Lecture #19.

Generating electricity from wind turbines.

We have been using the symbol u_0 for the ambient wind speed. For the present discussion, let's drop the subscript "o" and write the ambient wind speed as u.

We write $\Phi_u \Delta u$ = probability the wind speed lies between u and $u + \Delta u$

The mean, or average, wind speed is

$$\bar{u} = \sum u \Phi_u \Delta u$$

where Σ denotes summation over the full range of the wind speeds

The power of the wind is

$$P_{wind} = \frac{1}{2} \rho u^3 A_1$$

The mean, or average, power of the wind is

$$\begin{aligned} \bar{P}_{wind} &= \sum \frac{1}{2} \rho u^3 A_1 \Phi_u \Delta u \\ &= \frac{1}{2} \rho A_1 \sum u^3 \Phi_u \Delta u \\ &= \frac{1}{2} \rho \overline{u^3} A_1 \end{aligned}$$

It turns out, because of the type of distribution wind has, that

$$\overline{u^3} \cong 2\bar{u}^3$$

Thus

$$\bar{P}_{wind} \cong \rho \bar{u}^3 A_1$$

At one instant, the power extracted by the wind turbine is

$$P_T = \frac{1}{2}\rho u^3 A_1 C_p$$

The mean, or average, power extracted is

$$\bar{P}_T = \frac{1}{2}\rho A_1 \sum u^3 C_p \Phi_u \Delta u$$

Note C_p is a function of λ and thus a function of u, i.e.

$$C_p = C_p(\lambda) \quad \text{and} \quad \lambda = \frac{R\omega}{u}$$

If we could design and build wind turbines that run at the Betz criterion of $C_p = 0.59$, the

$$P_T = \frac{0.59}{2} \rho u^3 A,$$

and

$$\overline{P}_T \cong 0.59 \rho \overline{u}^3 A,$$

For a real wind turbine, we could try to run at the optimal λ, giving the maximum C_p, i.e. $C_{p\,max} < 0.59$.

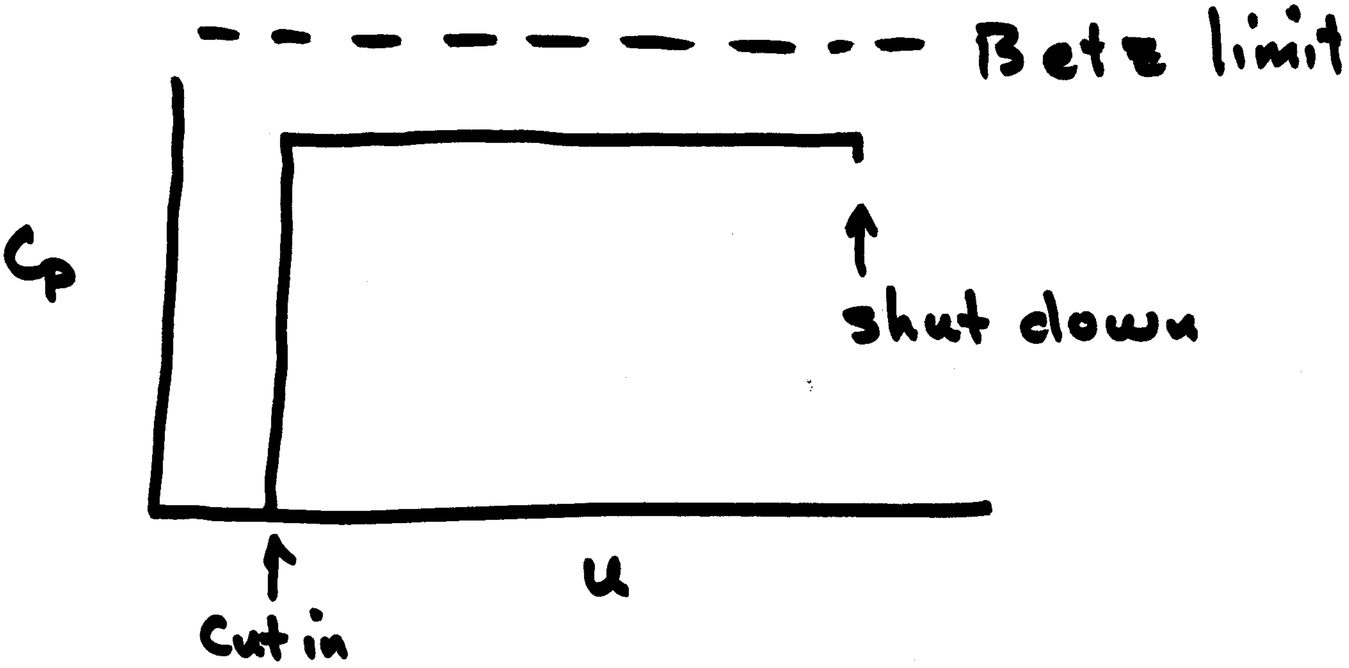
This means the rotational speed of the turbine would need to change as the wind speed changes in order to satisfy:

