PME4031
Engineering Optics

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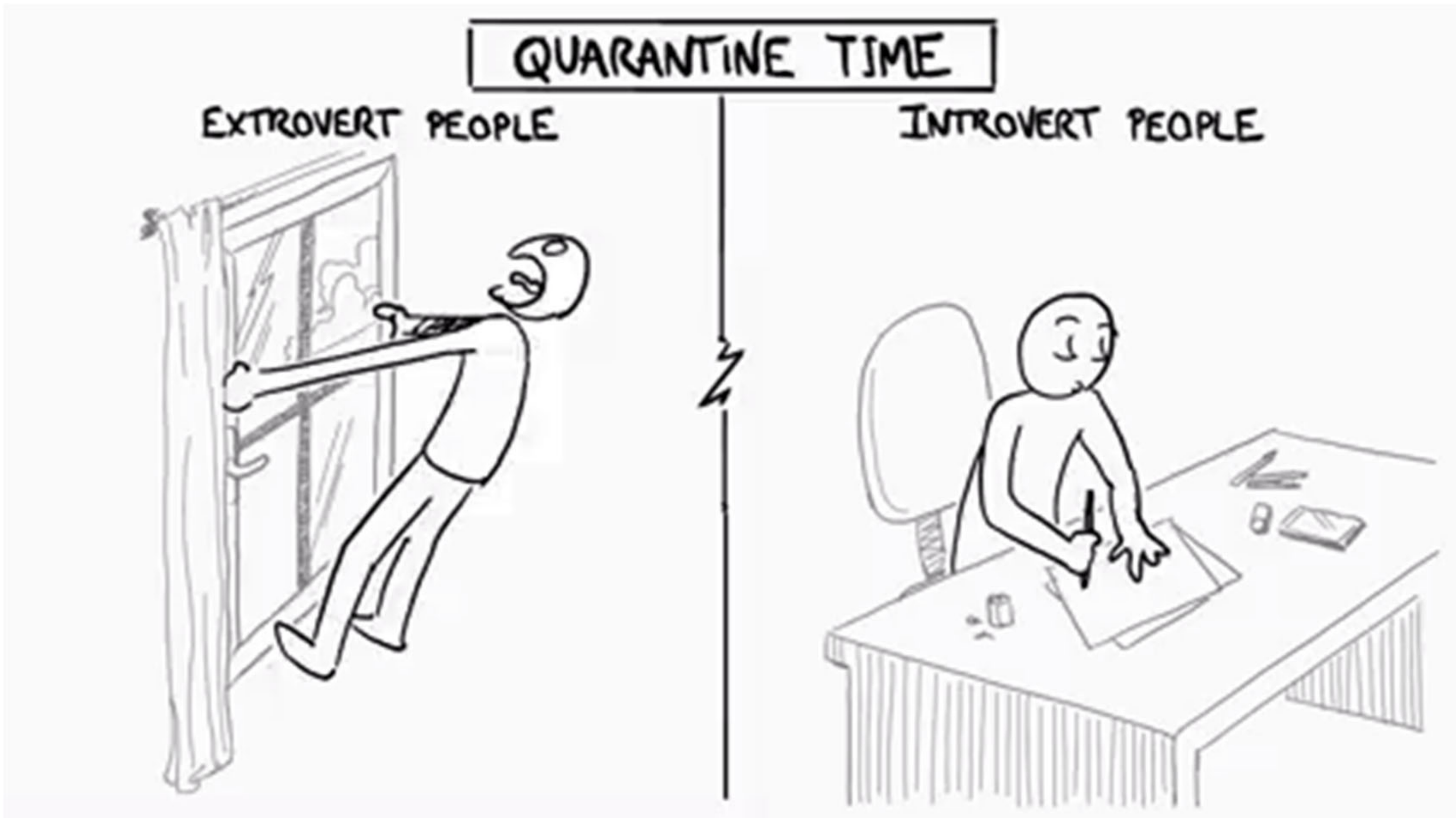
Department of Power Mechanical Engineering


QUARANTINE TIME
EXTROVERT PEOPLE
INTROVERT PEOPLE

Class Information

- Time: Lecture   M 1:20-3:10 PM (Engineering Building I Room 210, might move to Room 201)

