ME 354, MECHANICS OF MATERIALS LABORATORY

NOTES on Strain Gages

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RESISTANCE FOIL STRAIN GAGES

In 1856 Lord Kelvin reported that the electrical resistance of copper and iron wires increased when subjected to tensile stresses. This observation ultimately led to the development of the modern "strain gage" independently at California Institute of Technology and Massachusetts Institute of Technology in 1939. The underlying concept of the strain gage is very simple. In essence, an electrically-conductive wire or foil (i.e. the strain gage) is bonded to the structure of interest and the resistance of the wire or foil is measured before and after the structure is loaded. Since the strain gage is firmly bonded to the structure, any strain induced in the structure by the loading is also induced in the strain gage. This causes a change in the strain gage resistance thus serving as an indirect measure of the strain induced in the structure.

Originally, strain gages were made of wire and, in fact, wire strain gages are still in use under special circumstances. However, today foil strain gages are most widely used. A typical strain gage is shown in the sketch below. The strain sensing region of the strain gage is called the "gage grid." The grid is etched from a thin metallic foil. The orientation of the grid defines the strain sensing axis of the strain gage. Electrical connections are made by soldering lead wires to the strain gage "solder tabs." The entire strain gage is bonded to a thin polymeric backing which helps protect and support the delicate metal foil.

Foil strain gages are available in literally hundreds of shapes and sizes. The strain gage shown is called a "uniaxial strain gage." Other common strain gage configurations are:

Biaxial strain gages which consist of two individual strain gage elements oriented precisely 90° apart, allowing strain measurements in two orthogonal directions.

Rectangular, three-element strain gage rosettes which consist of three individual strain gage elements oriented precisely 45° apart, allowing the resolution of principal strains and principal directions regardless of the orientation of the rosette or the applied stress/strain.

Delta, three-element strain gage rosettes which consist of three individual strain gage elements oriented precisely 60° apart, allowing the resolution of principal strains and principal directions regardless of the orientation of the rosette or the applied stress/strain.

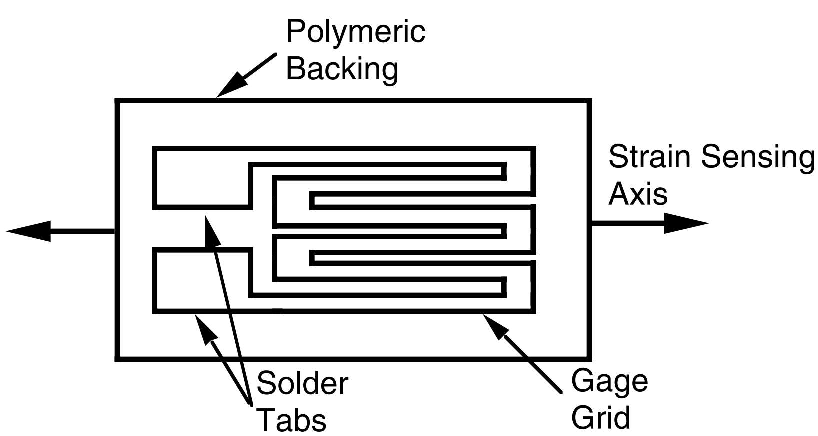

FIGURE 1 - Illustration of a typical, uniaxial resistance foil strain gage

STRAIN GAGE RESISTANCE

Strain gage manufacturers produce strain gages with three standard resistances: 120 Ω, 350 Ω, and 1000 Ω. The user specifies the desired resistance when ordering the strain gage. The 120 Ω resistance is the most commonly used, although 350 Ω and 1000 Ω strain gages are widely available.

As discussed previously, strains are sensed by bonding a strain gage to a structure of interest and subsequently measuring the strain gage resistance before and after the structure is loaded. Consider the magnitude of the resistance change which must typically be measured. Assume a measurement resolution of 10×10^{-6} m/m = 10 μm/m is required (a typical measurement). The change in resistance which corresponds to a strain of 10 μm/m can be calculated using Eq. 1:

$$\Delta R_g = (F_g)(R_g)(\varepsilon_m) = (2.00)(120\Omega)(10x10^{-6}\, m/m) = 0.0024\Omega \qquad (1)$$

where ΔR_g is the change in resistance, F_g is the gage factor, R_g is the initial gage resistance, and ε_m is the strain in the strain gage. Thus, a resistance change from 120 Ω to 120.0024 Ω must measured...a very small change indeed!!! In fact, it is very difficult to measure such small changes in resistance using "normal" ohmmeters. Instead, special "strain gage circuits" are used to measure these small resistance changes accurately and precisely. The most widely used strain gage circuit is the "Wheatstone bridge" and is described in the following section.

THE WHEATSTONE BRIDGE

As show in Fig. 2, the Wheatstone bridge circuit consists of four "arms." Each arm contains a resistance (i.e. resistances, R_1, R_2, R_3, and R_4). An excitation voltage V_{ex} (typically 2 to 10 volts) is applied across junction A-C and a voltmeter is used to measure the resulting potential across junctions B-D (voltage ΔE). If all the resistances are equal (i.e. $R_1 = R_2 = R_3 = R_4$) then $\Delta E = 0$ and the bridge is said to be balanced.

