Conservation of Momentum

$$\frac{D}{Dt}(M\vec{V})_{sys} = \sum \vec{F}_{ext} \quad : \text{Newton's } 2^{nd} \text{ Law}$$

$$\frac{D}{Dt}\int_{sys} \rho \vec{v}\, dV = \frac{\partial}{\partial t}\int_{c.v} \rho \vec{v}\, dV + \int_{c.s.} \rho \vec{v}\,(\vec{v}-\vec{v}_c)\cdot\vec{n}\, dA = \sum \vec{F}_{ext}$$

How do we calculate the external forces:

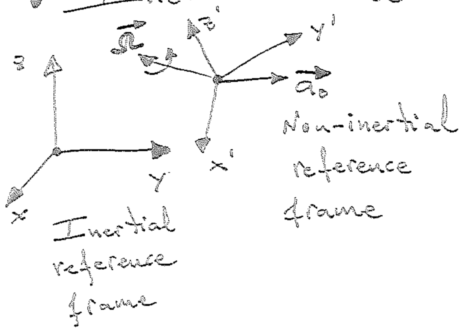
2 types: - Volume forces: gravity, electromagnetic, inerti.

- Surface forces: pressure and viscous forces

Volume Forces

- Gravity: $\int_{c.v.} \rho \vec{g}\, dV$

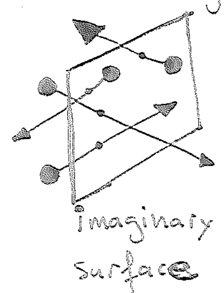
- Inertia: $\vec{a}' = \underbrace{\vec{a} - \vec{a}_0}_{\text{Linear acceleration of the reference frame}} - \underbrace{\vec{\Omega}\wedge\vec{\Omega}\wedge\vec{r}}_{\text{Centrifugal force}} - \underbrace{2\vec{\Omega}\wedge\vec{v}'}_{\text{Coriolis force}} - \underbrace{\frac{d\vec{\Omega}}{dt}\wedge\vec{r}}_{\text{angular acceleration of the reference frame.}}$



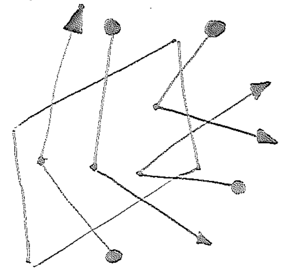
- Electromagnetic forces are outside the scope of this course.

SURFACE FORCES

- PRESSURE is the result of collisions between molecules which interchange momentum through those collissions.



pressure is a flux of momentum per unit area per unit time due ONLY to the fluctuating velocity of the molecules



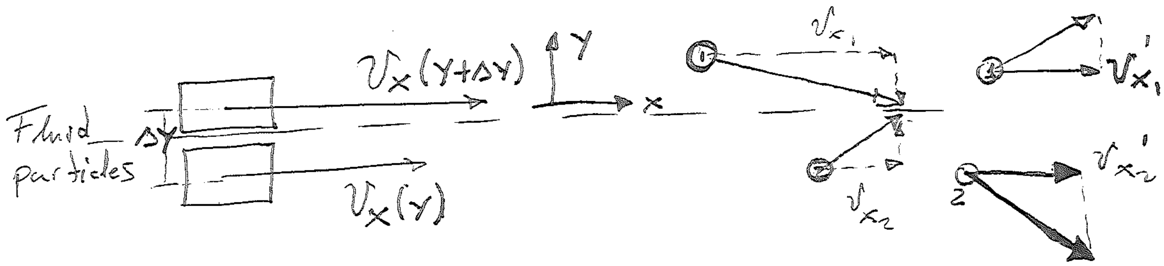
Next to a solid wall, the fluid velocity relative to the wall is zero and therefore the velocity of the molecules is only fluctuations

Pressure is the normal component of the momentum flux across a surface due to the fluctuating component of the velocity of the molecules. Resultant is given by:

$$-\int_{s.c.} p\,\vec{n}\,dA$$

- Viscous Forces are the result of a momentum exchange between molecules that have a net velocity gradient

If the collision is perfectly elastic the only force would be normal to the wall. But collisions are not



Because of inelastic collisions, momentum in the direction of motion is transferred through the collisions

$v_{x_1}' < v_{x_1}$ | $v_{x_1}' > v_{x_2}'$

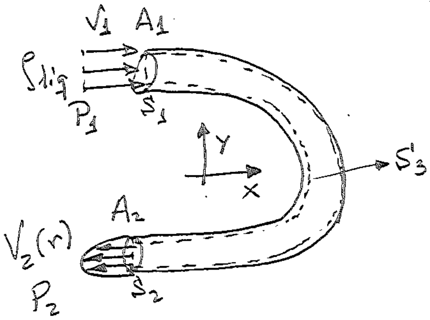
$v_{x_2}' > v_{x_2}$ | $v_{x_1} > v_{x_2}$

The transport of momentum in the x-direction due to a velocity difference in the fluid across the y direction gives rise to the shear stress on the fluid elements: $\tau_{xy}' \propto \frac{\Delta v_x}{\Delta y}$, the constant of proportionality is what we usually call μ, viscosity. We will see next week that things are much more complicated than that.

