lecture 13 Smits 6.9.2 Oct 24, 2018

Point (line) vortex = model

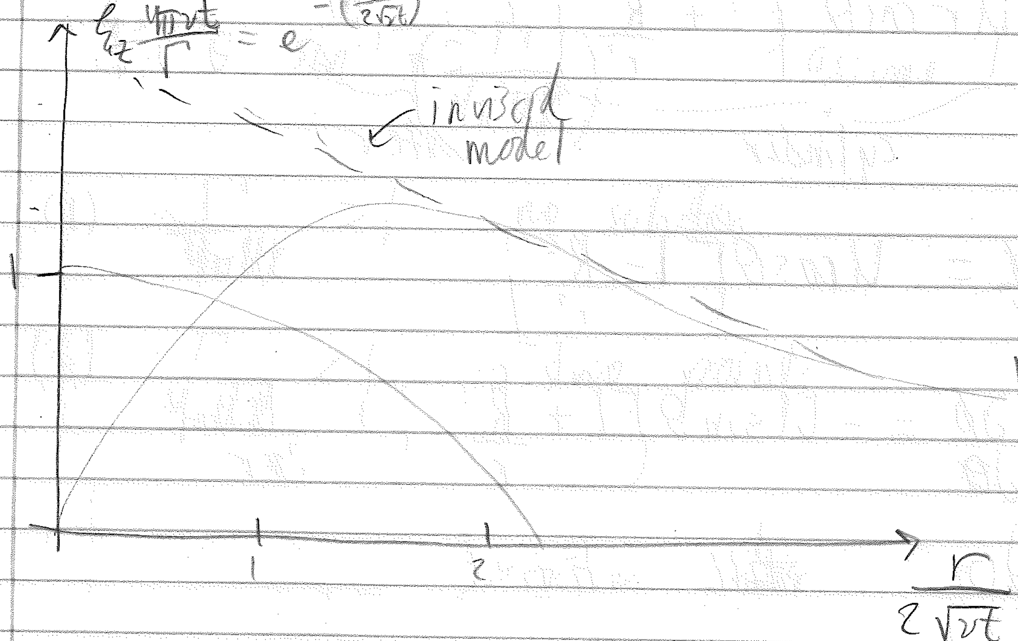
More realistic → viscosity
→ time evolution

$v_r = 0$

$v_\theta = \frac{\Gamma}{2\pi r}\left(1 - e^{\frac{-r^2}{4\nu t}}\right)$

$\nu = \frac{\mu}{\rho}$ [m^2/s] "pounders rate"

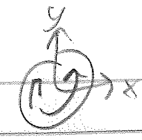
$\xi_z = \frac{1}{r}\left(\frac{\partial}{\partial r}(r v_\theta)\right) = \frac{\Gamma}{4\pi\nu t} e^{-\frac{r^2}{4\nu t}}$



Note: there is often an associated v_z velocity

- airplane → core velocity can be very high
  ↳ trade off between safety + intertakeoff time

high velocity → low pressure → condensation → ice crystals
↳ contrails!

Lift $\rightarrow$ 

e.g. flow around a cylinder + vortex

$$\nabla^2(\phi_1 + \phi_2) = \nabla^2 \phi_1 + \nabla^2 \phi_2 \quad + \text{B.C.s}$$

Laplace is linear $\rightarrow$ superposition

doesn't work for pressure

$$p \sim |v|^2$$

$$|v_1|^2 + |v_2|^2 \neq |v_1 + v_2|^2$$

So $\phi = \underbrace{U r \cos\theta \left[1 + \frac{R^2}{r^2}\right]}_{\text{cylinder}} + \underbrace{\frac{\Gamma}{2\pi}\theta}_{\text{vortex}}$

$$v_r = \frac{\partial \phi}{\partial r} = U\cos\theta\left[1 - \frac{R^2}{r^2}\right]$$

$$v_\theta = \frac{1}{r}\frac{\partial \phi}{\partial \theta} = -U\sin\theta\left[1 + \frac{R^2}{r^2}\right] + \frac{\Gamma}{2\pi r}$$

$v_r\Big|_{r=R} = 0$ still satisfied

$v_r \rightarrow U\cos\theta$ as $r \rightarrow \infty$ still satisfied

$v_\theta \rightarrow -U\sin\theta$

Stagnation point @ surface (r=R)

without vortex → $\theta = 0, \pi$

$$v_\theta \Big|_{r=R} = -U\sin\theta\,(1+1) + \frac{\Gamma}{2\pi R} = 0$$

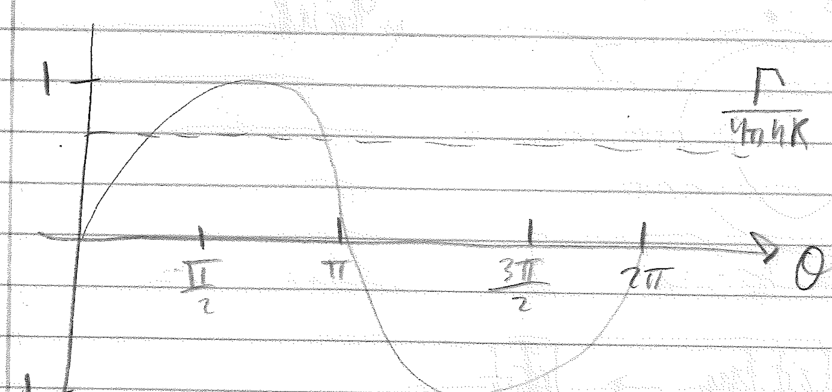
$$= -2U\sin\theta + \frac{\Gamma}{2\pi} R = 0$$

