phi$$

Force Coefficient   Lift Coefficient   Drag Coefficient

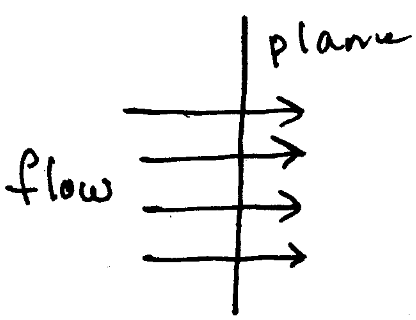
Today, we consider a different approach. This is the control volume approach.



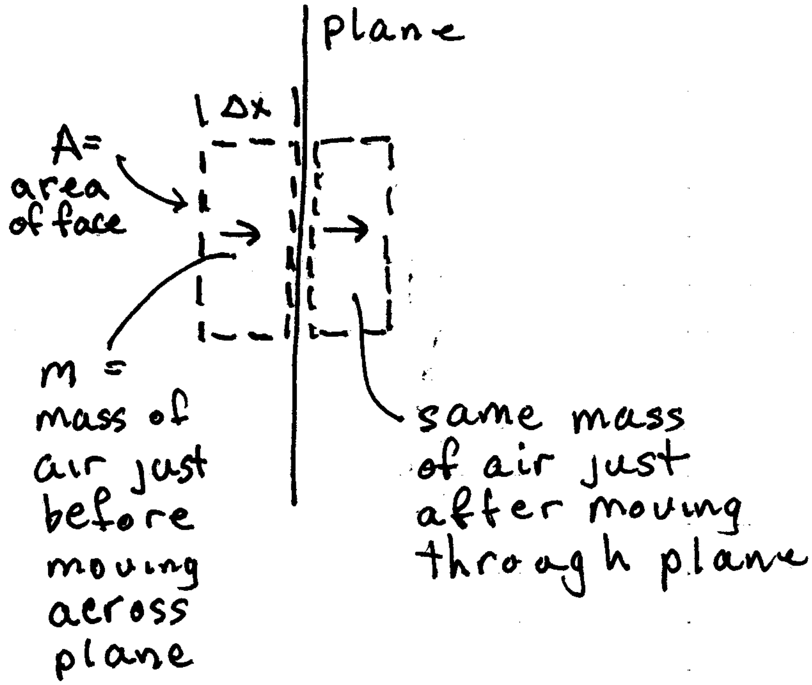
In this approach we need as expression for the mass flow rate:

$$\text{mass flow rate} = \dot{m} = \text{kg/s}$$

This is the mass of air per second passing through a plane normal to the flow:



In order to find the expression for $\dot{m}$ we consider the following



$$m = \rho V = \rho A \Delta x$$

ρ: density kg/m³

V: volume m³

The time, Δt, is the time it takes the mass to move through the plane

Thus

$$\dot{m} = \frac{\text{mass}}{\text{second}} = \rho A \left(\frac{\Delta x}{\Delta t}\right)$$

This is the velocity of the air normal to the plane $= u$

Finally

$$\boxed{\dot{m} = \rho A u}$$

Now we look at Conservation of Energy for the Control Volume.

The air has 3 energies:

Kinetic energy (KE)

Potential energy (PE)

Thermal energy

The energy that significantly changes between inlet (pt 0) and outlet (pt 2) is ~~the~~ energy focus upon. This the kinetic energy: $\frac{m}{2}u^2$

The potential energy doesn't change since the elevation doesn't change.

The thermal energy doesn't change since there are no significant changes in temperature of the air.

The KE per second entering is:

$$\frac{1}{2}mu_0^2/\Delta t = \frac{1}{2}\dot{m}u_0^2$$

The KE per second leaving is:

$$\frac{1}{2}mu_2^2/\Delta t = \frac{1}{2}\dot{m}u_2^2$$

The difference in the rate of KE entering versus leaving is the work/sec (or power) imparted to the turbine.

