# Lecture 27

Nov 30, 2018

$$\frac{\partial^2 u'}{\partial t^2} = c^2 \frac{\partial^2 u'}{\partial x^2}$$

$$c = \sqrt{\left.\frac{dp}{d\rho}\right|_{\rho_0}} \overset{gas}{\Rightarrow} \sqrt{\gamma R T_0}$$

## Solution

$u' = g(x \pm ct)$ with $g$ twice differentiable

$$\frac{\partial u'}{\partial t} = \pm c\, g'(x \pm ct)$$

$$\frac{\partial^2 u'}{\partial t^2} = c^2 g''(x \pm ct)$$

$$\frac{\partial^2 u'}{\partial x^2} = g''(x \pm ct)$$

$$\Rightarrow \frac{\partial^2 u'}{\partial t^2} = c^2 \frac{\partial^2 u'}{\partial x^2}$$

$x - ct$: wave $\longrightarrow$

$x + ct$: wave $\longleftarrow$

## Sine wave

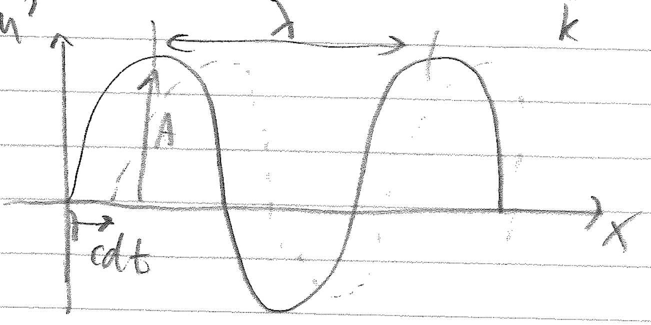
$$u'(x,t) = A \sin(k(x - ct))$$

$A$: amplitude; $k$: wave #

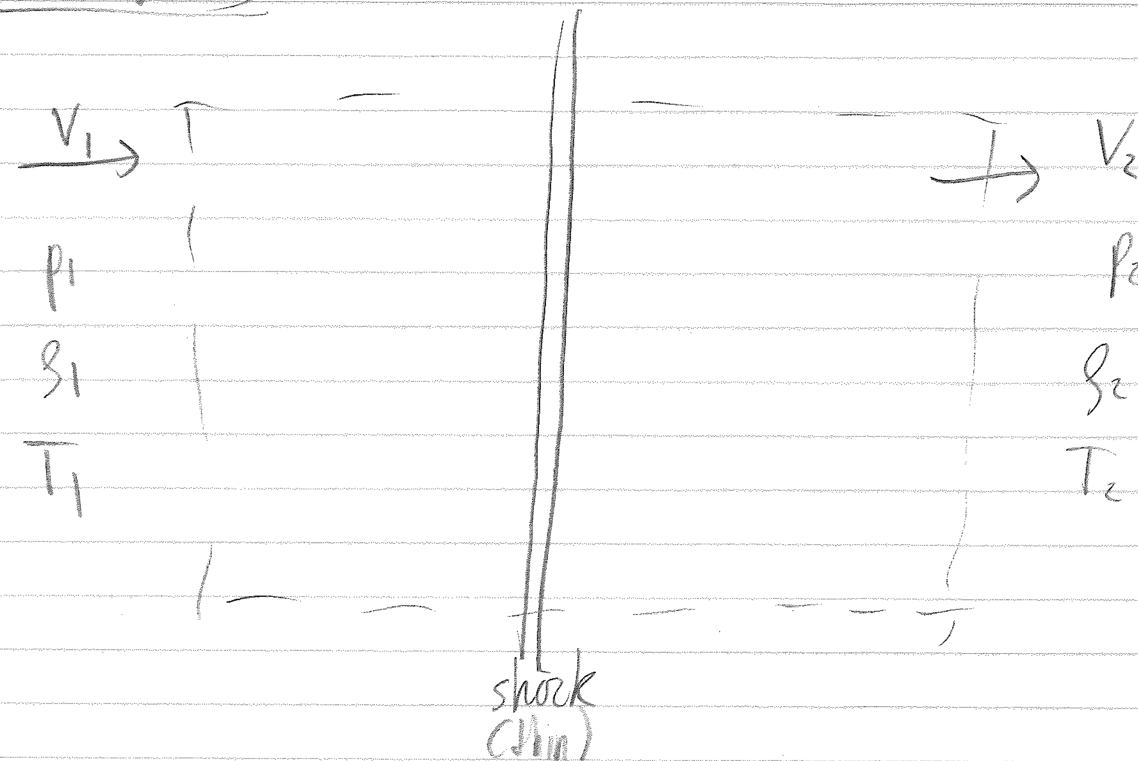
$\lambda = \frac{2\pi}{k}$

$c \neq f(\lambda)$ most media

$c = f(\lambda)$ dispersive media



# Shock waves




$M>1$ $\qquad$ $M<1$

discontinuity

CV moves with flow → steady

continuity

$$\underbrace{\cancel{\frac{d}{dt}\iiint_{CV} \rho \, dV}}_{\substack{0 \\ \text{steady}}} = -\iint_{CS} \rho \, \vec{V}\cdot\hat{n} \, dA$$

