

Thick Cylinder - Shrink Fit Example

A gun barrel has an ID of 150 mm. It is made by shrink fitting an outer sleeve of ID = 190.000 mm and OD of 210.0 mm over an inner sleeve of ID 150.0 mm and OD of 190.125 mm. The maximum pressure seen in the barrel is 50 MPa. The inner sleeve is made from steel with yield strength 320 MPa and the inner sleeve is made from steel with yield strength 670 MPa.

Determine the factor of safety using the von Mises yield criterion.

Determine the minimum temperature difference needed between the sleeves to permit shrink fitting to take place.

Steel properties: $E = 205 \text{ GPa}$. Thermal expansion coefficient = $13 \times 10^{-6}/\text{C}$

- a) find the interface pressure:

$$P_{\text{int}} = 205 \times 10^9 \times 0.000125 \left((0.105^2 - 0.095^2)(0.095^2 - 0.075^2) \right) / (2 \times 0.095^2 (0.105^2 - 0.075^2))$$

$$P_{\text{int}} = 11.29 \text{ MPa}$$

- b) find the stresses in each sleeve due to the interface pressure (all axial stresses = 0)

Inner sleeve: $\sigma_{\text{ci}} = -11.29 \times 2 \times 0.095^2 (0.095^2 - 0.075^2) = -59.94 \text{ MPa}$ (at $r = 75 \text{ mm}$)

$$\sigma_{\text{co}} = -11.29 (0.095^2 + 0.075^2) (0.095^2 - 0.075^2) = -48.65 \text{ MPa}$$
 (at $r = 95 \text{ mm}$)

Outer sleeve:

$$\sigma_{\text{ci}} = 11.29 (0.105^2 + 0.095^2) (0.105^2 - 0.095^2) = 113.19 \text{ MPa}$$
 (at $r = 95 \text{ mm}$)

$$\sigma_{\text{co}} = 11.29 \times 2 \times 0.105^2 (0.105^2 - 0.095^2) = 101.90 \text{ MPa}$$
 (at $r = 105 \text{ mm}$)

- c) find the stresses in the solid cylinder due to the internal pressure (all axial stresses = 0)

At $r = 75 \text{ mm}$ $\sigma_c = 50 (0.105^2 + 0.075^2) / (0.105^2 - 0.075^2) = 154.17 \text{ MPa}$

$$\sigma_r = -50 \text{ MPa}$$

At $r = 95 \text{ mm}$ $\sigma_c = 50 \times 0.095^2 (1 + 0.105^2 / 0.095^2) / (0.105^2 - 0.075^2) = 115.71 \text{ MPa}$

$$\sigma_r = 50 \times 0.075^2 (1 - 0.105^2 / 0.095^2) / (0.105^2 - 0.075^2) = -11.54 \text{ MPa}$$

At $r = 105 \text{ mm}$ $\sigma_c = 2 \times 50 \times 0.075^2 / (0.105^2 - 0.075^2) = 104.17 \text{ MPa}$

$$\sigma_r = 0 \text{ MPa}$$

- d) find the final stresses in the sleeves due to the internal pressure plus the shrink fit using linear superposition (all axial stresses = 0)

At $r = 75 \text{ mm}$ on inner sleeve $\sigma_c = 154.17 - 59.94 = 94.23 \text{ MPa}$

$$\sigma_r = -50 + 0 = -50 \text{ MPa}$$

At $r = 95 \text{ mm}$ on inner sleeve $\sigma_c = 115.71 - 48.65 = 67.06 \text{ MPa}$

$$\sigma_r = -11.29 - 11.54 = -22.83 \text{ MPa}$$

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At $r = 95$ mm on outer sleeve $\sigma_c = 115.71 + 113.19 = 228.90$ MPa

$$\sigma_r = -11.29 - 11.54 = -22.83 \text{ MPa}$$

At $r = 105$ mm on outer sleeve $\sigma_c = 104.17 + 101.90 = 206.07$ MPa

$$\sigma_r = 0 + 0 = 0 \text{ MPa}$$