Lecture 16 Nov 5, 2018

Turbulence

→ highly random ⇒ use averaging

→ unstable i.e. sensitive to small perturbations

→ 3D

→ high rates of mixing / diffusion

→ highly dissipative - mechanical energy ⇒ heat

→ satisfies NS eqns

Numerically:

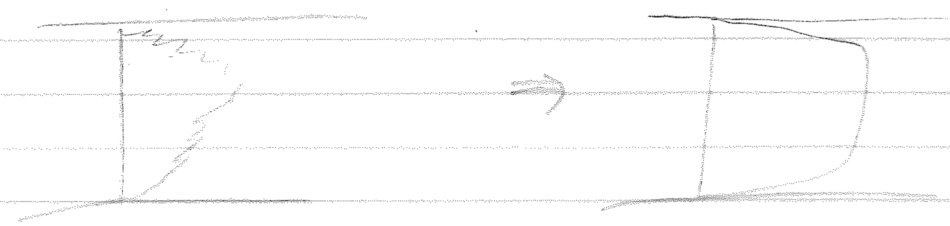
1. Direct numerical simulation (DNS)
   ↳ solve N-S in 3D over time

2. Large eddy simulation (LES)
   ↳ small scales are modelled, large scales are solved directly

3. Reynolds-Averaged Navier Stokes (RANS)
   ↳ solve time-averaged N-S

less computationally expensive

Averaging



+ fluctuations

$$u(x,y,z,t) = \frac{1}{T}\int_t^{t+T} u(x,y,z,t')\,dt' = \bar{u}(x,y,z)$$

T is large w.r.t. flow time scale
statistically steady

Properties of average

$\bar{c} = c$

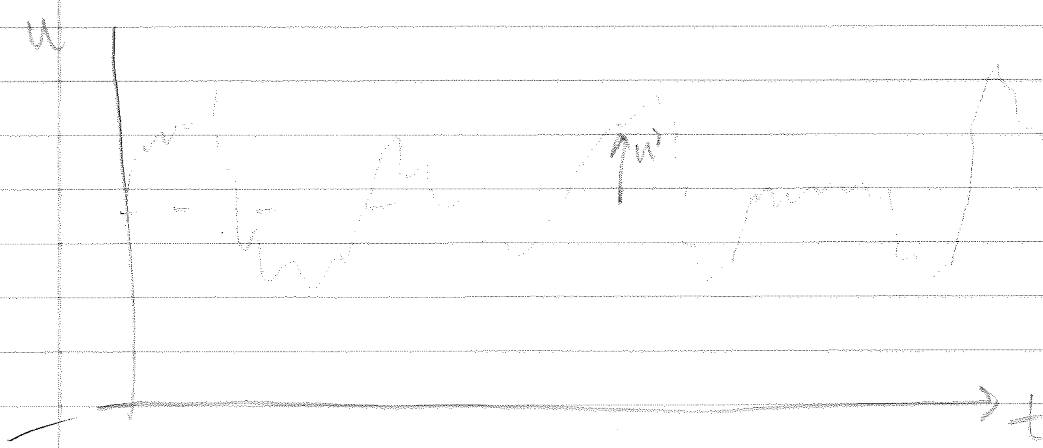
$\overline{cu} = c\,\bar{u}$

$\bar{\bar{u}} = \bar{u}$ (average of an average = average of a constant)

$\overline{\dfrac{\partial u}{\partial x}} = \dfrac{\partial \bar{u}}{\partial x}$

$\overline{a+b} = \bar{a} + \bar{b}$ ($\overline{ab} \neq \bar{a}\,\bar{b}$

Reynolds decomposition



$\bar{u}$

$u = \bar{u} + u'$

$$u = \bar{u} + u'$$
$$u' = u - \bar{u}$$

$$\bar{u} = \overline{\bar{u} + u'} = \bar{\bar{u}} + \overline{u'} = \bar{u} + \overline{u'}$$

$$\therefore \overline{u'} = 0$$

Time averaged equation of motion

$$\nabla \cdot \vec{v} = 0$$

$$\rho \frac{\partial \vec{v}}{\partial t} + \rho(\vec{v} \cdot \nabla)\vec{v} = \rho \vec{g} - \nabla p + \mu \nabla^2 \vec{v}$$

$$u = \bar{u} + u'$$
$$v = \bar{v} + v'$$
$$w = \bar{w} + w'$$

$$\overline{\frac{\partial(\bar{u} + u')}{\partial x}} = \overline{\frac{\partial u'}{\partial x}} + \overline{\frac{\partial \bar{u}}{\partial x}} = \frac{\partial \overline{u'}}{\partial x} + \frac{\partial \bar{u}}{\partial x}$$

$$\overline{\nabla \cdot \vec{v}} = 0 \Rightarrow \frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} = 0$$ averaged continuity

$$\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0$$

$$\rho \frac{\partial u}{\partial t} + \rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} + \rho w \frac{\partial u}{\partial z} = \rho g_x - \frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z}$$

$$\tau_{ij} = \mu\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)$$

$$\overline{u'} = 0$$

$$\overline{u'^2} \neq 0 \qquad \overline{u'u'} \neq 0 \qquad \overline{u'v'} \neq 0 \qquad \overline{u'\frac{\partial u'}{\partial x}} \neq 0$$

$$\frac{\partial(\bar{p}+p')}{\partial x} = \frac{\partial \bar{p}}{\partial x} + \overline{\frac{\partial p'}{\partial x}}^{\,=0}$$

$$\overline{\frac{\partial u}{\partial t}} = \frac{1}{T}\int_0^T \frac{\partial u}{\partial t}\,dt = \frac{u(T)-u(0)}{T} \xrightarrow[T\to\infty]{} 0$$