Thus:

$$P_T = \dot{m}\left(\frac{u_0^2}{2} - \frac{u_2^2}{2}\right)$$

The mass flow rate is steady:

$$\dot{m} = \dot{m}_0 = \dot{m}_1 = \dot{m}_2 = \text{const}$$

$$\dot{m} = \rho A_0 u_0 = \underbrace{\rho A_1 u_1}_{} = \rho A_2 u_2$$

we choose to express $\dot{m}$ with "1", since A_1 = area swept by the rotating turbine.

Thus:

$$P_T = \frac{\rho u_1 A_1}{2}\left(u_0^2 - u_2^2\right)$$

Rerranging

$$P_T = \frac{1}{2}\rho u_0^3 A_1 \left[\frac{u_1}{u_0}\left(1 - \frac{u_2^2}{u_0^2}\right)\right]$$

We assign the parameter, a :

$$a \equiv 1 - \frac{u_1}{u_0} \quad \text{or} \quad \frac{u_1}{u_0} = 1-a$$

We assume

$$u_1 = \frac{u_0 + u_2}{2}$$

that is, we assume the velocity at the turbine (just before the air goes around the turbine blades) is average of the inlet and outlet velocities. (For proof of this, see last year's web notes on wind turbines)

Combining !

$$\frac{u_1}{u_0} = \frac{1}{2}\left(1 + \frac{u_2}{u_0}\right) = 1-a$$

$$1 + \frac{u_2}{u_0} = 2 - 2a$$

$$\frac{u_2}{u_0} = 1 - 2a$$

Then:

$$P_T = \frac{1}{2}\rho u_0^3 A_1 \underbrace{\left[(1-a)\left(1-(1-2a)^2\right)\right]}_{4a(1-a)^2}$$

Thus $$P_T = \frac{1}{2}\rho u_0^3 A_1 \left[4a(1-a)^2\right]$$

In this equation, $\frac{1}{2}\rho u_0^3 A_1$ is the power of the wind (as found in an earlier lecture).

$4a(1-a)^2$ is called the power coefficient, C_p.

$$C_p = 4a(1-a)^2$$

The power coefficient represents the fraction of the power of the wind that is converted to the power of the turbine.

The maximum possible value of C_p occurs when

$$a = 1/3$$

Then $$C_p = C_{p_{max}} = 0.59$$

This is called the <u>Betz criterion</u>.

The other values at this condition are:

$$u_1/u_0 = 2/3$$

$$u_2/u_0 = 1/3$$

We write:

$$\boxed{P_T = \tfrac{1}{2}\rho u_0^3 A_1 C_P}$$

where $C_{P\,max} = 0.59$

Some rearrangement and interpretation

$$P_T = \underbrace{\tfrac{1}{2}\rho u_0^2 A_1}_{\text{a force, the max force}} \; u_0 \; C_P$$

$$P_T = F_{max} \; u_0 \; C_P$$

$$P_T = \underbrace{F_{max} R}_{\text{torque, max torque}} \; \frac{u_0}{R} \; C_P$$

where R = radius (hub to tip) of turbine

$$P_T = T_{max} \; \frac{u_0}{R} \; C_P$$

$$P_T = T_{max}\,\omega \; \frac{u_0}{R\omega} \; C_P$$

where ω = rotational speed in radians/sec

$$P_T = T_{max}\,\omega \; \frac{u_0}{V_{Tip}} \; C_P$$

$$\lambda = \frac{V_{Tip}}{u_0}$$

= tip speed ratio

$$P_T = T_{max}\,\omega \; \frac{C_P}{\lambda}$$

$$P_T = \underbrace{T_{max}\,\omega}_{\text{power, max power}} \underbrace{\frac{C_p}{\lambda}}_{C_T = \text{torque coefficient}}$$

$$\boxed{P_T = P_{T_{max}}\, C_T}$$

Thus, we have two ways to look at the same thing (P_T)

Although not seen in theory above, C_p varies with λ as follows: