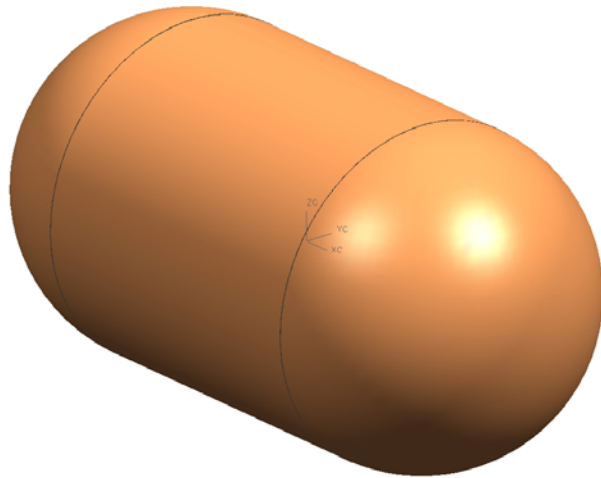
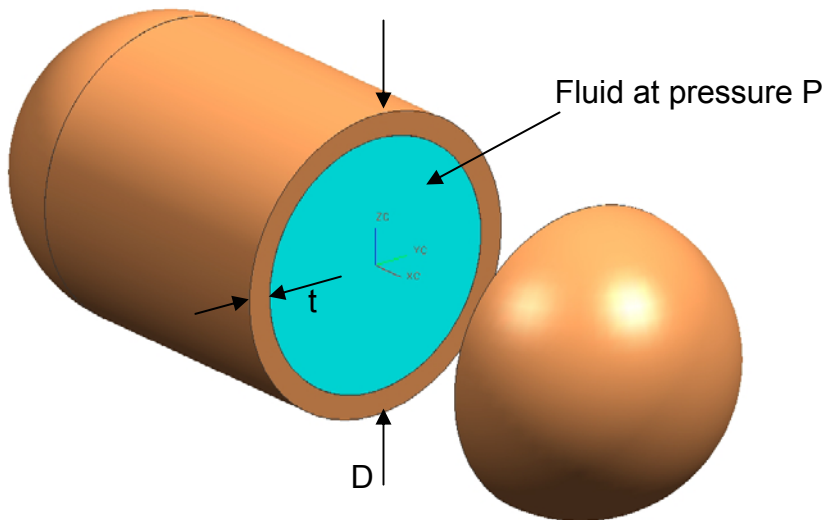
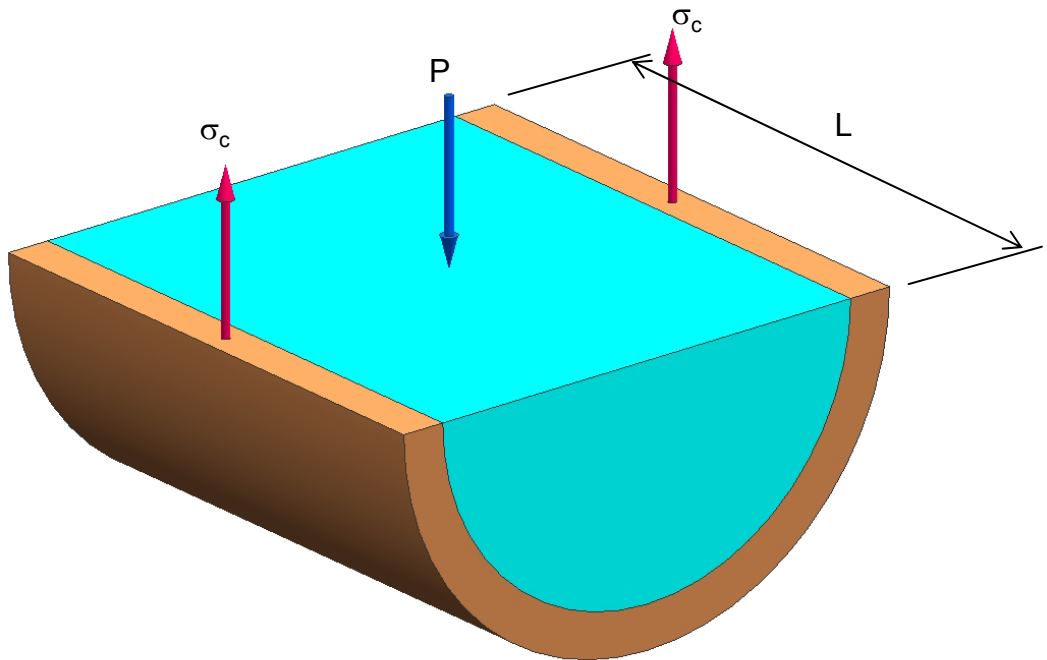


Thin Walled Pressure vessels



The cylindrical pressure vessel above has closed ends and contains a fluid at gauge pressure P as shown below. The outer diameter is D and the wall thickness is t . The term 'thin-wall' may be taken to mean that $D/t > 10$.

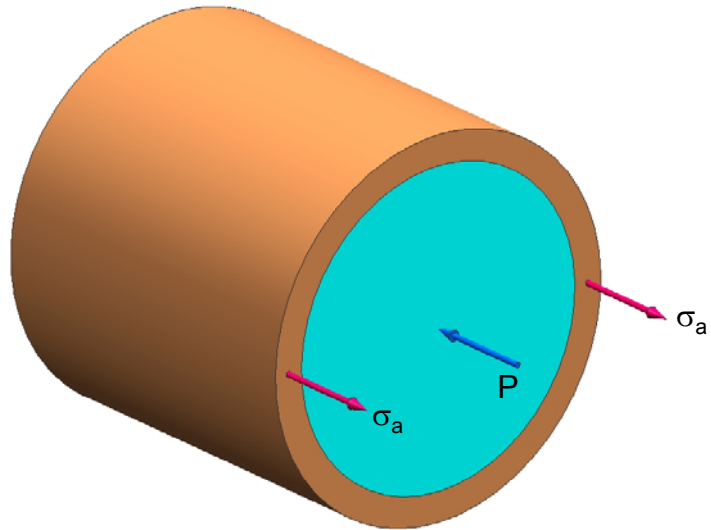




If we section the cylinder, of length L and its contents across its diameter as seen above, we see that we must have equilibrium of the forces due to the internal pressure P and the circumferential stress σ_c in the wall.

$PLD = 2\sigma_c Lt$ or, $\sigma_c = Pr/t$ is the circumferential stress in the wall.

Note that we have assumed that the stress is uniform across the thickness and that we have ignored the fact that the pressure acts on an area defined by the inner diameter. These are only acceptable if $D/t > 10$.



If the cylinder has closed ends, the axial stress σ_a in the wall I found in a similar way by considering a transverse section as shown above. Equilibrium of forces gives:

$$P\pi r^2 = \sigma_a \pi D t \quad \text{and thus the axial stress } \sigma_a = Pr/2t$$

The same assumptions apply.

Note that σ_c and σ_a are principal stresses and remember that the third principal stress $\sigma_3 = 0$. The maximum shear stress is thus

$$\tau_{\max} = |\sigma_1 - \sigma_3|/2 = pr/2t$$

A thin-wall spherical vessel can be analyzed in the same way and it is easily seen that σ_c and σ_a are equal and equal to $pr/2t$. Thus the principal stresses σ_1 and σ_2 are equal and $\sigma_3 = 0$. The maximum shear stress is $\tau_{\max} = |\sigma_1 - \sigma_3|/2 = pr/4t$