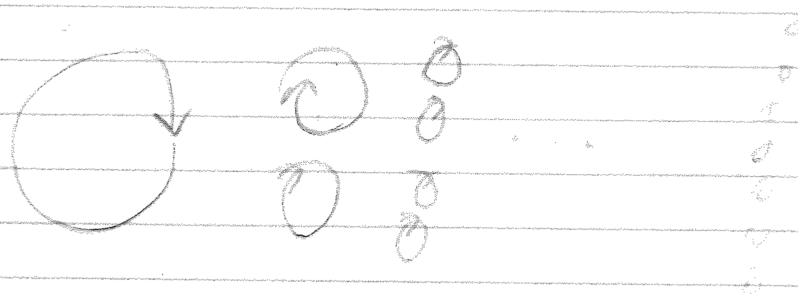
Lecture 19

$U\frac{1}{D}U$ $\qquad$ $\nu\frac{1}{D}U$ $\qquad$ Nov 14, 2015

$$\frac{\partial \vec{v}}{\partial t} + \underbrace{(\vec{v}\cdot\nabla)\vec{v}}_{\text{advection of momentum}} = -\frac{\nabla p}{\rho} + \underbrace{\nu\nabla^2\vec{v}}_{\text{viscous diffusion of momentum}}$$

$$Re = \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{UD}{\nu} = \frac{\rho U D}{\mu} \qquad \frac{U\frac{1}{D}U}{\nu\frac{1}{D^2}U} = \frac{UD}{\nu}$$

Turbulent cascade



$Re = \frac{UD}{\nu}$ $\qquad$ $Re = 1$

viscous effects dominate
dissipate energy to heat

Kolmogorov (1941)

small scale eddies η

2 variables: ν, ε
$\left[\frac{m^2}{s}\right]$ $\left[\frac{m^2}{s^3}\right]$

$$\eta = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4} \qquad Re = \frac{\left(\frac{\eta}{\tau_\eta}\right)\eta}{\nu} = 1$$

$$\tau_\eta = \left(\frac{\nu}{\varepsilon}\right)^{1/2}$$

e.g. propeller

$R = 10\,cm$ $\qquad \varepsilon = 5\,\frac{m^2}{s^3}$

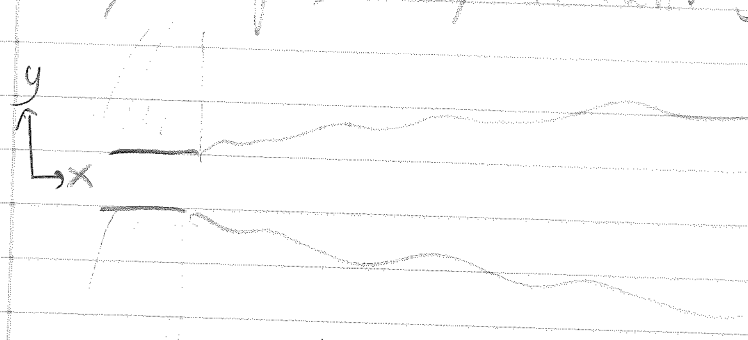
$f = 4\,Hz$ $\qquad \nu = 10^{-6}\,\frac{m^2}{s}$ $\Rightarrow \eta = 20\,\mu m$

$\frac{2R}{\eta} = 10^4$ cells across to resolve smallest eddies

in 3D: $(10^4)^3 = 10^{12}$ cells $\qquad$ that's a lot!

Boundary layer equations

2D, incompressible, turbulent jet from slot



Define length scales

l_x : length over which $\bar{u}$ changes along x significantly

$$\bar{u} \xrightarrow{l_x} \frac{\bar{u}}{2}$$

l_y: change of $\bar{u}$ along y by same amount

$\frac{l_y}{l_x} << 1$ $\qquad$ thin jet

(1) $\frac{l_y(x)}{l_x(x)} \ll 1$ locally thin

$Re(x) = \frac{U(x)\, l_y(x)}{\nu} \gg 1$ local high Re

where $U(x)$ = peak velocity (centerline)

continuity: $\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} = 0 \qquad \frac{U}{l_x} = \frac{V}{l_y} \qquad V = U\frac{l_y}{l_x}$ (V is small)

x-comp: $\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} + \nu\left(\frac{\partial^2 \bar{u}}{\partial x^2} + \frac{\partial^2 \bar{u}}{\partial y^2}\right) - \frac{\partial \overline{u'^2}}{\partial x} - \frac{\partial \overline{u'v'}}{\partial y}$

$\frac{U^2}{l_x} \qquad \frac{U\cancel{V}}{l_x}\left(\frac{U}{l_y}\right) \qquad \frac{1}{\rho}\frac{\rho U^2}{l_x} \qquad \frac{\nu U}{l_x^2} \qquad \frac{\nu U}{l_y^2} \qquad \frac{u^2}{l_x} \qquad \frac{u^2}{l_y}$

$\qquad\qquad \frac{U^2}{l_x} \qquad\qquad \frac{U^2}{l_x}$

$\div \frac{U^2}{l_x} \qquad 1 \qquad 1 \qquad 1 \qquad \frac{\nu}{U l_y}\frac{l_y}{l_x} \qquad \frac{\nu}{U l_y}\frac{l_x}{l_y} \qquad \frac{u^2}{U^2} \qquad \frac{u^2}{U^2}\frac{l_x}{l_y}$

$\frac{1}{Re} \ll 1$ assume u^2 is small; smaller than this

$\frac{u^2}{U^2}\frac{l_x}{l_y} \sim 1$ for turbulence to be relevant

$\frac{u}{U} \sim \sqrt{\frac{l_y}{l_x}}$

(2) $\bar{u}\frac{\partial \bar{u}}{\partial x} + \bar{v}\frac{\partial \bar{u}}{\partial y} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial x} - \frac{\partial \overline{u'v'}}{\partial y}$

→ viscous stress is much less than Reynolds stress by a ratio of $\frac{1}{Re}$

→ normal Reynolds stress << shear Reynolds stress
$\overline{u'^2}$ $\overline{u'v'}$

$$\text{comp.}\quad u\frac{\partial \overline{v}}{\partial x} + v\frac{\partial \overline{v}}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 \overline{v}}{\partial x^2} + \frac{\partial^2 \overline{v}}{\partial y^2}\right) - \frac{\partial \overline{u'v'}}{\partial x} - \frac{\partial \overline{v'^2}}{\partial y}$$

$$\frac{Ul_y}{l_x}\quad \frac{U^2}{l_y}\left(\frac{l_y}{l_x}\right)^2\quad \frac{U^2}{l_y}\left(\frac{l_y}{l_x}\right)^2\quad \frac{U^2}{l_y}\quad \frac{\nu U l_y}{l_x^2}\quad \frac{\nu U}{l_x l_y}\quad \frac{u^2}{l_x}\quad \frac{u^2}{l_y}$$

$$\frac{U^2}{l_y}\quad \left(\frac{l_y}{l_x}\right)^2\quad \left(\frac{l_y}{l_x}\right)^2\quad 1\quad \frac{\nu}{Ul_y}\left(\frac{l_y}{l_x}\right)^2\quad \frac{\nu}{Ul_y}\left(\frac{l_y}{l_x}\right)\quad \left(\frac{l_y}{l_x}\right)^2\quad \left(\frac{l_y}{l_x}\right)$$

$$\frac{\partial \overline{p}}{\partial y} = 0$$

$$p = p(x)$$