$$-\left(-\rho_1 V_1 A + \rho_2 V_2 A\right) = 0$$

$$\boxed{\rho_1 V_1 = \rho_2 V_2}$$

conserve momentum

④ steady 1D momentum

$$\underbrace{\cancel{\frac{d}{dt}\int_{cv} \rho u \, d\forall}}_{\substack{0 \\ \text{steady}}} = \underbrace{F_x}_{\substack{\text{neglect} \\ \text{gravity} \\ \& \\ \text{viscous effects}}} - \int_{cs} \rho u (\vec{v}\cdot\hat{n}) dA$$

$$0 = (p_1 - p_2)A - (-\rho_1 V_1 V_1 A + \rho_2 V_2 V_2 A)$$

$$\boxed{p_1 + \rho_1 V_1^2 = p_2 + \rho_2 V_2^2}$$ (not Bernoulli)

④ energy

$$e = u + \tfrac{1}{2}|\vec{v}|^2$$

$$\underbrace{\cancel{\frac{d}{dt}\int_{cv} \rho e \, d\forall}}_{\substack{0 \\ \text{steady}}} = \underbrace{\cancel{\dot{Q}}}_{\substack{0 \\ \text{no heat} \\ \text{production}}} + \underbrace{\dot{W}}_{\substack{\text{pressure work} \\ \text{neglect viscous effects}}} - \int_{cs} \rho e (\vec{v}\cdot\hat{n}) dA$$

$$0 = -\int_{cs} \rho\left(e + \frac{p}{\rho}\right)(\vec{v}\cdot\hat{n}) dA$$

$$e + \frac{p}{\rho} = u + \tfrac{1}{2}|\vec{v}|^2 + \frac{p}{\rho} = h + \tfrac{1}{2}|\vec{v}|^2$$

$$h_1 + \tfrac{1}{2} V_1^2 = h_2 + \tfrac{1}{2} V_2^2$$

ideal gas $h = C_p T$

$$\boxed{T_1 + \frac{V_1^2}{2C_p} = T_2 + \frac{V_2^2}{2C_p}}$$

(*) need extra eqn

ideal gas law $\boxed{p_2 = \rho_2 R T_2}$

$M_1 = \frac{V_1}{c}$ , $M_2 = \frac{V_2}{c}$ $\quad c = \sqrt{\gamma R T}$

$\gamma = \frac{C_p}{C_v} = 1.4$

$$p_1 + \rho_1 V_1^2 = p_2 + \rho_2 V_2^2$$

$$p_1 + \gamma\left(\frac{p_1}{\gamma R T_1}\right) V_1^2 = p_2 + \gamma\left(\frac{p_2}{\gamma R T_2}\right) V_2^2$$

$$\boxed{\frac{p_1}{p_2} = \frac{1 + \gamma M_1^2}{1 + \gamma M_2^2}}$$

if $M_1 > 1$ , $p_2 > p_1$

$$\boxed{\frac{T_2}{T_1} = \frac{1 + \left(\frac{\gamma - 1}{2}\right) M_1^2}{1 + \left(\frac{\gamma - 1}{2}\right) M_2^2}}$$

Continuity + gas law

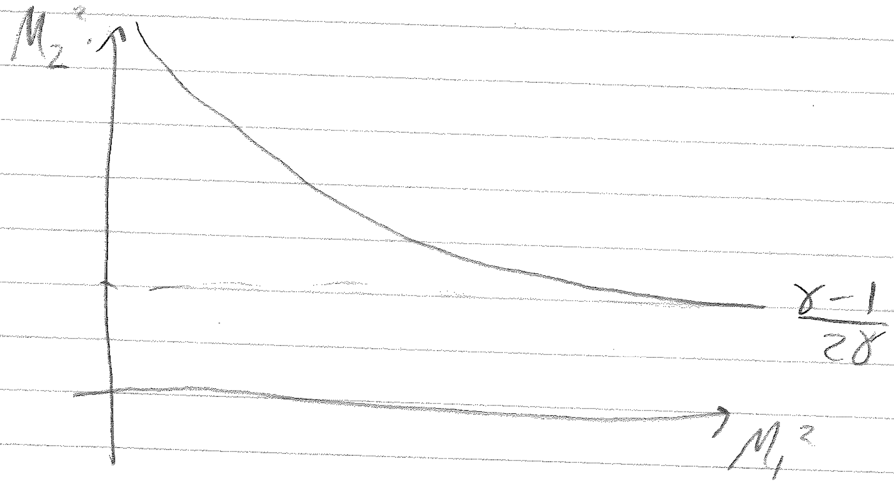
$$\frac{p_1}{p_2}\frac{M_1}{M_2} = \sqrt{\frac{T_1}{T_2}}$$

2 solutions

① $M_1 = M_2$, $p_1 = p_2$, $T_1 = T_2$

② $M_2^2 = \dfrac{(\gamma-1)M_1^2 + 2}{2\gamma M_1^2 - (\gamma-1)}$

if $M_1 > 1$, $M_2 < 1$




with $\gamma = \frac{7}{5}$

$$\frac{p_2}{p_1} = 1 + 1.17(M_1^2 - 1)$$

$$\frac{\Delta p}{p_1} = \frac{2\gamma}{\gamma+1}(M_1^2 - 1) \quad \text{for } M_1 < 2$$

"shock strength"