ME 431/599F — Handout — 1 February 2008

Solution of the Heat Equation

Summary of Explicit Finite-Difference Equations

1. Basic Mathematical Problem

Conservation of energy: $\dfrac{\partial T}{\partial t} = k\dfrac{\partial^2 T}{\partial y^2}$

Initial conditions: $T(y,0) = F(y)$

Boundary conditions: $T(0,t) = T_0$, $T(L,t) = T_L$

2. Numerical Solution

With $t_n = (n-1)\Delta t$, $n \geq 1$, $y_i = (i-1)\Delta y$, $1 \leq i \leq N$, $\Delta y = L/(N-1)$, $T_i^n = T(y_i, t_n)$

Initialization

$$T_i^1 = F(y_i) = F_i \, , \, 1 \leq i \leq N$$

Time-stepping: step n to step $n+1$, $R = \dfrac{k\Delta t}{(\Delta y)^2}$

$$\begin{aligned} T_1^{n+1} &= T_0 \\ T_i^{n+1} &= T_i^n + R(T_{i+1}^n - 2T_i^n + T_{i-1}^n) \, , \, 2 \leq i \leq N-1 \\ T_N^{n+1} &= T_L \end{aligned}$$

3. Basic Matlab Script

For this example: $F(y) = T_m \exp(-y^2/d^2)$.

% Initialization

for $i = 1 : N$
$T(i,1) = T_m \exp(-((i-1) * dy)^2/d^2)$;
end

% Time-stepping

for $n = 2 : M$

% Set boundary conditions

$T(1,n) = T_0$
$T(N,n) = T_L$

% Main solution loop

for $i = 2 : (N-1)$
$T(i,n) = T(i,n-1) + R * (T(i+1,n-1) - 2 * T(i,n-1) + T(i-1,n-1))$
end
% Finished main loop
end
% Finished time-stepping