

Notes on some relations in plasticity.

The load in a uniaxial tensile test is given by $P = \sigma'A$ and differentiating gives:
 $dP = \sigma'dA + Ad\sigma' = 0$ at the point of maximum load (i.e. the start of neck formation).

Hence, we get $\frac{d\sigma'}{\sigma'} = -\frac{dA}{A}$

Volume is constant during plastic deformation, i.e. $AL = \text{Const}$ and hence,

$$AdL + LdA = 0 \quad \text{or} \quad \frac{dL}{L} = -\frac{dA}{A} = \frac{d\sigma'}{\sigma'} \quad \text{from above.}$$

By definition, the incremental (logarithmic, or true) strain is given by $d\varepsilon' = \frac{dL}{L}$ and,

substituting in the above, we get $\frac{d\sigma'}{d\varepsilon'} = \sigma'$ at the maximum load point.

Given the empirical relation $\sigma' = K\varepsilon'^n$, we get $\frac{d\sigma'}{d\varepsilon'} = nK\varepsilon'^{(n-1)}$ and, combining these

gives $\sigma' = K\varepsilon'^n = \frac{d\sigma'}{d\varepsilon'} = nK\varepsilon'^{(n-1)}$ and, hence we find $\varepsilon' = n$ at the maximum load

point, i.e. the true strain at the point of neck formation is equal to the strain hardening exponent. This is important in the analysis of metal forming operations as this strain is the forming limit of the material, i.e. the point at which deformations become localized, not uniform.