  Lab   Th 1:10-2:10 PM (TBA)
- Instructor: Wei-Chih Wang

  office: Delta 319

  course website: http://depts.washington.edu/me557/optics (subject to change)
- Suggested Textbooks:
  - Applied Electromagnetism, Liang Chi Shen, Weber&Schmidt Dubury
  - Fundamentals of Photonics, B. Saleh, John Wiley& Sons.
  - Optical Methods of Engineering Analysis, Gary Cloud, Cambridge University Press.
  - Handbook on Experimental Mechanics, Albert S. Kobayashi, society of experimental mechanics.
  - Optoelectronics and Photonics: Principles and Practices, S. O. Kasap, Prentice Hall.
  - Fiber optic Sensors, E. Udd, John Wiley& Sons
  - Selected papers in photonics, optical sensors, optical MEMS devices and integrated optical devices.

Class information

- Grading

  Homework and Lab assignments 80% (2 assignments and 2 design projects and 2 labs)

  Final Project 20% (10% hardware, 10% final report+ final presentation)

- Final Project:

  - Choose topics related to simple free space optics design, fiberopic sensors, waveguide sensors or geometric Moiré, Moiré interferometer, photoelasticity for mechanical sensing or simple optical design.

  - Details of the project will be announced in mid quarter

  - three to four people can work as a team on a project, but each person needs to turn in his/her own final report (due last date of the final's week).

  - Oral presentation will be held sometimes in **week 17** (no final).

Week 1

- Lecture notes can be downloaded from:

  http://courses.washington.edu/me557/optics/

- Reading Materials:

  - Week 1 additional reading materials can be found:

  http://courses.washington.edu/me557/readings/

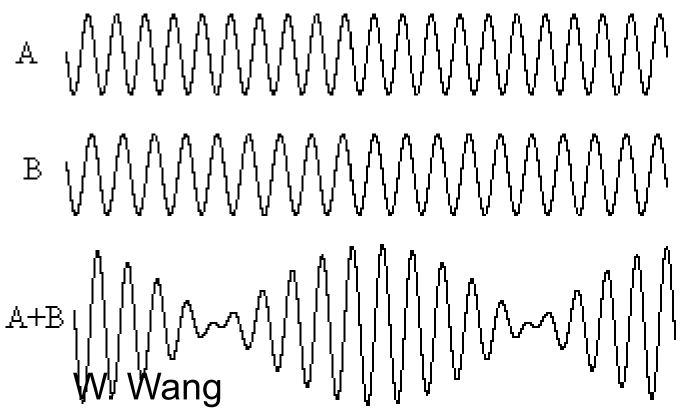
- Useful website: http://refractiveindex.info/?shelf=organic&book=polycarbonate&page=Sultanova
- Website updated weekly, make sure you upload the latest lecture notes and homework
- Week 5 no class (attending conference that week)
- There will be makeup classes on Thursday. I will let you know
- Week 17 Mon (6/10, Final Presentation)

Objectives

The main goal of this course is to introduce the characteristics of light that can be used to accomplish a variety of engineering tasks especially in physical sensing and mechanical analysis.

Manipulate phase modulation for mechanical measurement:

monitoring changes in interference pattern due to a mechanical modulation



$$\sin A+\sin B = 2\sin(A+B)/2 * \cos(A-B)/2$$

Let $A = k_1x+\omega_1t+ \phi_1 \qquad k_1 = 2\pi n_1/\lambda$

$B = k_2x+\omega_2t+ \phi_2 \qquad k_2 = 2\pi n_2/\lambda$

Course Outline

GOALS: To develop student understanding of

Week 1-4 Ray-Optics Approach (Snell's law, Geometric optics, thin lens, matrix method) and Light sources and photodetectors

Week 5-7 Electromagnetic-Wave Approach (wave equation, polarization, diffraction, interference, grating)

Week 8-9 Electromagnetic-Wave Approach (wave equation, polarization, diffraction, interference, diffraction grating, waveplate, Jones matrix)

