What is Fluid Mechanics?

The discipline, within applied mechanics, that studies the behaviour of fluids: liquids and gases.

What is a Fluid?

The definition of a fluid is based on mechanical behaviour, not on molecular structure. It is based on the response to a certain force. Specifically:

A fluid is a state of matter that deforms continuous when it is subjected to shear stress.

Alternatively, it can be defined as a state where matter cannot withstand a shear stress without continuous deformation.

When we apply force on a body, it is subjected to a distribution of stresses (force per unit area). If the force has a tangential component, part of those stresses will be shearing (and typically part will be normal stresses).

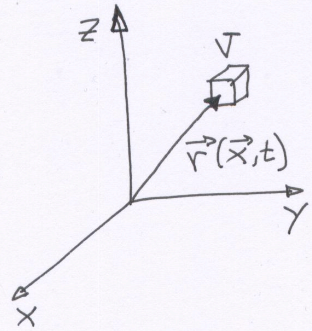
A solid responds to a stress distribution with a distribution of strain (deformation). A typical solid responds in an elastic way: the deformation is linearly proportional to the stress; $\epsilon = \frac{1}{E}\sigma$ where the constant of proportionality is the inverse of the elastic (or Young's) modulu

In parallel with this theory, a fluid responds with a distribution of rate of strain to the application of a shear stress distribution. If the response is linear $\sigma = \mu \dot{\gamma}$ then the fluid is called Newtonian (in parallel to a Hookean solid) and the constant of proportionality is the coefficient of shear viscosity (μ).

CONTINUUM HYPOTHESIS

The study of fluid mechanics relies on the assumption that we can characterize the behaviour of matter without tracking the properties of individual molecules. By averaging over a large number of molecules that occupy a small region of space in a given time, we obtain fluid properties that are meaningful and well defined (from a mathematical standpoint) Rigorously, the definition of how large a number of molecules over the small averaging volume is needed depends on the value of Knudsen number:

$$\frac{KT}{\sqrt{2}\,\pi P d^2 L} = K_n = \frac{\lambda \rightarrow \text{mean free path between collisions}}{L \rightarrow \text{characteristic length scale of the flow}}$$

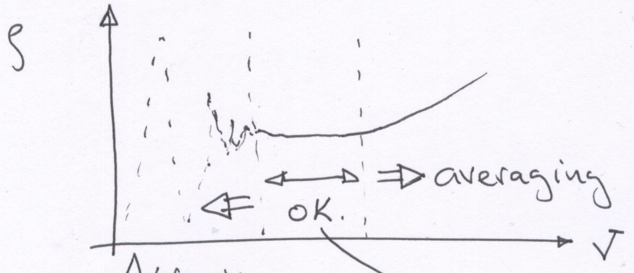


$\rho = \frac{1}{V}\sum_{i=1}^{N} M_i$: density as average mass per unit volume

$\vec{v} = \frac{1}{N}\sum_{i=1}^{N} \vec{v}_i$: fluid velocity as an average of the molecules' individual velocity

$\frac{3K}{2}T = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\|\vec{v}-\vec{v}_i\|^2}$: temperature as an average of the fluctuating Kinetic energy of the molecules with respect to the average fluid velocity

~~If~~ we look at any of these properties, for example density:

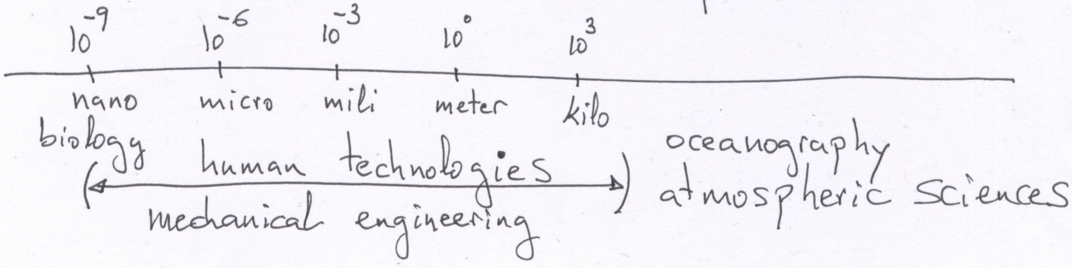


⇒ averaging over large volumes start to drift because of macroscopic gradients

Averaging over small volumes captures individual jumps due to molecules coming in or out of the averaging volume

There is a range where the continuum description is valid and meaningful.

We need a relatively large number of molecules ($(10^6$ within a relatively small volume of interest:



Gas at standard conditions (288 K, 1 atm) contains 10^{16} molecules per cubic milimeter.

- Outer atmosphere / space applications: although the length scale of interest is large, the density (number of molecules per unit volume) is so low that the Continuum Hypothesis fail $\Rightarrow$ Theory of rarefied gases

- Nano/micro technologies: although density is not extremely low, the lengthscale of interest is so small (10^{-9} - 10^{-6} m) that Continuum Hypothesis may fail.

To formulate problems in fluid mechanics we will use concepts from
- mechanics: Newton's laws $\frac{d(m\vec{v})}{dt} = \Sigma \bar{F}$
- thermodynamics: $-dW + \delta Q = dE$

  $T\,dS = \delta Q$
- physical chemistry: $P/\rho = R_g T$
- generalized thermodynamic relations:

  $\left.\frac{\partial P}{\partial \rho}\right)_s = a^2, \dots$

In order to apply thermodynamics information to fluid mechanics, we need to reconcile the different treatment of time in both disciplines.

Thermodynamic theory assumes that systems are in equilibrium and describes the evolution between equilibrium states (and therefore take infinite time by definition).
Mechanics describes the evolution in finite time and therefore needs the relationships from thermodynamics to apply to systems that are changing in finite time.
The key is that timescales for thermodynamics and mechanics are different. The evolution describe in mechanics are between equilibrium states and therefore thermodynamic relations are valid to describe the time evolution of the system.

For this to be justified the molecules in the system need to collide a sufficient number of times in the course of their motion to exchange momentum so that the average propertie of the fluid are in equilibrium. This short time scale for collisions to establish thermodynamic equilibrium justifies that for the time scale of mechanical evolution, the system is always in a state described by thermodynamic laws:

The evolation of the fluid properties are a sequence of infinitesimal steps between equilibri states.