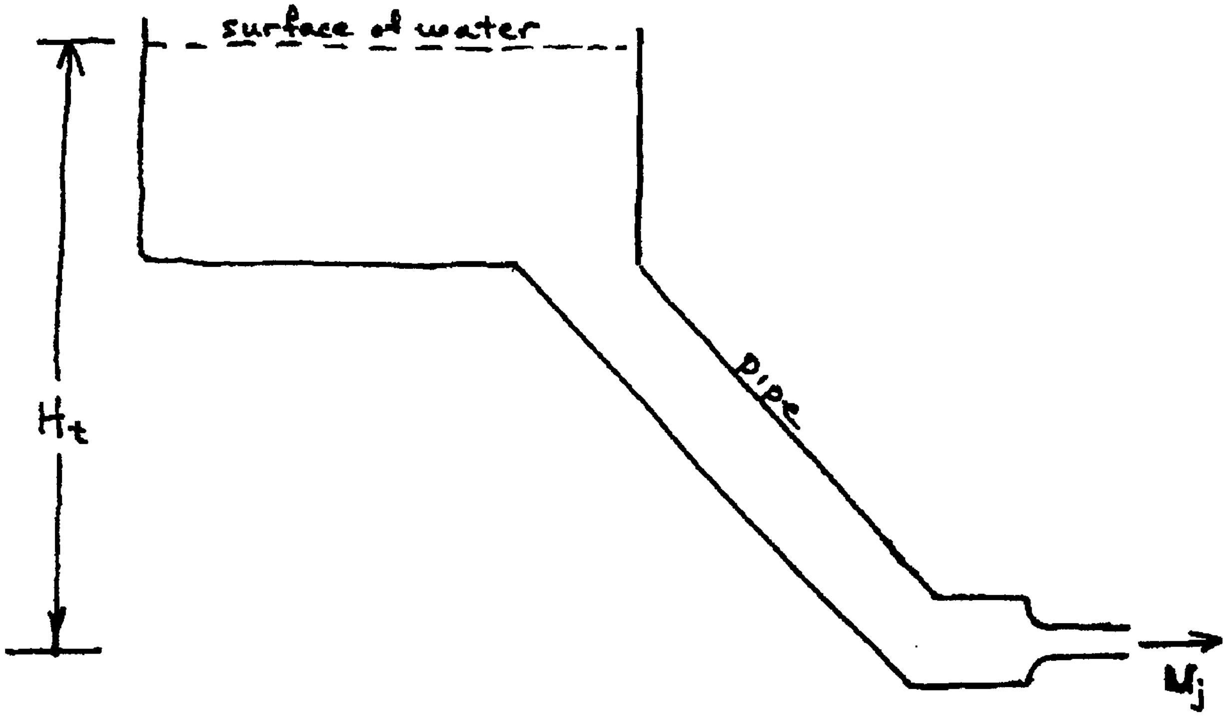
Lecture #21

HYDROELECTRICITY

The first step in the creation of hydroelectricity is the conversion of the potential energy of water (in the reservoir) to kinetic energy, i.e. to a jet (or jets) of water ready to drive the turbine. This is depicted below:



The system converts Potential Energy into Kinetic Energy

jet of water driving turbine wheel

The total head (H_t) is the elevation of the surface of the water (in the reservoir) relative to the elevation of the jet (just before it strikes the turbine)

Thus, the potential energy of the water is:

$$PE = m g H_t$$

where m = mass of water being considered

g = acceleration of gravity = $9.81\ m/s^2$

H_t = head in meters

The power available is

$$P_{water} = \dot{m} g H_t$$

where $\dot{m}$ = mass flow rate of water (kg/s)

Normally, for hydroelectric systems we write:

$$\dot{m} = \rho Q$$

where ρ = density of water (1000 kg/m^3)

Q = volume flow rate (m^3/s)

The power available is :

$$\boxed{P_{water} = \rho g Q H_t}$$

Not all of this power reaches the electricity end-users.
Power is lost at the following points :

1) Friction in the pipe (in the penstock) connecting the reservoir to the jet)
This effectively reduces the head :

$$H_a = H_t - H_f$$

where H_f = amount of the head used to overcome friction in the penstock
H_a = available head remaining to drive the jet