Week 10 Optical Components (optical materials, coatings, filters, mirrors, lenses, prisms and polarizing optics)

Week 11 light sources

Week 12 detectors

Week 13 Geometric Moiré: In-plane displacement measurement

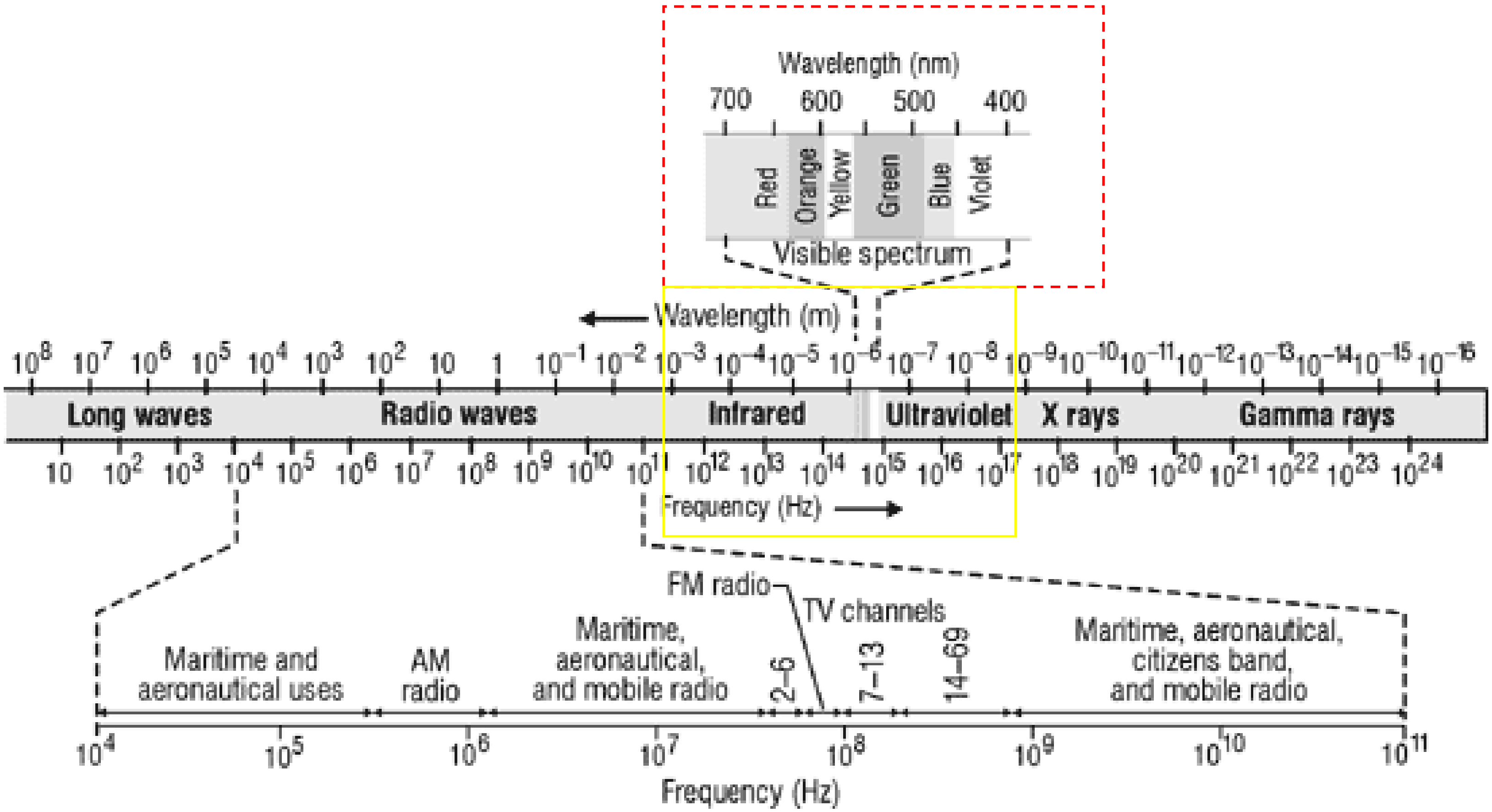
Week 14 Geometric Moiré: out of plane displacement measurement