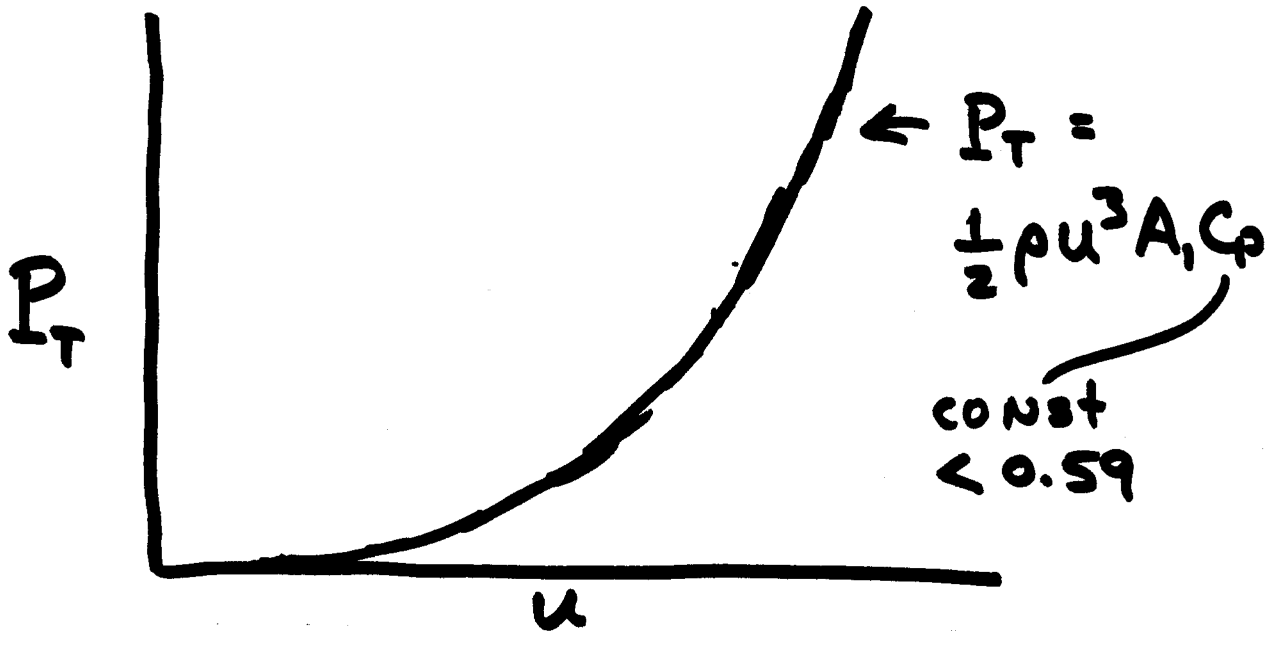
$$\lambda = \lambda_{optimal} = \text{constant} = \frac{R\omega}{u}$$

The electricity produced by such a system could be used for resistive heating (which isn't sensitive to frequency), or after conversion to DC, could be used for battery charging. See Fig 9.23a, p 247, of text.

