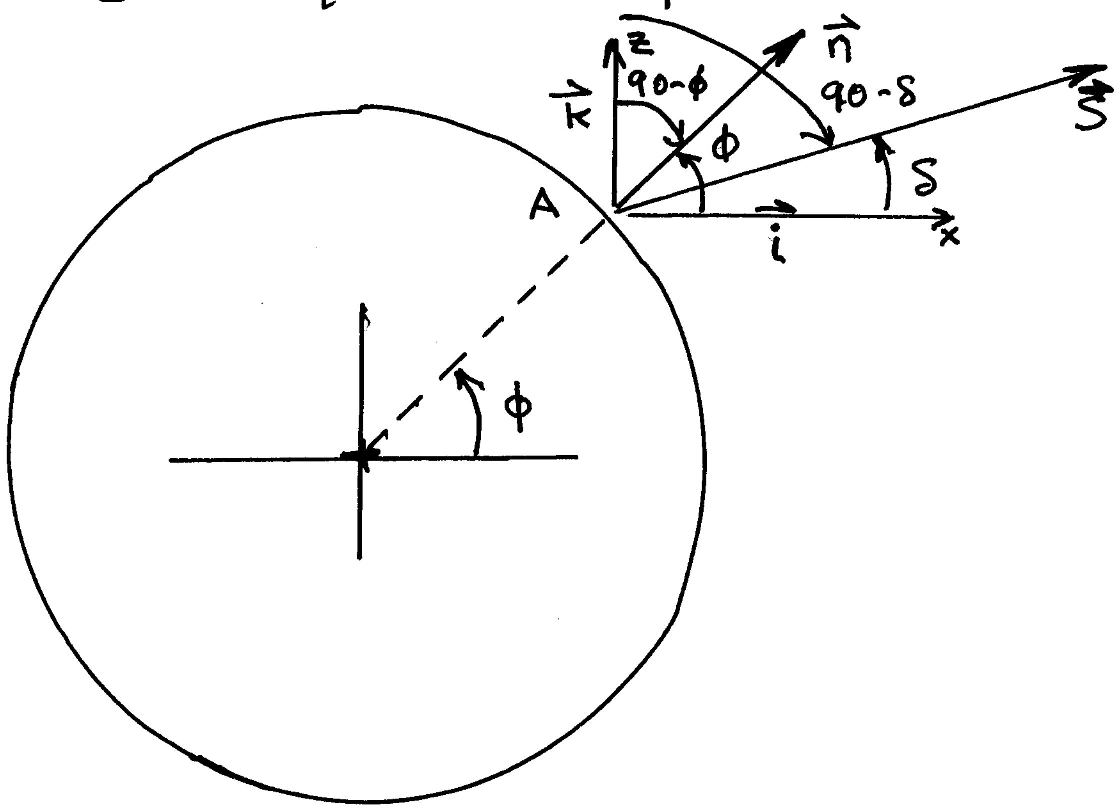
Addendum to Lecture 4

Derivation from scratch of:

$$\cos\theta_z = \sin\phi \sin\delta + \cos\phi \cos\delta \cos\omega$$



The unit vector normal to the ground at the location of interest (A) is

$$\vec{n} = n_x \vec{i} + n_y \vec{j} + n_z \vec{k}$$

The coordinate system is:

x direction is to right from point A

y direction is into the page (to the east)

z direction is vertical from point A

The direction cosines are

n_x, n_y, n_z

$n_x = \cos\phi$

$n_y = 0$

$n_z = \cos(90-\phi)$

Thus

$\vec{n} = \cos\phi\,\vec{i} + \sin\phi\,\vec{k}$

The unit vector from point A to the sun is $\vec{S}$

$\vec{S} = S_x\vec{i} + S_y\vec{j} + S_z\vec{k}$

where S_x, S_y, S_z are the direction cosines

At solar noon

$S_x = \cos\delta$

$S_y = 0$

$S_z = \cos(90-\delta) = \sin\delta$

As the day progresses, the vector $\vec{S}$ rotates about the z direction. Thus, in general