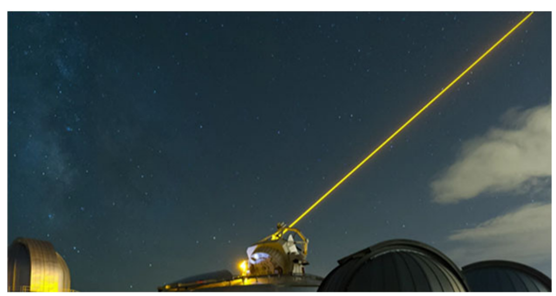
Week 15 Moire Interferometry: Interference and Diffraction, Grating fabrication

Week 16 Photoelasticity, Fiberoptic and polymer waveguide sensors

Week 16 Final project presentation

Electromagnetic Spectrum





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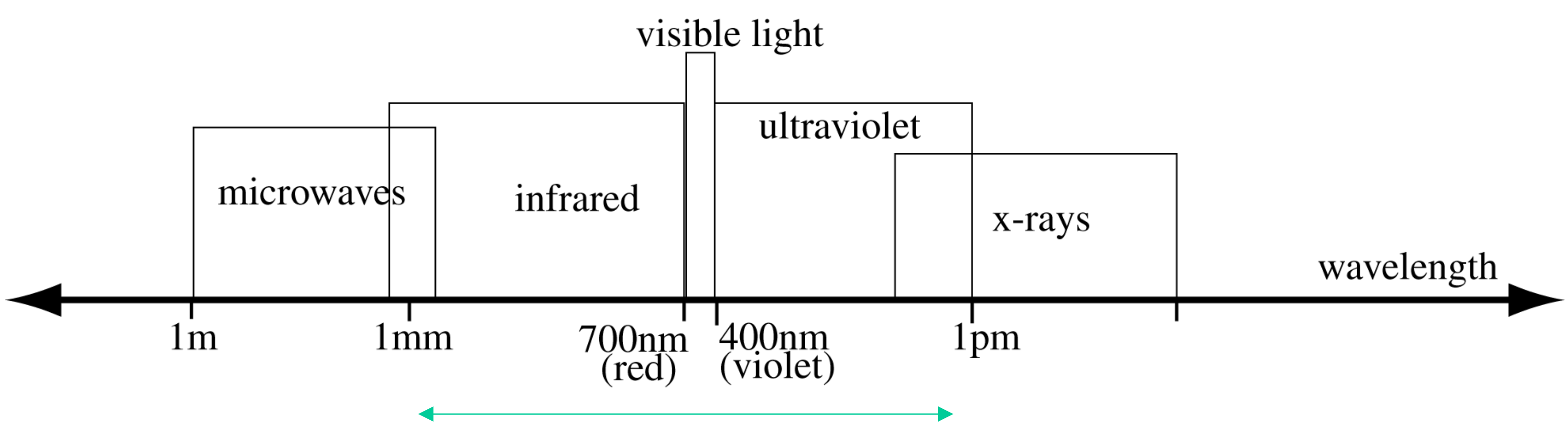
Spectrum of "optical" radiation

- Nomenclature:
  - Visible light
  - Infrared radiation (not infrared "light")
  - Ultraviolet radiation (not UV "light")
- Ranges shown are approximate and somewhat arbitrary



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Infrared radiation

- Approximate spectrum
  - 1mm (300 GHz) to 700nm (430 THz)
- Meaning: below red (in terms of frequency)
- Near infrared (closer to visible light)
- Far infrared (closer to microwaves)
- Invisible radiation, usually understood as **"thermal" radiation**
- 1nm=10^{-9}m 1GHz=10^{9} Hz, 1THz=10^{15} Hz

Visible light

- Approximate spectrum
  - 700nm (430 THz) to 400nm (750 THz)
- Based on our eye's response
- From red (low frequency, long wavelength)
- To violet (high frequency, short wavelength)
- Our eye is most sensitive in the middle (green to yellow)
- Optical sensors may cover the whole range, may extend beyond it or may be narrower