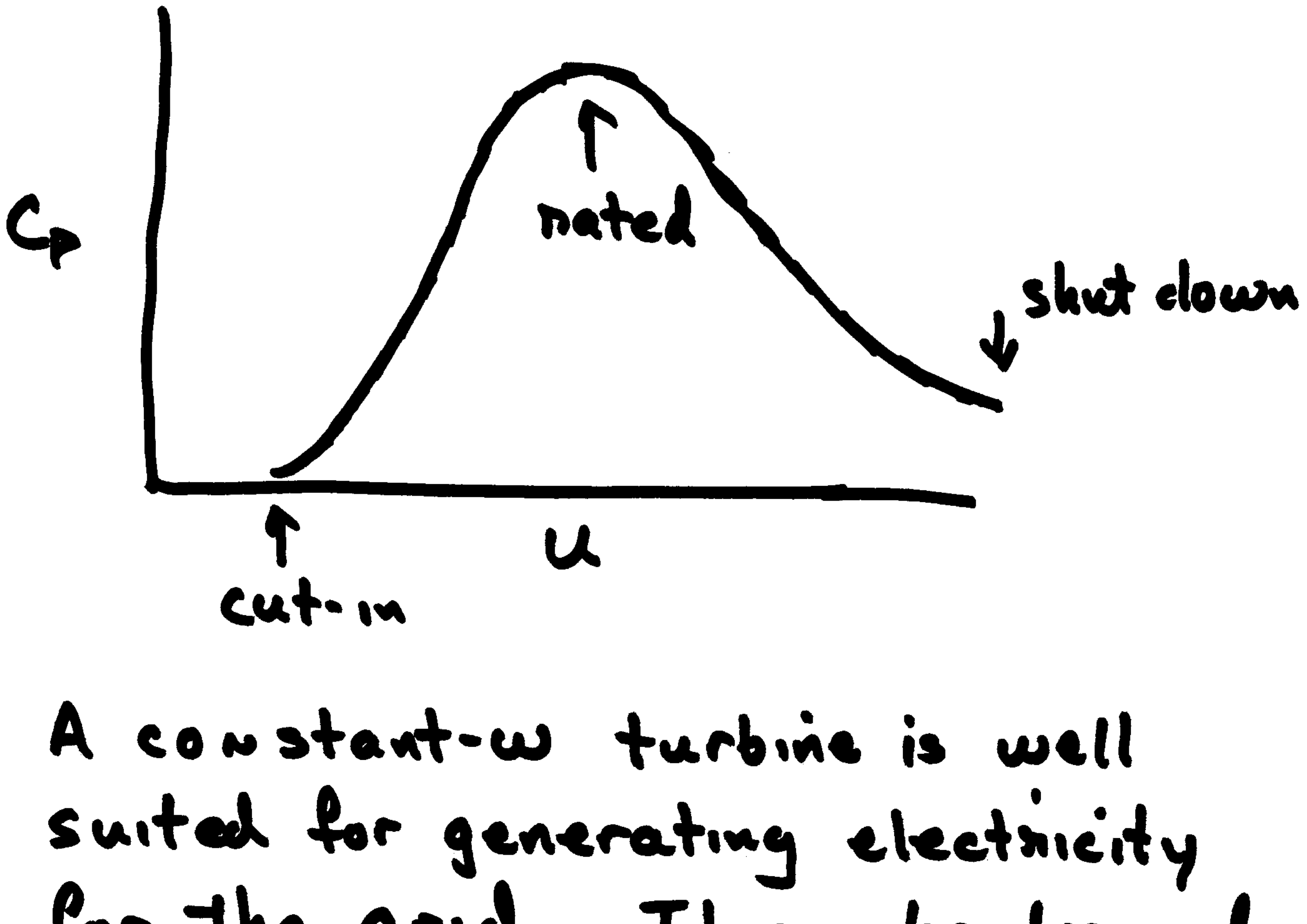
Also, at some point the turbine will reach the rated, or maximum, rotational speed.

