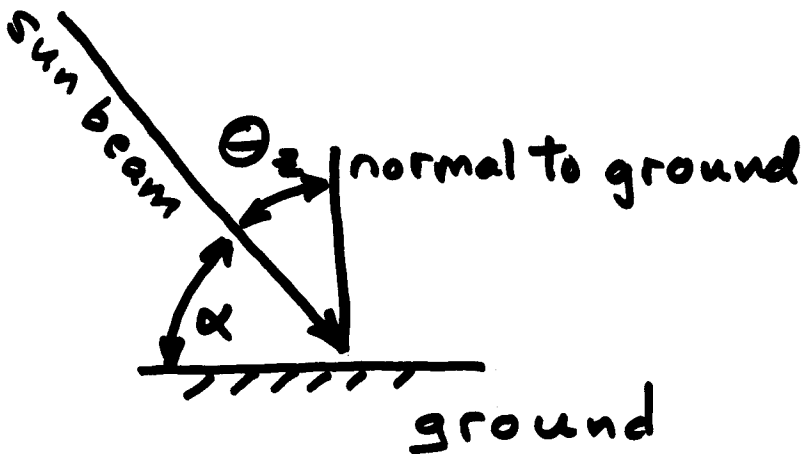
Lecture #4

Solar incidence angle

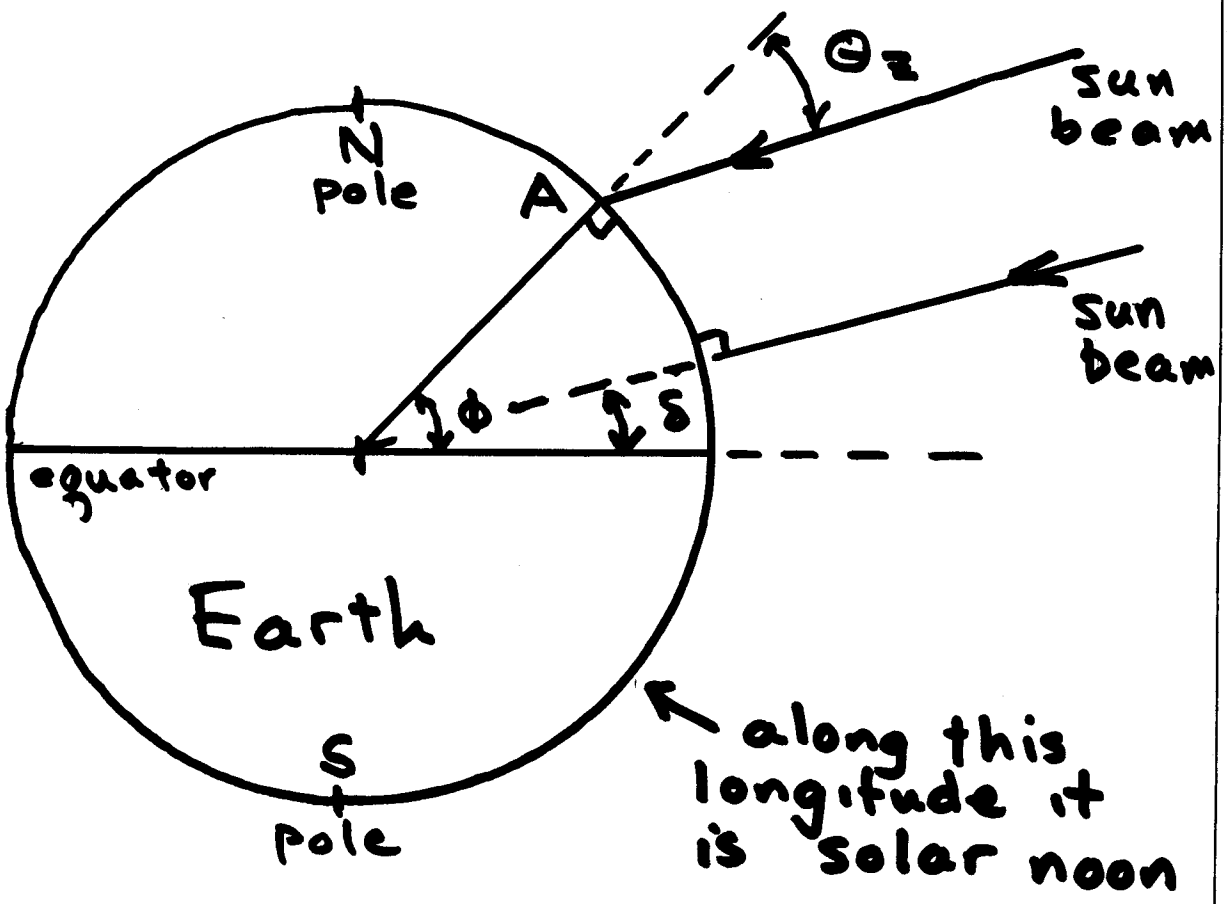


θ_z = solar incidence angle
= angle between sun's rays (a beam) and the perpendicular (a normal) to the ground

α = solar altitude angle
$= 90 - \theta_z$

We need an equation for the solar incidence angle.

Let's start with simple case, and build on that.



The latitude angle of the location of interest (A) is ϕ.

For example, for Seattle, $\phi = 47.5$ degrees.

The declination angle, δ, is the angle between the sun rays and the equation. δ depends on the day of the year.

The range is

$$-23.5^\circ \leq \delta \leq 23.5^\circ$$

↑ South of equator

↑ North of equator

The equation for δ is

$$\delta = 23.5^\circ \sin\left[360^\circ (284+n)/365\right]$$

where n = day of year starting with n=1 for Jan 1.

By this equation:

n = 81 (Mar 22)

$\delta = 0^\circ$ spring equinox

n = 172 (Jun 21)

$\delta = 23.5^\circ$ summer solstice

n = 264 (Sep 21)

$\delta = 0^\circ$ autumn equinox

n = 355, $\delta = -23.5^\circ$, winter solstice

By the figure on page 2:

Along the solar noon trace:

$$\Theta_z = \phi - \delta$$

Thus, for Seattle:

Spring & Autumn Equinox:
$\delta = 0$, $\Theta_z = 47.5°$

Summer Solstice
$\delta = 23.5$, $\Theta_z = 24.0°$

Winter Solstice
$\delta = -23.5$, $\Theta_z = 71.0°$

The cosine of Θ_z gives the component of the sun beam normal to the ground

$$\cos\Theta_z \Big)_{\text{solar noon}} = \sin\phi\sin\delta + \cos\phi\cos\delta$$

The earth makes one revolution about its axis in 24 hr. That is, it rotates 360° in 24 hr, or 15° in 1 hr.

The hour angle is defined relative to solar noon for the longitude of interest.

That is,

$$\omega = 15^\circ t$$

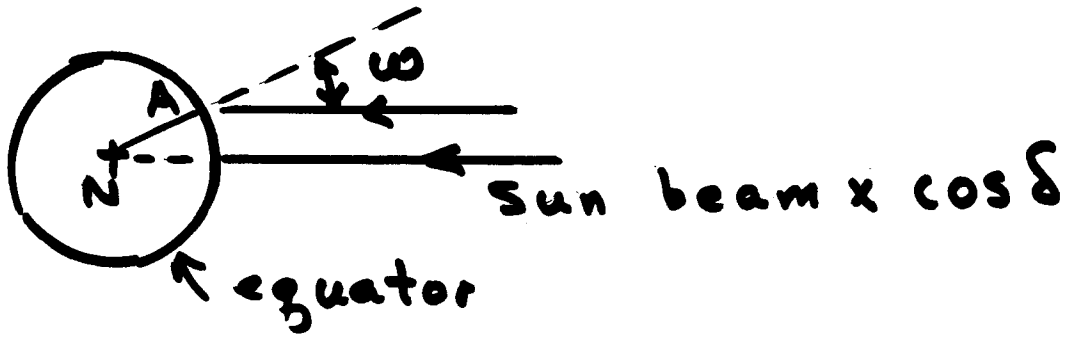
t ← time in hours past solar noon (at solar noon, $\omega = 0$)

Consider a location on the equator, i.e. $\phi = 0$.

Then $\cos\theta_z)_{\substack{\text{solar} \\ \text{noon}}} = \cos\delta$

For another time, for this site on the equator:

$$\cos\Theta_z\Big)_{equator} = \cos\delta\cos\omega$$



The general equation for the solar incidence angle is:

$$\cos\Theta_z = \sin\phi\sin\delta + \cos\phi\cos\delta\cos\omega$$