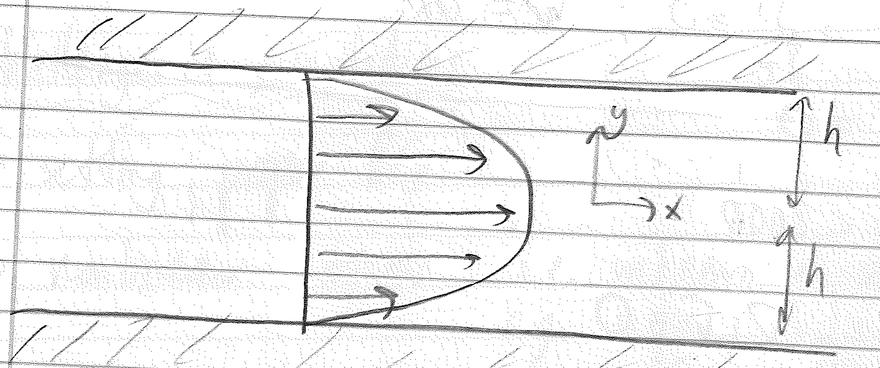
Lecture 7

Oct 10, 2018



Continuity

$$\underbrace{\cancel{\frac{\partial u}{\partial x}}}_{\text{fully developed}} + \frac{\partial v}{\partial y} + \underbrace{\cancel{\frac{\partial w}{\partial z}}}_{\text{2D}} = 0$$

$$\frac{\partial v}{\partial y} = 0 \Rightarrow v(y) = C$$

no slip BC: $v(y = \pm h) = 0$

$$\vec{V} = (u(y), 0, 0)$$

x: $$\rho\left(\underbrace{\cancel{\frac{\partial u}{\partial t}}}_{\text{steady}} + \underbrace{\cancel{u\frac{\partial u}{\partial x}}}_{\text{fully dev}} + \underbrace{\cancel{v\frac{\partial u}{\partial y}}}_{v=0} + \underbrace{\cancel{w\frac{\partial u}{\partial z}}}_{w=0}\right) = -\frac{\partial p}{\partial x} + \mu\left(\underbrace{\cancel{\frac{\partial^2 u}{\partial x^2}}}_{\text{fully dev}} + \frac{\partial^2 u}{\partial y^2} + \underbrace{\cancel{\frac{\partial^2 u}{\partial z^2}}}_{\text{2D}}\right) + \underbrace{\cancel{\rho g_x}}_{\text{no body force}}$$

y: $$\rho\left(\cancel{\frac{\partial v}{\partial t}} + \cancel{u\frac{\partial v}{\partial x}} + \cancel{v\frac{\partial v}{\partial y}} + \cancel{w\frac{\partial v}{\partial z}}\right) = -\frac{\partial p}{\partial y} + \mu\left(\cancel{\frac{\partial^2 v}{\partial x^2}} + \cancel{\frac{\partial^2 v}{\partial y^2}} + \cancel{\frac{\partial^2 v}{\partial z^2}}\right) + \cancel{\rho g_y}$$

$v = 0$

$$\frac{\partial p}{\partial y} = 0$$

pressure is uniform along cross section

$$\therefore \quad p = p(x)$$

x: $\quad \frac{dp}{dx} = \mu \frac{d^2u}{dy^2} = \text{constant}$

$\frac{dp}{dx} = \text{constant} < 0$ for flow in $+x$ direction
i.e. P must linearly decrease