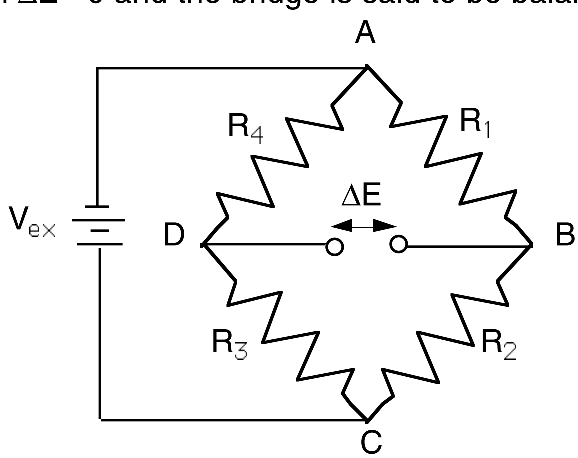

FIGURE 2 - Schematic Diagram of a Wheatstone Bridge

Typically, a 1/4 (quarter) arm Wheatstone bridge circuit is used for individual strain gages where resistance R_1 is the strain gage (i.e., $R_1 = R_g$) and the other three resistances are precision resistors equal to the nominal resistance of R_g (e.g. $R_2 = R_3 = R_4 = 120\ \Omega$)[1] . If the strain gage experiences a strain, the strain gage resistance changes, causing the bridge to become unbalanced. The resulting voltage, ΔE is given by:

$$\Delta E = \frac{V_{ex}}{4}\left[\frac{\Delta R_g}{R_g}\right] \qquad (2).$$

Combining Eqs. 1 and 2 yields:

$$\varepsilon_m = \frac{4}{F_g}\left[\frac{\Delta E}{V_{ex}}\right] \qquad (3)$$

Equation 3 is an important result. It shows that the strain in the strain gage, ε_m, is related to the quantities, F_g, ΔE, and V_{ex}. Generally though, Eq. 3 is not applied directly, Instead, strain gage amplifiers which have been calibrated according to Eq. 3 are used to provide a direct readout of strain.

THE GAGE FACTOR

Strain gage manufacturers perform standard calibration measurements for each lot of strain gages they produce. When a user purchases a strain gage, the manufacturer provides the results of these measurements in the form of several calibration constants. One of these constants is the "gage factor." The gage factor allows the user the convert the change in gage resistance to the corresponding strain level. Specifically, the strain measured in an individual strain gage is related to the change in the strain gage resistance such that:

$$\varepsilon_m = \frac{1}{F_g}\left[\frac{\Delta R_g}{R_g}\right] \qquad (4)$$

The value of the gage factor depends on the metallic alloy used and varies slightly from lot to lot, typically being in the range of 2.0 to 2.1.

THREE-ELEMENT STRAIN GAGE ROSETTES

Three-element strain gage rosettes are used when it is desired to measure ε_x, ε_y, and γ_{xy}, induced at a point (or equivalently, when the principal strains and principal directions are unknown). Referring to the x-y coordinate system in Fig. 3, recall that the normal strain induced in an arbitrary direction from the x-axis defined by angle θ (strain ε_θ) is related to the strains in the x-y coordinate system according to:

$$\varepsilon_\theta = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\cos\theta\sin\theta \qquad (5)$$

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[1] Actually, the resistances need not be identical. Instead, all that is required is that $(R_1/R_2) = (R_3/R_4)$. However, for the present purposes, it is sufficient to assume $(R_1=R_2= R_3=R_4)$

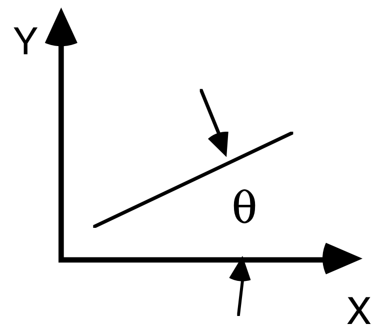


FIGURE 3 - Single Strain Arbitrarily Oriented to the X-Y Coordinate System.