The constant-λ turbine would need to shut down when the maximum ω (and u) is reached. The performance would be as follows:



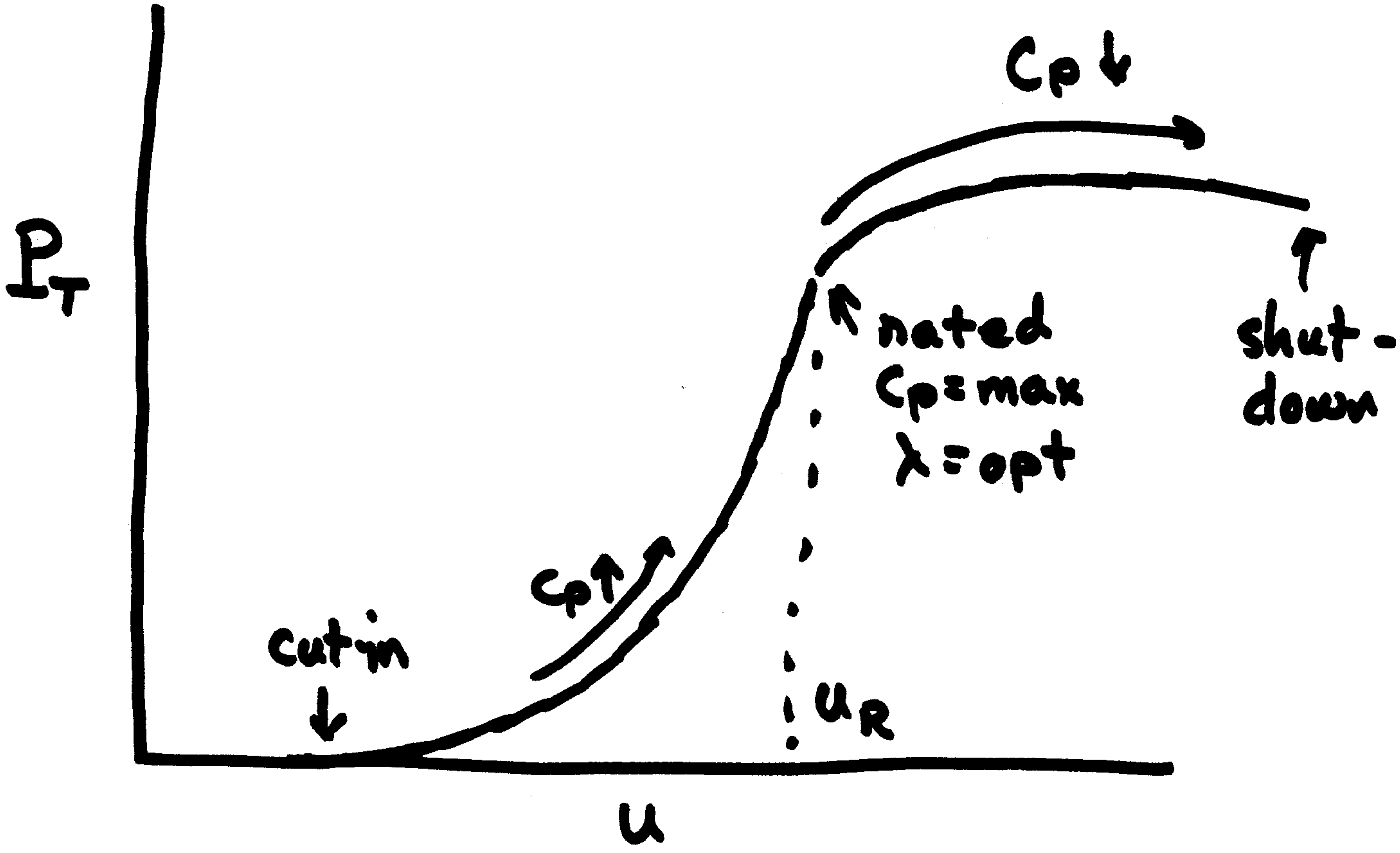


On the other hand, what if the wind turbine is maintained at ω = constant. Then λ will be optimal only at one wind speed, the rated wind speed. C_p will vary as follows:



A constant-ω turbine is well suited for generating electricity for the grid. It can be designed to produce 50 or 60 cycle/sec electricity.

The power curve might look like:



Note: For $u > u_R$, the angles ϕ and α will become large. The blade may stall, unless the blade angle γ is adjusted to reduce α.