$$\frac{d^2u}{dy^2} = \frac{1}{\mu}\frac{dp}{dx}$$

$$\frac{du}{dy} = \frac{1}{\mu}\frac{dp}{dx}y + A$$

$$u(y) = \frac{1}{2\mu}\frac{dp}{dx}y^2 + Ay + B$$

no slip: $u(y=\pm h) = 0$

$$\frac{1}{2\mu}\frac{dp}{dx}h^2 + Ah + B = 0$$

$$-\left(\frac{1}{2\mu}\frac{dp}{dx}(-h)^2 - Ah + B = 0\right)$$

$$2Ah = 0$$

$$A = 0$$

Σ eqns: $\quad \frac{1}{\mu}\frac{dp}{dx}h^2 + 2B = 0$

$$B = \frac{-h^2}{2\mu}\frac{dp}{dx}$$

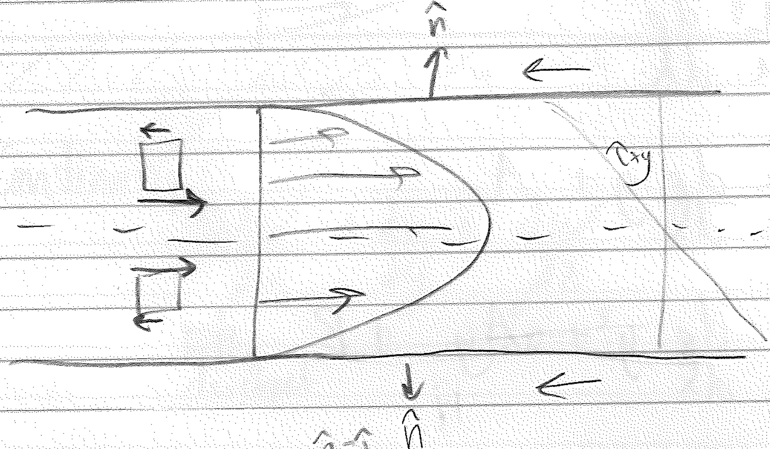
$$u(y) = \frac{h^2}{2\mu}\frac{dP}{dx}\left(\frac{y^2}{h^2} - 1\right)$$

$$= u_m\left(1 - \frac{y^2}{h^2}\right) \qquad u_m = \frac{-h^2}{2\mu}\frac{dP}{dx} > 0$$

There is only one shear stress:

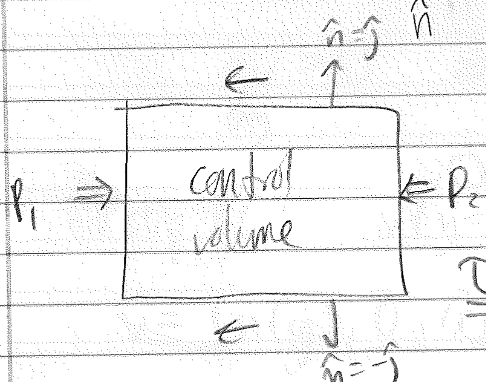
$$\tau_{xy} = \tau_{yx} = \mu\left(\frac{du}{dy} + \frac{dv}{dx}\right) = -2\mu U_m \frac{y}{h^2}$$

$$= \frac{dP}{dx}\, y \qquad \frac{dP}{dx} < 0$$



$y > 0,\ \tau_{xy} < 0$

$y < 0,\ \tau_{xy} > 0$



$$\vec{F}_\mu = \iint_A \underline{\underline{\tau}} \cdot \hat{n}\, dA$$

$$\underline{\underline{\tau}} \cdot \hat{j} = \tau_{yx}\,\hat{i} + \cancel{\tau_{yy}}\,\hat{j} + \cancel{\tau_{yz}}\,\hat{k}$$

$$\vec{F}_\mu = \iint_A \left(\tau_{yx}\, dA\right)\hat{i}$$

At the wall: $\tau_{yx}(h) = -2\mu \frac{U_m h}{h^2}$

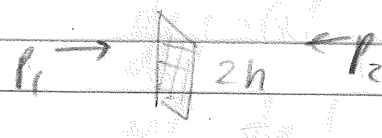
upper: $\int \tau_{yx}\, dx = -\frac{2\mu U_m \ell}{h}$

lower: $\tau_{yx}(-h) = -2\mu \frac{U_m(-h)}{h^2} = \frac{2\mu U_m h}{h^2}$

$\tau_{yx} \cdot -\hat{j} = -\frac{2\mu U_m \ell}{h}$

total: $= -\frac{4\mu U_m l}{h}$

⊛ pressure



$$(P_1 - P_2)\, 2h = \left(\frac{-dp}{dx}\, l\right) 2h = \left(\frac{2\mu U_m}{h^2}\right) l\, 2h = \frac{4\mu U_m l}{h}$$