$S_x = \cos\delta\cos\omega$

$S_y = \cos\delta\cos(90-\omega) = \cos\delta\sin\omega$

$S_z = \sin\delta$ (doesn't change)

Thus:

$$\vec{S} = \cos\delta\cos\omega\,\vec{i} + \cos\delta\sin\omega\,\vec{j} + \sin\delta\,\vec{k}$$

The (dot) product of the two unit vectors $\vec{n}$ and $\vec{s}$ gives the cosine of the angle between them. That is:

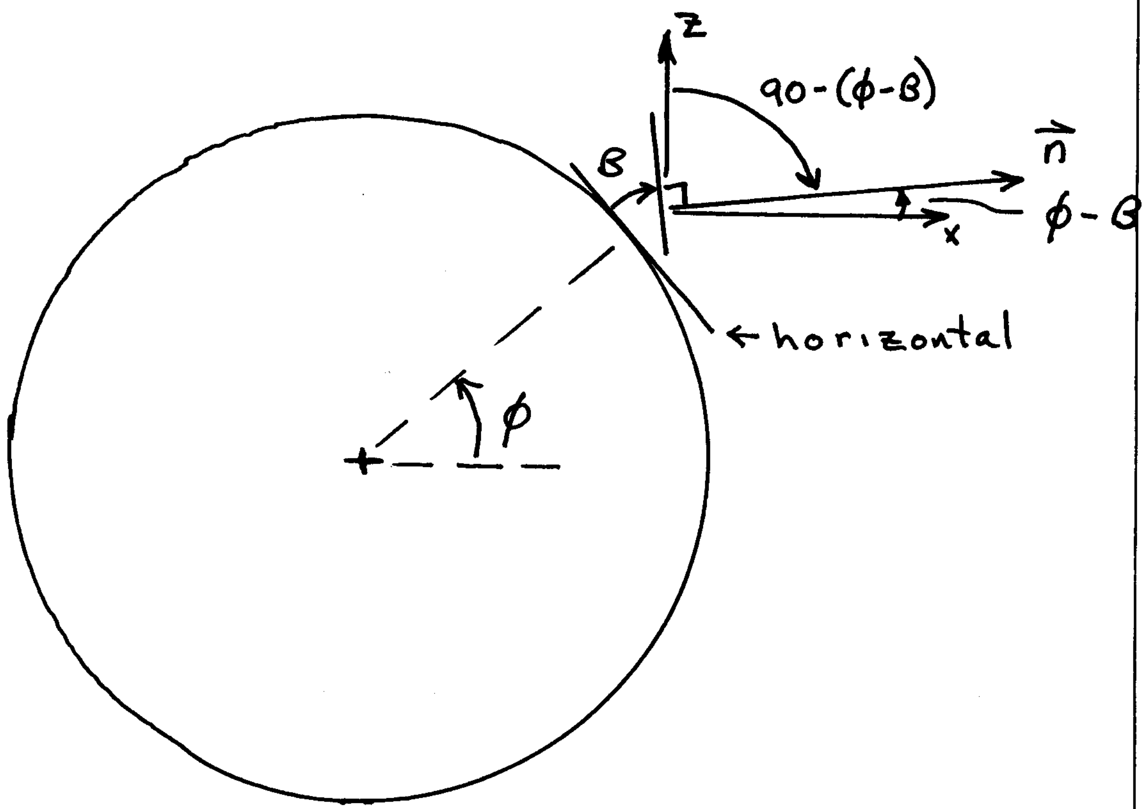
$$\cos\theta_z = \vec{n}\cdot\vec{S} = n_x S_x + n_y S_y + n_z S_z$$

$$= \cos\phi\cos\delta\cos\omega + 0\,(\cos\delta\sin\omega) + \sin\phi\sin\delta$$

$$\cos\theta_z = \sin\phi\sin\delta + \cos\phi\cos\delta\cos\omega$$

This equation is complete, however it is restricted to a horizontal surface on the ground (or parallel to the ground).

