lecture 5

Oct 5, 2018

→ Fluids cannot support a shear stress without continuously deforming.

→ Static fluids can only support normal stresses

Normal stress

1) compressive (opposite to the outward normal)

2) isotropic (same for any surface oriented in any direction)

3) usual static pressure

$\therefore \quad \sigma_{xx} = \sigma_{yy} = \sigma_{zz} = -P$ when the fluid is static

$\tau_{xy} = \tau_{yz} = \ldots = 0$ i.e. shear stresses are zero

The same pressure is assumed to exist for a moving fluid.

take it at $\quad \underline{\underline{\sigma}} = \begin{pmatrix} -P+\tau_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & -P+\tau_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & -P+\tau_{zz} \end{pmatrix}$

$\sigma_{xx} = -P + \tau_{xx}$

$\underline{\underline{\tau}}$ is the remainder of the surface stresses that are due to viscosity

For solid ⇝ strain e.g. Hooke's law: stress $\propto$ strain

For fluid ⇝ stress $\propto$ rate of strain

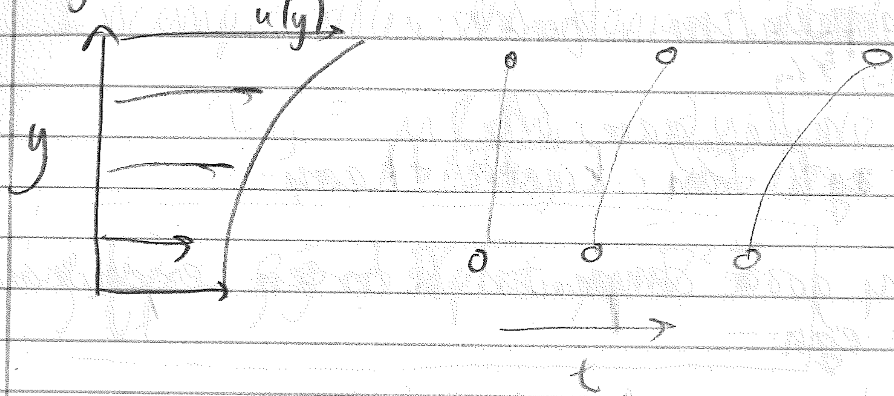
i.e. rate of change of relative displacement

$\iint \vec{V} \cdot d\vec{A}$ $\quad \frac{m^3}{s}$

$\vec{V}$

$d\vec{A} = dA \cdot \hat{n}$

e.g. shear flow



rate of strain is related to $\frac{du}{dy}$

Newtonian fluids: stress is a linear fn of rate of strain

For incompressible fluids → relationship between $\underline{\tau}$ and $\vec{v}$

$$\underbrace{\tau_{xx} = 2\mu \frac{\partial u}{\partial x}}_{\text{normal rate of strain}} \qquad \tau_{yy} = 2\mu \frac{\partial v}{\partial y} \qquad \tau_{zz} = 2\mu \frac{\partial w}{\partial z}$$

$$\tau_{xy} = \tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$\tau_{xz} = \tau_{zx} = \mu\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right)$$

$$\tau_{yz} = \tau_{zy} = \mu\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)$$

$\underline{\tau}$ is a symmetric tensor

$$\frac{\mu\partial}{\partial x}\left(2\frac{\partial u}{\partial x}\right) = 2\mu\frac{\partial^2 u}{\partial x^2}$$

$$\frac{\mu\partial}{\partial y}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) = \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 v}{\partial x \partial y}$$

$$\frac{\mu\partial}{\partial z}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right) = \frac{\partial^2 w}{\partial x \partial z} + \frac{\partial^2 u}{\partial z^2}$$

$$\mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) + \frac{\partial}{\partial x}\underbrace{\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right)}_{\nabla \cdot v = 0}$$

* symmetric nature of τ comes from angular momentum balance

* in case of simple gases (H_2)
  ↳ derive eqn from kinetic theory

* generally, very good comparison between experiments and the eqn

⇒ plug that into momentum eqn
  ↳ $M_a < 0.2$

$\rho = const.$
$\mu = const.$

$$\rho \frac{Du}{Dt} d\forall = \rho g_x + \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z}$$

$$x: \rho \frac{Du}{Dt} = \rho g_x - \frac{\partial p}{\partial x} + \mu \underbrace{\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right)}_{\nabla^2 u}$$

$$\nabla \cdot (\nabla u) = \nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$$

$$y: \rho \frac{Dv}{Dt} = \rho g_y - \frac{\partial p}{\partial y} + \mu \nabla^2 v$$

$$z: \rho \frac{Dw}{Dt} = \rho g_z - \frac{\partial p}{\partial z} + \mu \nabla^2 w$$