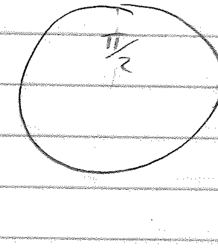
$$\sin\theta = \frac{\Gamma}{4\pi U R}$$

$$\theta = \sin^{-1}\left(\frac{\Gamma}{4\pi U R}\right)$$ assume $\Gamma > 0$

(1) $\frac{\Gamma}{4\pi U R} > 1$ no solution

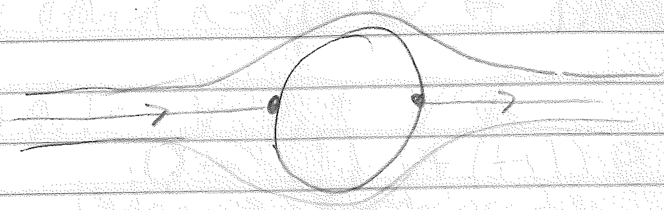
(2) $\frac{\Gamma}{4\pi U R} < 1$ → two points





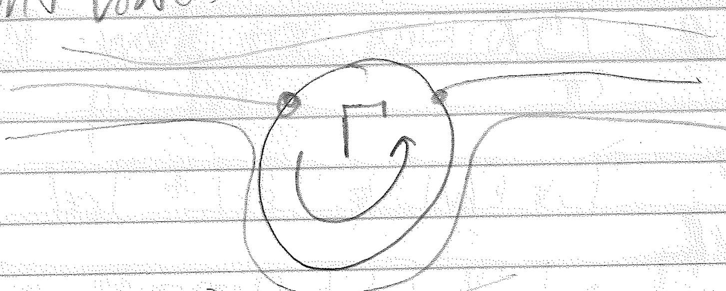
Uniform

$$\frac{\Gamma}{4\pi UR} = 0$$

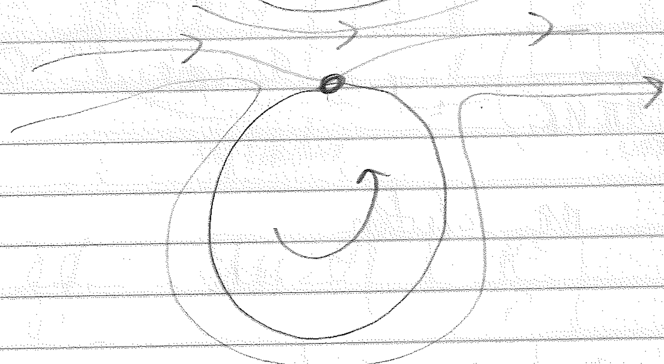


Uniform + vortex

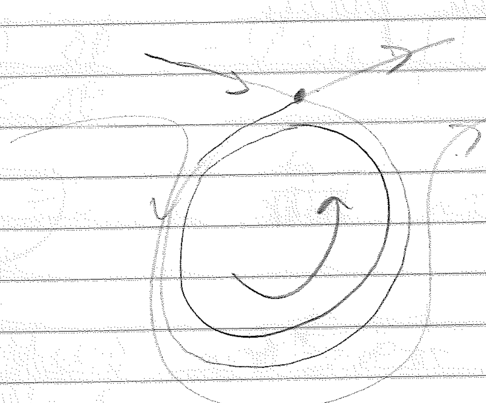
$$\frac{\Gamma}{4\pi UR} < 1$$



$$\frac{\Gamma}{4\pi UR} = 1$$



$$\frac{\Gamma}{4\pi UR} > 1$$



asymmetry top/bottom → lift

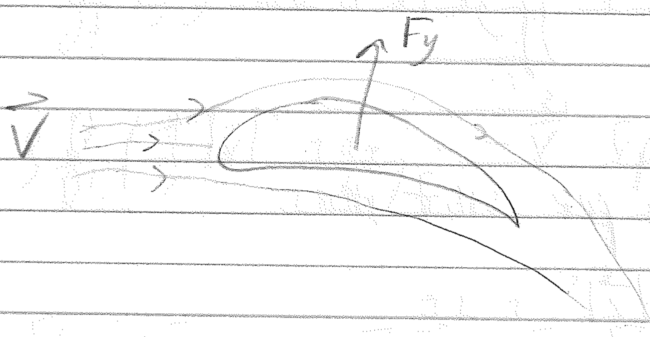
symmetry front/back → no drag

Smits 6.10

Pressure from Bernoulli → compute force

$$F_y = -\rho U \Gamma$$

(1) Related to 2D flow over airfoil



$F_y = \rho U \Gamma$ ($F_y < 0$)

(2) Related to flow over a spinning cylinder

↳ Magnus effect

$\Gamma > 0$

$F_y = -\rho U \Gamma < 0$