The resultant force from viscous stresses on a surface can be computed by:

$$\int_{s.c.} \bar{\bar{\tau}}' \cdot \vec{n}\, dA$$

In fact, surface forces can be combined as the force due to the stress tensor at each element of a surface: $\int \bar{\bar{T}} \cdot \vec{n}\, dA$ where $\bar{\bar{T}} = -p\,\bar{\bar{I}} + \bar{\bar{\tau}}'$



Conservation of mass

$$\underbrace{\cancel{\frac{\partial}{\partial t}\int_{c.v.}\rho\, dV}}_{\text{STEADY}} + \int_{c.s.}\rho(\vec{v}-\cancel{\vec{v}_c})\cdot\vec{n}\,dA = 0$$

$$\cancel{\int_{\text{SOLID WALL}+S_3}\vec{v}\cdot\vec{n}\,dA} = 0$$

$$\rho_{liq}\left[\int_{S_1}\vec{v}\cdot\vec{n}\,dA + \int_{S_2}\vec{v}\cdot\vec{n}\,dA\right] = 0$$

$$\rho_{liq}\left[\int v_1\vec{i}\cdot(-\vec{i})\,dA + \int_0^R v_2(r)(-\vec{i})\cdot(-\vec{i})\,dA\right] =$$

$$-\rho_{liq}v_1A_1 + \rho_{liq}\int_0^R K\left[1-\left(\frac{r}{R}\right)^2\right]2\pi r\,dr = 0$$

$$\rho_{liq}v_1A_1 = \rho_{liq}K\,2\pi\left[\frac{r^2}{2}-\frac{r^4}{4R^2}\right]_0^R$$

$$\cancel{\rho_{liq}}\,v_1\,\cancel{\pi}\cancel{R^2} = \cancel{\rho_{liq}}\,K\,\cancel{2\pi}\,\frac{\cancel{R^2}}{\cancel{4}}$$

$$\underline{\underline{K = 2v_1}}$$

Conservation of momentum

$$\underbrace{\cancel{\frac{\partial}{\partial t}\int_{c.v.}\rho\vec{v}\,dV}}_{\text{STEADY}} + \int_{c.s.}\rho\vec{v}(\vec{v}-\cancel{\vec{v}_c})\cdot\vec{n}\,dA = -\int_{c.s.}p\vec{n}\,dA + \int_{c.s.}\bar{\bar{\tau}}\cdot\vec{n}\,dA + \int_{c.v.}\rho\vec{g}\,dV$$

$$\rho_{liq}\left[\int_{S_1}v_1\vec{i}\;v_1\vec{i}\cdot(-\vec{i})\,dA + \rho_{liq}\int_0^R v_2(r)(\vec{i})\,v_2(r)(-\vec{i})\cdot(-\vec{i})\,dA + \int_{S_3}\rho\vec{v}\;\underbrace{\cancel{\vec{v}\cdot\vec{n}}}_{0\ \text{SOLID WALL}}\,dA\right] =$$

$$-\int_{S_1}P_1\vec{n}\,dA - \int_{S_2}P_2\vec{n}\,dA - \int_{S_3}p\vec{n}\,dA + \int_{S_3}\bar{\bar{\tau}}\cdot\vec{n}\,dA + \underbrace{\cancel{\int_{S_1+S_2}\bar{\bar{\tau}}\cdot\vec{n}\,dA}}_{\text{negligible inlets and outlets.}} + \int_{c.v.}\rho\vec{g}\,dV$$

$$+\int_{S_1}P_{atm}\vec{n}\,dA + \int_{S_2}P_{atm}\vec{n}\,dA + \int_{S_3}P_{atm}\vec{n}\,dA = -\vec{F}_{pipe}$$

$$-\vec{F}_{fluid\to pipe}$$

$$-\vec{F}_{atm\to pipe}$$

$$\vec{F}_{pipe} = +\int_{S_1}(P_1-P_{atm})\underbrace{\vec{n}}_{(+\vec{i})}\,dA + \int_{S_2}(P_2-P_{atm})\underbrace{\vec{n}}_{(+\vec{i})}\,dA + \int_{c.v.}\rho\vec{g}\,dV$$