For a modern hydroelectric facility, $H_f \lesssim 0.1\ H_t$, so that $H_a \gtrsim 0.9\ H_t$

2) Imperfect conversion of the kinetic energy of the jet (or the flow at the bottom of the penstock) to the mechanical energy of the turbine.
For a well engineered and operated hydroelectric facility, the efficiency of turbine is about

$$\eta_m \cong 0.9$$

3) The turbine drives an electrical generator. Typically, electrical generators have high efficiencies:

$$\eta_g \gtrsim 0.95$$

4) Now the electricity leaves the hydroelectric plant and flows through high voltage transmission lines (765 to about 135 KV).

On average, about 5% of the electrical power is lost (to I^2R heating) in the transmission lines.

5) Finally, the electricity is distributed in the local community. The pole power lines run about 26 kV and the voltage is stepped down (in transformers) to 240 v for homes.
On average, about another 5% of the electrical power is lost.

Thus, the overall efficiency of hydroelectricity can be:

$$\eta \cong \underset{\substack{\text{friction}\\ \text{in penstock}}}{.9} \times \underset{\text{turbine}}{.9} \times \underset{\text{gen}}{1.0^{-}} \times \underset{\text{trans}}{.95} \times \underset{\text{dist}}{.95}$$

$$\eta \cong 0.70$$

η can also be as low as about 0.50.

Between the top of the reservoir (i.e. the surface) and bottom of the penstock PE is converted to KE.

Thus:

$$mgH_a = \frac{1}{2} m u_j^2$$

where u_j = velocity of jet(s)

$$\boxed{u_j = \sqrt{2gH_a}}$$

Thus, the power of the jet(s) may be written

$$P_{jet} = \frac{\dot{m}}{2} u_j^2$$

$$\boxed{P_{jet} = \frac{1}{2} \rho Q u_j^2}$$

Rearranging with $\dot{m} = \rho Q = \rho u_j A_j$
we get:

$$\boxed{P_{jet} = \frac{1}{2} \rho A_j u_j^3}$$

where A_j = cross-sectional area of jet(s)

and since $u_j = \sqrt{2 g H_a}$
we may write

$$\boxed{P_{jet} = \frac{1}{2} \rho A_j \left(2 g H_a\right)^{3/2}}$$

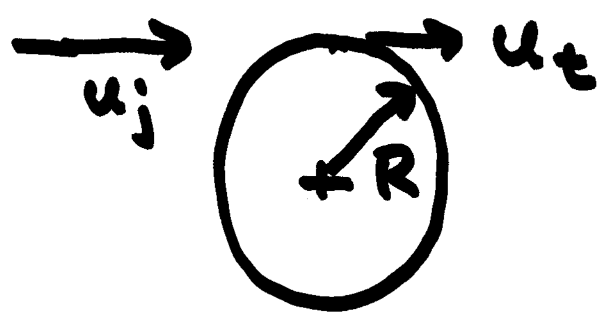
We would like to convert all of the power of the jet to the mechanical power of the (rotating) turbine. Water turbines have several types, but one of more basic types in the Pelton wheel turbine -- see Fig 5.27, p 205, in text.

In this case, the jet of water strikes cups (one at a time) on the wheel. See Figure 5.30, p 206, in text. With a little bit of elementary physics, you should be able to convince yourself that all of the power of the jet is converted to the power of the turbine if:

$$u_t = u_j / 2$$

where u_t = tangential velocity of turbine.

In this case, the water is left with no velocity in its original direction (or opposite to its original direction). See Figure 5.30.



Then, the power of the turbine is

$$P_{mech} = P_{jet} = \frac{1}{2} \rho A_j (2g H_a)^{3/2}$$

However, in practice there will be some losses:

1) $u_t = u_j/2$ may not be exactly obtained for all of the water
2) some mechanical friction occurs in the turbine -- in its bearings
3) some aerodynamic loss occurs as the cups rotate

So, we write

$$\boxed{P_{mech} = \frac{1}{2} \eta_m \rho A_j (2g H_a)^{3/2}}$$

where η_m = efficiency of converting jet power to turbine power

Now we come to the interesting stuff -- how big to make the turbine, and what type of turbine to use, for the particular situation.

