Energy and Environment II
HW#9
Due Friday, March 8, 2002

1. Off the southern coast of Australia, the annual average wave power (per crest length) is about 75 kw/m. Assuming these waves have a wavelength of 100 m, find the following:
   a. The period of the waves (T, seconds).
   b. The wave speed (V, m/s)
   c. The height of the waves (H = 2a, m)
   d. The maximum speed of the water particles (u, m/s)
   Clearly show all steps in your work.

2. Box 5.8 on page 209 of the text contains some important equations for hydropower. Start with the equation:
   $$P \text{ (kw)} \times 1000 \text{ (w/kw)} = \eta \times \rho \text{ (kg/m}^3) \times g \text{ (m/s}^2) \times Q \text{ (m}^3\text{/s)} \times H \text{ (m)}$$
   Assume H is the effective (or working) head, that is, the head after the effect of the friction in the penstock as been taken into account. The mechanical efficiency of the turbine is η. The power output of the turbine is P.
   a. Derive the first equation in the box, ie, the first equation for $P/H^2H^{1/2}$. Note $v_w = u_j$ (the jet velocity, m/s), and $r = r_j$ (jet radius, m). There is only one jet. The density of water is 1000 kg/m^2 and the acceleration of gravity is 9.81 m/s^2. DO NOT ASSUME g = 10 m/s^2 AS THE TEXT HAS DONE. ALSO NOTE THERE IS AN ERROR IN THE TEXT EQUATION.
   b. Now derive the second equation in the box, ie, the second equation for $P/H^2H^{1/2}$. Retain the efficiency η in your equation. The constant in your equation should reflect the correct value for the acceleration of gravity.
   c. Now derive the third equation in the box, ie, the equation for N_s. Retain the efficiency η in your equation. The constant in your equation should reflect the correct value for g. Note n is the revolutions per minute of the turbine, v_B is the tip speed of the turbine, and R is the radius of the turbine. Note this equation holds for all types of water turbines.
   d. For a particular application: the total head is 3 m, the flow rate is 1.0 m^3/s, and mechanical power output of the turbine is 20 kw. Find the velocity of the jet, the radius of the jet, and the mechanical efficiency of the turbine. State any assumptions used.
   e. Would you use a Pelton wheel turbine for this application? Explain. What would be the rotational speed of your turbine (n, rev/min)?

3. We have studied some methods of obtaining energy from water. Which of the methods studied do you believe offers the best ratio of "benefit to environmental-impact"? Explain. Type your answer double-spaced on one page.

Energy and Environment II

Recommended Solution to HW #9

1. Wave Energy

Given $P = 75$ kw/m (Power/crest length)

$L = 100$ m (wavelength)

a. Period

$L = 2\pi g/\omega^2$ with $L = 100$ m and $g = 9.81$ m/s^2

gives $\omega = 0.785$ radians/s

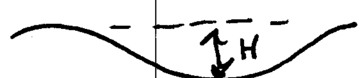
$2\pi N = \omega$ gives $N = 0.125\ s^{-1}$

Thus $T = 1/N = \underline{\underline{8.0 \text{ sec}}}$

b. $V = L/T = 100/8 = \underline{\underline{12.5 \text{ m/s}}}$

Wave Speed

c. Height of Waves



Since $P = \dfrac{\rho g^2 H^2 T}{32\pi}$

we have sufficient info to solve for H

$$\underset{kw}{75 \times 1000}\ \underset{w/kw}{=}\ \underbrace{\underset{kg/m^3}{1000} \times \underset{(\frac{m}{s})^2}{(9.81)^2} \times \underset{m^2}{H^2} \times \underset{s}{8.0}}\ /32\pi$$

$$\underbrace{\frac{kg}{m^3}\ \frac{m^2}{s^4}}\ m^2\ s$$

$$\underbrace{kg\ \frac{m}{s^2}}_{N}\ \frac{m}{\cancel{m}}\ \frac{1}{s}$$

$$\frac{N\text{-}m}{m}\ \frac{1}{s} = \frac{J}{s}\ \frac{1}{m} = w/m$$

$H = \underline{\underline{3.13 \text{ m}}}$

d. water particle velocity is max at top of wave

$u = a\omega = \dfrac{H}{2}\omega = \dfrac{3.13}{2} \times .785 = \underline{\underline{1.23 \text{ m/s}}}$

2. Hydropower

Start with $P = \eta \rho g Q H$

with $P = kW$, $\rho = kg/m^3$, $g = m/s^2$, $Q = \frac{m^3}{s}$, $H = m$

need to multiply P by 1000 W/kW

$P \times 1000 = \eta \rho g Q H$ (H = effective head)

a. Derive first equation in Box 5.8 of text

Use $Q = A_j u_j = \pi r_j^2 u_j$

Then

$$\frac{P}{H} = \frac{\eta \rho g \pi r_j^2 u_j}{1000}$$

$$\frac{P}{H^2 \sqrt{H}} = \frac{\eta \rho g \pi r_j^2 u_j}{1000 \; H \sqrt{H}}$$

