$$\overline{u'} = 0$$

$$\overline{u'^2} \neq 0 \qquad \overline{u'u'} \neq 0 \qquad \overline{u'\frac{\partial u'}{\partial x}} \neq 0$$

$$\overline{u'v'} \neq 0$$

$$\frac{\partial \overline{(\bar{p} + p')}}{\partial x} = \frac{\partial \bar{p}}{\partial x} + \frac{\partial \overline{p'}}{\partial x} \leftarrow 0$$

$$\overline{\frac{\partial u}{\partial t}} = \frac{1}{T}\int_0^T \frac{\partial u}{\partial t}\,dt = \frac{u(T) - u(0)}{T} \xrightarrow[T \to \infty]{} 0$$

Lecture 17 Nov 7, 2018

Project: 100 pts

* Presentation (30 pts)
  - make it interesting to read
  - provide engineering context
  - cosmetics

* Problem difficulty (10 pts)

* Simulation skills (25 pts)
  - mesh refinement

* Analysis and conclusions (35 pts)
  - compare to experimental data

Review

$$\overline{u} = \frac{1}{T}\int_0^T u(x,y,z,t)\,dt$$

Reynolds decomposition

$$u = \overline{u} + u'$$

$$\overline{u'} = 0$$

$$\nabla \cdot \overline{u} = 0$$

$$\nabla \cdot \overline{u'} = 0$$

$$\frac{\partial \bar{u}}{\partial t} \rightarrow 0$$

$$\frac{\partial(\bar{p}+p')}{\partial x} = \frac{\partial \bar{p}}{\partial x}$$

$$\overline{(\bar{u}+u')\frac{\partial(\bar{u}+u')}{\partial x}} = \overline{\bar{u}\frac{\partial \bar{u}}{\partial x}} + \overline{u'\frac{\partial \bar{u}}{\partial x}} + \overline{\bar{u}\frac{\partial u'}{\partial x}} + \overline{u'\frac{\partial u'}{\partial x}}$$

$$\bar{u}\frac{\partial \bar{u}}{\partial x} + \cancel{\overline{u'}}^{\,0}\frac{\partial \bar{u}}{\partial x} + \bar{u}\cancel{\frac{\partial \overline{u'}}{\partial x}}^{\,0} + \overline{u'\frac{\partial u'}{\partial x}}$$

$$= \bar{u}\frac{\partial \bar{u}}{\partial x} + \overline{u'\frac{\partial u'}{\partial x}}$$

$$\overline{(v+v')\frac{\partial(\bar{u}+u')}{\partial y}} = \bar{v}\frac{\partial \bar{u}}{\partial y} + \overline{v'\frac{\partial u'}{\partial y}}$$

$$\overline{(w+w')\frac{\partial(\bar{u}+u')}{\partial z}} = \bar{w}\frac{\partial \bar{u}}{\partial z} + \overline{w'\frac{\partial u'}{\partial z}}$$

$$\boxed{\overline{u'\frac{\partial u'}{\partial x}} + \overline{v'\frac{\partial u'}{\partial y}} + \overline{w'\frac{\partial u'}{\partial z}} =}$$

reverse product rule

$$\nabla \cdot \vec{v}' = 0$$

$$\frac{\partial \overline{u'u'}}{\partial x} + \frac{\partial \overline{u'v'}}{\partial y} + \frac{\partial \overline{u'w'}}{\partial z} - \cancel{u'\left(\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z}\right)}$$

Plug into N-S

$$x: \rho\bar{u}\frac{\partial \bar{u}}{\partial x} + \rho\bar{v}\frac{\partial \bar{u}}{\partial y} + \rho\bar{w}\frac{\partial \bar{u}}{\partial z} = \rho g_x - \frac{\partial \bar{p}}{\partial x}$$

$$+ \frac{\partial}{\partial x}\left(\tau_{xx} - \boxed{\rho\overline{u'u'}}\right) + \frac{\partial}{\partial y}\left(\tau_{xy} - \boxed{\rho\overline{u'v'}}\right) + \frac{\partial}{\partial z}\left(\tau_{xz} - \boxed{\rho\overline{u'w'}}\right)$$

$$\overline{\tau_{ij}} = \mu\left(\frac{\partial \overline{u_j}}{\partial x_i} + \frac{\partial \overline{u_i}}{\partial x_j}\right)$$

y: $\rho\bar{u}\frac{\partial\bar{v}}{\partial x} + \rho\bar{v}\frac{\partial\bar{v}}{\partial y} + \rho\bar{v}\frac{\partial\bar{v}}{\partial z} = \rho g_y - \frac{\partial\bar{p}}{\partial y}$
$+\frac{\partial}{\partial x}\left(\overline{\tau_{yx}} - \rho\overline{u'v'}\right) + \frac{\partial}{\partial y}\left(\overline{\tau_{yy}} - \rho\overline{v'v'}\right) + \frac{\partial}{\partial z}\left(\overline{\tau_{yz}} - \rho\overline{v'w'}\right)$

z: $\rho\bar{u}\frac{\partial\bar{w}}{\partial x} + \rho\bar{v}\frac{\partial\bar{w}}{\partial y} + \rho\bar{w}\frac{\partial\bar{w}}{\partial z} = \rho g_z - \frac{\partial\bar{p}}{\partial z}$
$+\frac{\partial}{\partial x}\left(\overline{\tau_{zx}} - \rho\overline{u'w'}\right) + \frac{\partial}{\partial y}\left(\overline{\tau_{zy}} - \rho\overline{v'w'}\right) + \frac{\partial}{\partial z}\left(\overline{\tau_{zz}} - \rho\overline{w'w'}\right)$

Equation for $\bar{u}, \bar{v}, \bar{w}, \bar{p}$ are the same as for u, v, w, p except for the Reynolds stress term:

Reynolds stress $\underline{\mathbb{R}} = -\rho\begin{pmatrix} \overline{u'u'} & \overline{u'v'} & \overline{u'w'} \\ \overline{v'u'} & \overline{v'v'} & \overline{v'w'} \\ \overline{w'u'} & \overline{w'v'} & \overline{w'w'} \end{pmatrix}$ where $\overline{u'v'} = \overline{v'u'}$ $\therefore$ symmetric

4 eqns (3 mom + continuity)

$\bar{u}, \bar{v}, \bar{w}, \bar{p}$ + 6 Reynolds stresses

→ closure problem

hypothesis for turbulence modeling

k-ε equation

dynamic viscosity μ [Pa·s]

kinematic viscosity $\nu = \frac{\mu}{\rho}$ [m^2/s]

Motivation: turbulence modeling → relate u' to $\bar{u}$
v' $\bar{v}$
w' $\bar{w}$

$$-\overline{u'v'} = \nu_T\left(\frac{\partial \bar{u}}{\partial y} + \frac{\partial \bar{v}}{\partial x}\right)$$

turbulent viscosity $\nu_T = C_T U_T L_T$ [m^2/s]

constant velocity + length
characteristic
of turbulence