Note: $U_t = R\omega$

where R = radius of turbine

ω = angular frequency of turbine $= 2\pi N$, where N = rev/s

and $A_j = n_j \pi r_j^2$

where n_j = number of jets per turbine wheel

r_j = radius of jet

Example: suppose $H_t = 100$ m. Then our estimate of H_a is $H_a = 0.9\, H_t = 90$ m

Then $u_j = \sqrt{2 g H_a} = \sqrt{2 \times 9.81 \frac{m}{s^2} \times 90 m}$

Thus, $u_j = 42.02$ m/s

We wish for the turbine power to be 200 kW. We assume a mechanical efficiency of $\eta_m = 0.9$. Thus, from

$$P_{mech} = \frac{1}{2}\eta_m \rho A_j u_j^3$$

we find the jet size

$$200\,\frac{kJ}{s} \times 1000\,\frac{J}{kg}$$

$$= \frac{1}{2} \times 0.9 \times 1000\,\frac{kg}{m^3} \times A_j(m^2) \times \left(42.02\,\frac{m}{s}\right)^3$$

This gives:

$A_j = 0.0060\ m^2$

Assuming one jet, we find

$r_j = 0.0437$ m $= 4.37$ cm

The flow rate is

$$Q = u_j A_j = 42.02 \frac{m}{s} \times .0060 \, m^2$$
$$= 0.252 \, m^3/s$$

In order to select the turbine, we look at Figure 5.32, p 208.

Our situation has:

1) A large value for the head (100 m)
2) A very low value for the flow rate ($0.25 \, m^3/s$)

This is Pelton wheel territory.

Pelton wheels work best if

$r_j \ll R$, i.e. about $\frac{r_j}{R} \cong \frac{1}{12}$

Assuming this, we obtain

$$R = 12 \times .0437 = 0.524 \, m$$

and $\omega = u_t/R = u_j/2R = 40.1 \frac{rad}{s}$

$$N = 6.4 \, rev/s$$

Note $N = 6.4$ rev/s is not a nice multiple of 60 rev/s. Thus, we might wish to adjust our R to give $N = 6$ rev/s.

We can rearrange our equation

$$P_{mech} = \frac{1}{2}\eta_m \rho A_j (2gH_a)^{3/2}$$

into a shape factor that permits us to assume the type of turbine.

$$P_{mech} = \frac{1}{2}\eta_m \rho \underbrace{n_j \pi r_j^2}_{A_j} \frac{u_t^2}{u_t^2} (2gH_a)^{3/2}$$

use $u_t = u_j/2 = \frac{1}{2}\sqrt{2gH_a}$

and $u_t = R\omega$

Then

$$P_{mech} = \frac{1}{2}\eta_m \rho n_j \pi \left(\frac{r_j}{R}\right)^2 \frac{1}{\omega^2} \left(\frac{1}{2}\right)^2 (2gH_a)^{5/2}$$

Rearranging

$$P_{mech} = 2.22\, \eta_m \rho n_j \left(\frac{r_j}{R}\right)^2 \frac{(gH_a)^{5/2}}{\omega^2}$$

The shape factor ς is defined as:

$$\varsigma = \sqrt{\frac{P_{mech}}{\rho}} \frac{\omega}{(g H_a)^{5/4}}$$

This may also be written as

$$\varsigma = \frac{r_j}{R} \sqrt{2.22 \eta_m r_j}$$

Single jet Pelton wheel turbines obtain best efficiency ($\eta_m \cong 0.9$) when $r_j / R \cong 0.08$. Thus, $\varsigma \cong 0.11$. This small value ς implies a large H_a. However, the small r_j implies a low Q.

Francis turbines (which you can view in the text, and have a broad range, see Figure 5.32), have $\eta_m \gtrsim 0.9$ for $\varsigma \cong 1$ to 2. Propeller-type water turbines exhibit $\eta_m \gtrsim 0.9$ for $\varsigma \cong 2$ to 4.