Lecture 6 — Oct 8, 2018

Incompressible Newtonian fluids

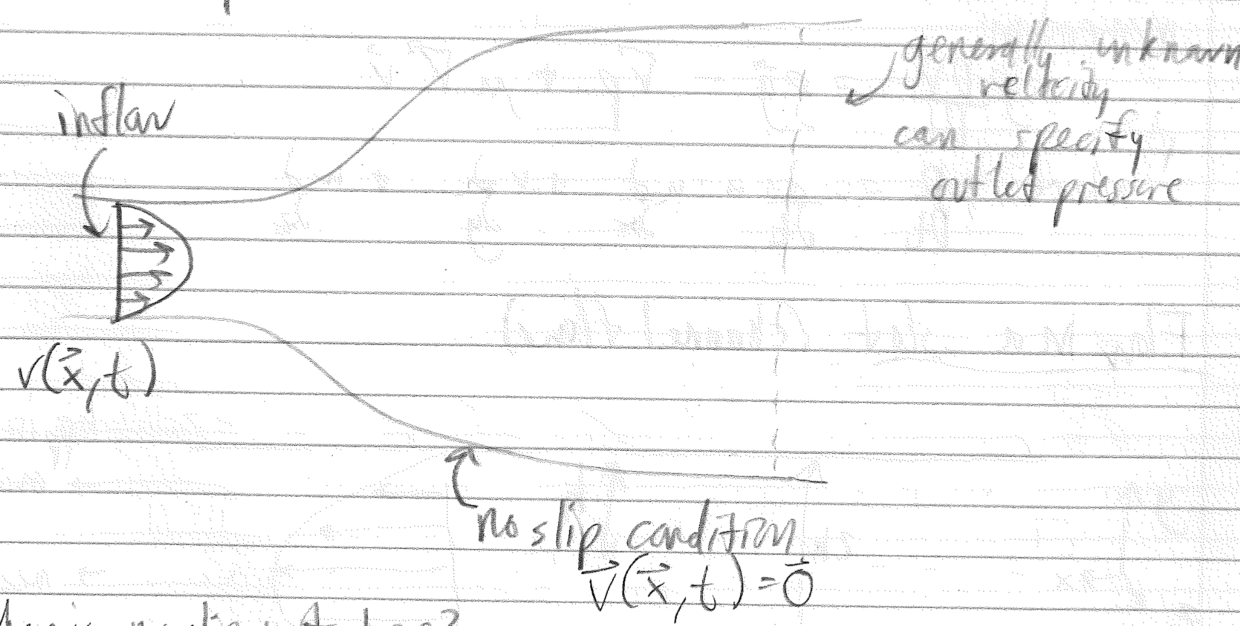
$$\tau_{ij} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

$$\boxed{\rho \frac{D\vec{v}}{Dt} = \rho\vec{g} - \nabla p + \mu \nabla^2 \vec{v}} \qquad \boxed{\nabla \cdot \vec{u} = 0}$$

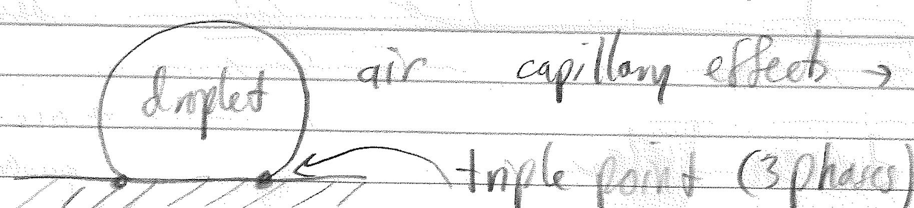
4 eqns + initial + boundary conditions

* Initial conditions → needed for unsteady flow $\frac{d}{dt} \neq 0$
  → guess for steady flow simulation

* Boundary conditions



When is no slip not true?



capillary effects → $\vec{v}_w \neq 0$

Challenges in simulations:

→ setting proper BCs

→ nonlinear terms i.e. unknown × unknown
e.g. $u\frac{\partial u}{\partial x}$, $v\frac{\partial u}{\partial y}$, $w\frac{\partial v}{\partial z}$ etc.

Review

continuity:

$$\frac{\partial \rho}{\partial t} + \nabla\cdot\vec{v} = 0 \qquad \text{incompressible} \Rightarrow \nabla\cdot\vec{v} = 0$$

stream function Ψ:

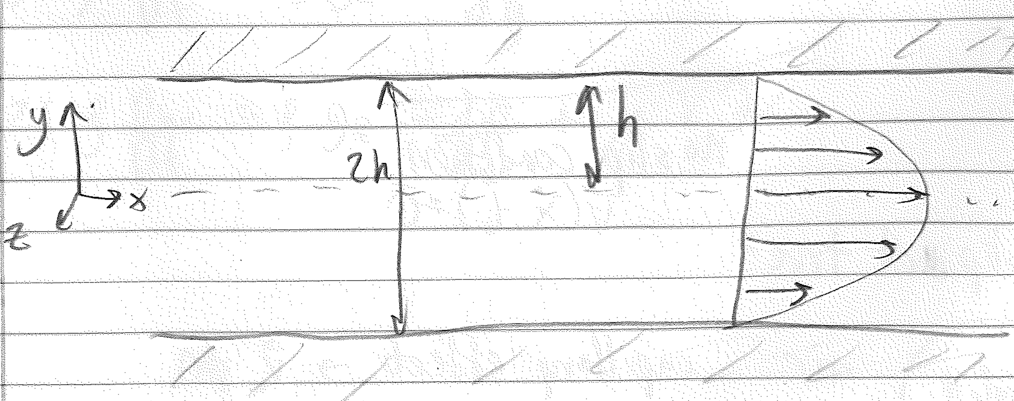
$$u = \frac{\partial \Psi}{\partial y} \qquad v = -\frac{\partial \Psi}{\partial x}$$

momentum:

$$\rho\frac{D\vec{v}}{Dt} = \rho\vec{g} - \nabla p + \mu\nabla^2\vec{v}$$

where $$\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}$$

Flow in a slot (channel flow) Smits 8.5.1



Applications
→ microfluidics
→ mechanical electromagnetic devices
→ oil between piston + cylinder

Pressure driven flow

Two dimensional: $\frac{\partial}{\partial z} = 0$, $w = 0$

Steady: $\frac{\partial}{\partial t} = 0$

Laminar: $Re < 2000$

Incompressible: $\nabla \cdot \vec{V} = 0$

Fully developed: far from the inlet $\therefore \frac{\partial \vec{V}}{\partial x} = 0$

Neglecting gravity: $\rho \vec{g} = 0$