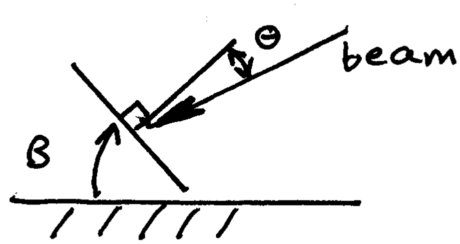
Many times collectors are tilted to face the sun. The tilt angle is β as shown in the picture below



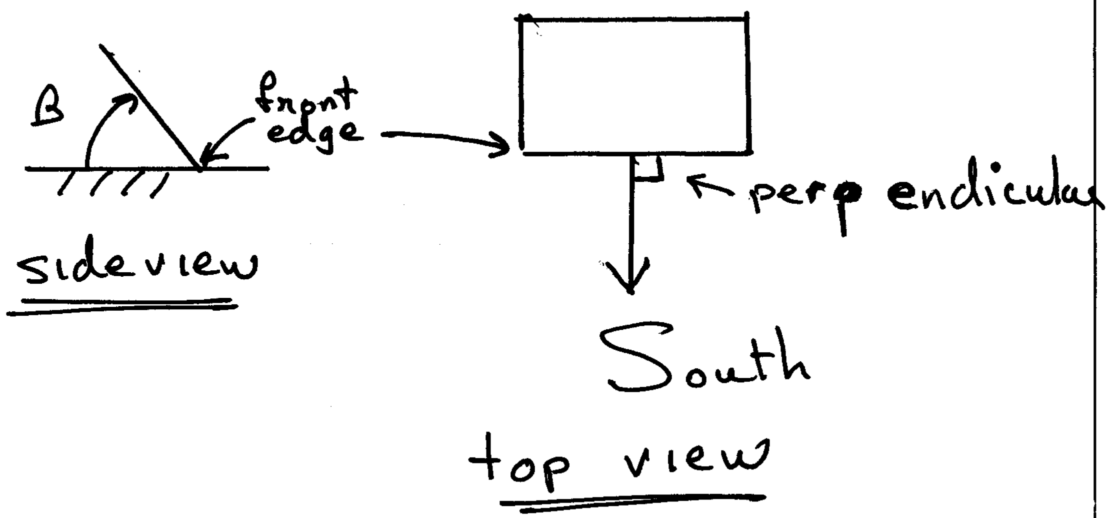
Everything works in our equation if we replace ϕ with $\phi - \beta$. Thus

$$\cos\theta = \sin(\phi-\beta)\sin\delta + \cos(\phi-\beta)\cos\delta\cos\omega$$

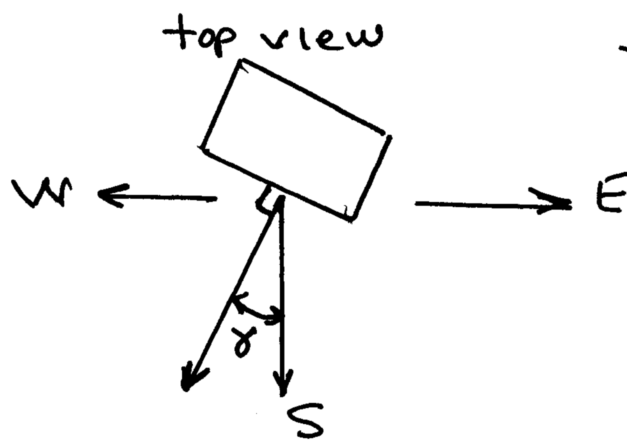
θ is the angle between the perpendicular to the collector and the sun beam



Note, the front edge of the collector is perpendicular to true south (i.e, solar south) — or to true north



Suppose, instead, that the front edge of the tilted collector is not perpendicular to true south. Suppose we have:



$\gamma > 0$ if front edge points west

$\gamma < 0$ if front edge points east

Our complete equation is then:

$$\cos\Theta = A\sin\delta + \cos\delta\sin\beta\sin\gamma\sin\omega + B\cos\delta\cos\omega$$

where:

$$A = \sin\phi\cos\beta - \cos\phi\sin\beta\cos\gamma$$

and

$$B = \cos\phi\cos\beta + \sin\phi\sin\beta\cos\gamma$$