Error in text: text has extra square here

but $H = u_j^2/2g$, so that

$$\frac{P}{H^2 \sqrt{H}} = \frac{\eta \rho g \pi r_j^2 u_j}{1000 \; u_j^2 \sqrt{u_j^2}} \; 2g \sqrt{2g}$$

Since $\rho = 1000 \; kg/m^3$ and $g = 9.81 \; m/s^2$ we have

$$\frac{P}{H^2 \sqrt{H}} = \frac{\eta \; g^2 2\pi \sqrt{2g} \; r_j^2 \cancel{u_j}}{u_j^2 \cancel{\sqrt{u_j^2}}}$$

$$\frac{P}{H^2 \sqrt{H}} = 2678 \; \eta \left(\frac{r_j}{u_j}\right)^2$$

with $r_j = r$ and $u_j = V_w$, we arrive at the text expression (with a more accurate constant)

$$\boxed{\frac{P}{H^2 \sqrt{H}} = 2678 \; \eta \left(\frac{r}{V_w}\right)^2}$$

where $P = kW$
$H = m$
$r = m$
$V_w = m/s$

b. $$\boxed{\sqrt{\frac{P}{H^2\sqrt{H}}} = 51.75\sqrt{\eta}\left(\frac{r}{V_w}\right)}$$

c. Specific speed

$$N_s = n\sqrt{\frac{P}{H^2\sqrt{H}}}$$ where n = turbine rotational speed in rev/min

turbine tip speed $= V_B = 2\pi R N$ ← rev/s

R: radius of turbine

$V_B = 2\pi R \frac{n}{60}$ ← rev/min

$n = 60\frac{V_B}{2\pi R}$

multiply by n gives

$$N_s = \frac{60\,V_B}{2\pi R}\sqrt{\frac{P}{H^2\sqrt{H}}}$$

$$N_s = \frac{60\,V_B}{2\pi R}\,51.75\sqrt{\eta}\left(\frac{r}{V_w}\right)$$

$$\boxed{N_s = 494\sqrt{\eta}\left(\frac{r}{R}\right)\left(\frac{V_B}{V_w}\right)}$$ ← text equation with efficiency of turbine η included and more accurate constant

d. A particular case:

$H_t = 3\,m$, $Q = 1.0\ m^3/s$, $P = 20\ kW$

This is a low head, low flow case

Assume $H = H_a = 0.9\,H_t \Rightarrow H = 2.7\,m$

Then $u_j = V_w = \sqrt{2gH} = \underline{\underline{7.278\ m/s}}$

$Q = u_j A_j \Rightarrow A_j = 0.1374\ m^2$

and $A_j = \pi r_j^2 \Rightarrow r_j = r = \underline{\underline{0.2091\,m}}$

$P \times 1000 = \eta\,\rho g Q H = \eta \times 1000 \times 9.81 \times 1 \times 2.7$

$\Rightarrow \underline{\underline{\eta = 0.755}}$

e. Could a Pelton wheel turbine be used?
For a Pelton wheel turbine
$r/R \cong 0.1$ and $\frac{V_B}{V_w} = \frac{u_t}{u_j} = \frac{1}{2}$ for best efficiency

Thus $N_s \cong 494 \sqrt{.755} \times 0.1 \times 0.5$
$N_s \cong 21.5$
According to Table 5.3, p 208
we are in the Pelton wheel
turbine range of $N_s = 10$ to 35

However, with $\frac{r}{R} \cong .1$, we have $R \cong 2m$.
The Pelton wheel diameter $\cong 4$ m. Big
Further
$N_s \cong 21.5 = n\sqrt{\frac{P}{H^2\sqrt{H}}} = n\sqrt{\frac{20}{2.7^2\sqrt{2.7}}}$
$\Rightarrow n \cong 16.6$ rev/min or $N \cong 0.28$ rev/s
We have a big wheel moving fairly slowly. This is not the best situation for steadiness.
Most important, Fig 5.32, p 208 in text indicates we are outside the recommended range. The head is too low. The jet velocity is too low.
Fig 5.32 indicates for the this low head, low Q case a propeller type turbine should be used. By Table 5.3, N_s should be in 350-1000 range. Suppose we select
$n = 600$ rev/min. Then $N_s = n\sqrt{\frac{P}{H^2\sqrt{2.7}}}$
gives $N_s = 775$ and by $N_s = 494\sqrt{\eta}\,\frac{r}{R}\frac{V_B}{V_w}$
we have $\frac{r}{R}\frac{V_B}{V_w} \cong 1.80$. According to Box 5.8, we have a typical Kaplan turbine application – a type of propeller turbine.