The strain, ε_θ, can be measured by simply mounting a strain gage in the direction defined by angle θ. In solving Eq. 5, ε_θ and θ are known but there are still three unknowns, ε_x, ε_y, and γ_{xy}. Thus, to solve for the unknowns, two more equations are required for a total of three equations (i.e. three equations, three unknowns). If a total of three strain gages are mounted in three distinct but arbitrary directions (θ_1, $\theta 2$, and θ_3) as shown in Fig. 4, then Eq. 5 can be applied three times such that:

$$\varepsilon_{\theta 1} = \varepsilon_x \cos^2\theta_1 + \varepsilon_y \sin^2\theta_1 + \gamma_{xy}\cos\theta_1\sin\theta_1 \tag{6a}$$

$$\varepsilon_{\theta 2} = \varepsilon_x \cos^2\theta_2 + \varepsilon_y \sin^2\theta_2 + \gamma_{xy}\cos\theta_2\sin\theta_2 \tag{6b}$$

$$\varepsilon_{\theta 3} = \varepsilon_x \cos^2\theta_3 + \varepsilon_y \sin^2\theta_3 + \gamma_{xy}\cos\theta_3\sin\theta_3 \tag{6c}$$

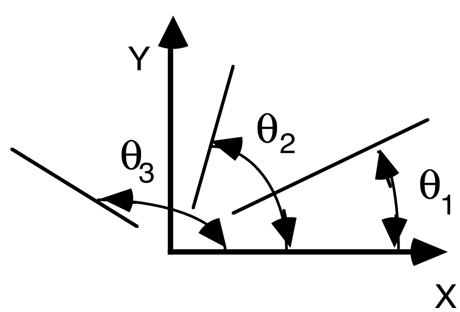


FIGURE 4 - Three Strains Arbitrarily Oriented to the X-Y Coordinate System.

Equations 6a-6c are called the general rosette equations. If θ_1 is set equal to 0°, θ_2 to 45° and θ_3 to 90° the resulting strain gage configuration is called a rectangular rosette as shown in Fig. 5a. The resulting equations for a rectangular rosette are:

$$\varepsilon_{0^\circ} = \varepsilon_x\cos^2(0^\circ) + \varepsilon_y\sin^2(0^\circ) + \gamma_{xy}\cos(0^\circ)\sin(0^\circ) \tag{7a}$$

$$\varepsilon_{45^\circ} = \varepsilon_x\cos^2(45^\circ) + \varepsilon_y\sin^2(45^\circ) + \gamma_{xy}\cos(45^\circ)\sin(45^\circ) \tag{7b}$$

$$\varepsilon_{90^\circ} = \varepsilon_x\cos^2(90^\circ) + \varepsilon_y\sin^2(90^\circ) + \gamma_{xy}\cos(90^\circ)\sin(90^\circ) \tag{7c}$$

which can be reduced to:

$$\varepsilon_x = \varepsilon_{0^\circ} \tag{8a}$$

$$\varepsilon_y = \varepsilon_{90^\circ} \tag{8b}$$

$$\gamma_{xy} = 2\varepsilon_{45^\circ} - (\varepsilon_{0^\circ} + \varepsilon_{90^\circ}) \tag{8c}$$

Similarly, if θ_1 is set equal to 0°, θ_2 to 60° and θ_3 to 120°, the resulting strain gage configuration is called a delta rosette as shown in Fig. 5b. The resulting equations for a delta rosette are:

$$\varepsilon_{0^\circ} = \varepsilon_x\cos^2(0^\circ) + \varepsilon_y\sin^2(0^\circ) + \gamma_{xy}\cos(0^\circ)\sin(0^\circ) \tag{9a}$$

$$\varepsilon_{60^\circ} = \varepsilon_x\cos^2(60^\circ) + \varepsilon_y\sin^2(60^\circ) + \gamma_{xy}\cos(60^\circ)\sin(60^\circ) \tag{9b}$$

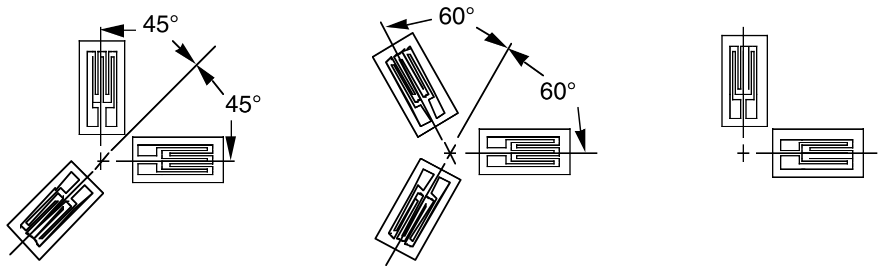
$$\varepsilon_{120^\circ} = \varepsilon_x\cos^2(120^\circ) + \varepsilon_y\sin^2(120^\circ) + \gamma_{xy}\cos(120^\circ)\sin(120^\circ) \tag{9c}$$

which can be reduced to:

$$\varepsilon_x = \varepsilon_{0^\circ} \tag{10a}$$

$$\varepsilon_y = \frac{1}{3}\left[2(\varepsilon_{60^\circ} + \varepsilon_{120^\circ}) - \varepsilon_{0^\circ}\right] \tag{10b}$$

$$\gamma_{xy} = \frac{2\sqrt{3}}{3}\left[\varepsilon_{60^\circ} - \varepsilon_{120^\circ}\right] \tag{10c}$$



a) Rectangular Rosette b) Delta Rosette c) Biaxiial Strain Gage

FIGURE 5 - Rectangular and Delta Rosettes as well as a Biaxial Strain Gage.