Lecture 4 Oct 3, 2018

$m\vec{a} = \sum F_i$

$\rho \frac{D\vec{V}}{Dt} d\forall = \sum_i F_i$

1) Body forces → proportional to mass

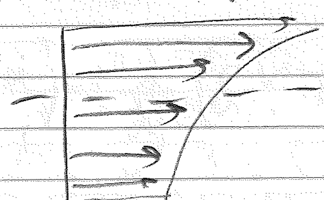
e.g. gravity, magnetic/electric

only gravity $\vec{F}_B = m\vec{g}$ $\vec{g} = (0, 0, -g)$

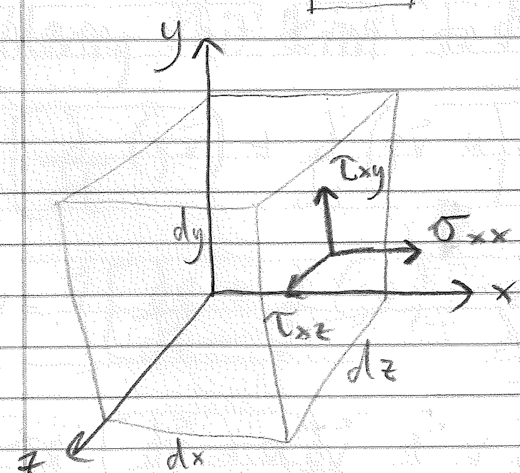
C.V. $F_B = \iiint_{C.V.} \rho \vec{g} \, d\forall = \rho \vec{g} \, dx dy dz$

2) Surface forces → proportional to surface area

e.g.



"sliding" ⇒ shear forces



x-surface: τ_{xi} ← unit normal of surface; i ← direction of stress

σ_{xx}

Each surface
3 components
↳ 1 normal σ
↳ 2 tangential τ

9 component matrix (tensor)

$$\underline{\underline{\sigma}} = \begin{pmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{pmatrix} \leftarrow x \text{ surface}$$

surface stress

x direction

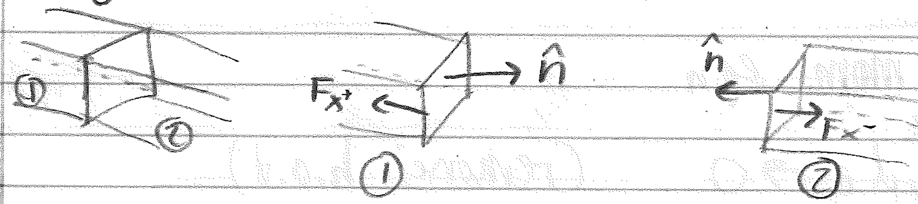
(1) $\tau_{xy}\, dydz$: force in y direction on surface of area $dydz$ with normal $\hat{n} = \hat{x}$

(2) diagonal : $\sigma_{xx}, \sigma_{yy}, \sigma_{zz} \Rightarrow$ normal stresses

off diagonal: $\tau_{ij} \Rightarrow$ shear stresses

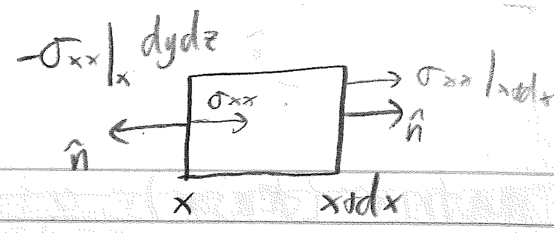
(3) sign convention: a component of $\underline{\sigma}$ is positive if the force vector component and the area normal ($\hat{n}$) are in the same direction (both pos. or both neg.)

e.g. $\sigma_{xx} < 0$



$F_{x^-} = -F_{x^+}$ 3rd law

Note: pressure applies inward, normal is outward $\Rightarrow -p$ always



6 surfaces:

x direction

The force on the left face:

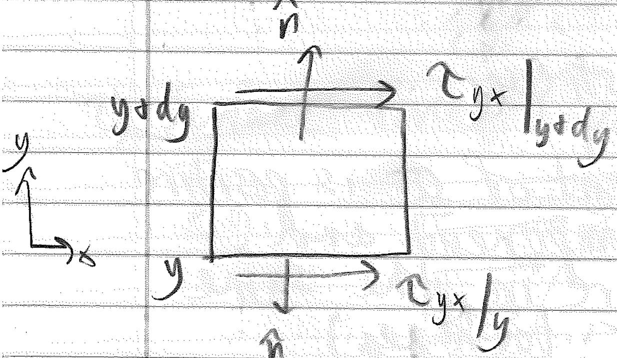
$$-\sigma_{xx}\Big|_x dydz$$

" " right face:

$$\sigma_{xx}\Big|_{x+dx} dydz = \left[\sigma_{xx}\Big|_x + \frac{d\sigma_{xx}}{dx}dx\right] dydz$$

Combining:

$$\frac{\partial \sigma_{xx}}{\partial x} dxdydz$$



$$\left(\frac{\partial \tau_{yx}}{\partial y} dy\right) dxdz$$

$$\left(\frac{\partial \tau_{zx}}{\partial z} dz\right) dxdy$$

x-comp of mom. eqn

$\div dxdydz$

$dx, dy, dz \to 0$ (removes h.o.t)

$$\underbrace{\rho \frac{Du}{Dt}}_{\text{rate of change that follows a fluid particle}} = \underbrace{\rho g_x}_{\text{body force}} + \underbrace{\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}}_{\text{surface forces}}$$

$$\rho \frac{Dv}{Dt} = \rho g_y + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z}$$

$$\rho \frac{Dw}{Dt} = \rho g_z + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z}$$

Equations

$$\left.\begin{array}{ll} \text{conservation of mass} & 1\text{ eqn} \\ \text{momentum balance} & 3\text{ eqn} \\ \text{conservation of energy} & 1\text{ eqn} \\ \text{(angular momentum balance)} & 3\text{ eqns} \\ \quad\hookrightarrow \text{leads to symmetry of stress tensor} & \end{array}\right\} 8\text{ eqns}$$

Unknowns: $u, v, w, \rho, \sigma_{xx}, \tau_{xy}, \tau_{xy} \ldots \Rightarrow$ 13 unknowns

Properties of a fluid

1) Constitutive eqn (mechanical properties)

2) Gas law - thermodynamics