Ultraviolet (UV) radiation

- Approximate spectrum
  - 400nm (750 THz) to 400pm (300 PHz)
- Meaning - above violet (in terms of frequency)
- Understood as "penetrating" radiation
- Only the lower end of the UV spectrum is usually sensed
- Exceptions: radiation sensors based on ionization

SI multiple for hertz (Hz)

SI multiples for hertz (Hz)

Submultiples				Multiples		
Value	SI symbol	Name		Value	SI symbol	Name
10^{-1} Hz	dHz	decihertz		10^{1} Hz	daHz	decahertz
10^{-2} Hz	cHz	centihertz		10^{2} Hz	hHz	hectohertz
10^{-3} Hz	mHz	millihertz		10^{3} Hz	**kHz**	**kilohertz**
10^{-6} Hz	µHz	microhertz		10^{6} Hz	**MHz**	**megahertz**
10^{-9} Hz	nHz	nanohertz		10^{9} Hz	**GHz**	**gigahertz**
10^{-12} Hz	pHz	picohertz		10^{12} Hz	**THz**	**terahertz**
10^{-15} Hz	fHz	femtohertz	➡	10^{15} Hz	PHz	petahertz
10^{-18} Hz	aHz	attohertz		10^{18} Hz	EHz	exahertz
10^{-21} Hz	zHz	zeptohertz		10^{21} Hz	ZHz	zettahertz
10^{-24} Hz	yHz	yoctohertz		10^{24} Hz	YHz	yottahertz

Common prefixed units are in bold face.

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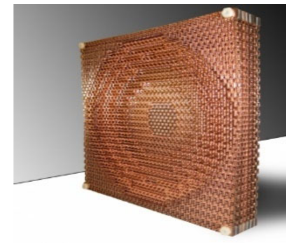
Difference between RF and Optical System

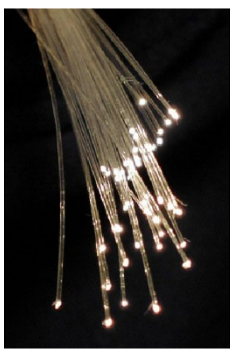
- Frequency, bandwidth

Materials and operating frequencies

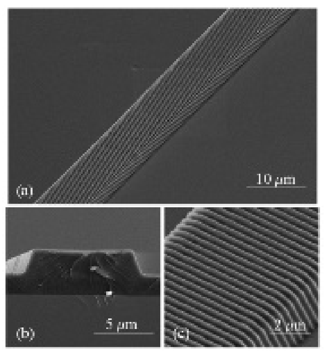
Free Space Optics transceiver system



New metamaterial lens focuses radio waves | MIT News
MIT News - Massachusetts Institute of Technology



Optical fiber



Brgg grating waveguide
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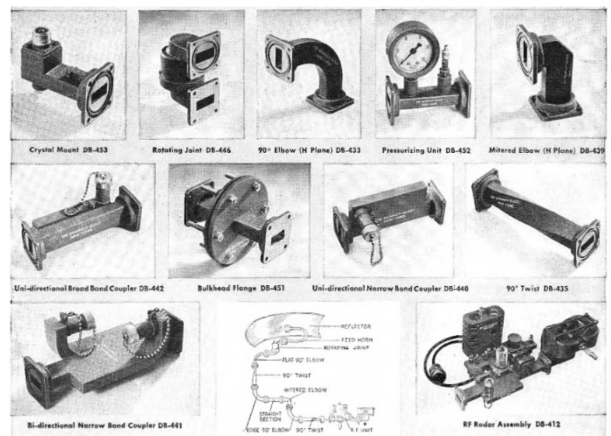


Metal waveguide Wikipedia

- Dielectric instead of metal materials (lower attenuation)

Waveguide Structure

- Metallic waveguide (hallow metal waveguide, coaxial cable, micro strip)
- Dielectric waveguide (optical fiber, integrated waveguide)

Optical fiber

Coaxial cable

Differences Between Metallic and Dielectric Waveguides

Losses are from real and imaginary parts of epsilon- real part from Snell's law and imaginary part from conductivity (dielectric not a function of wavelength).

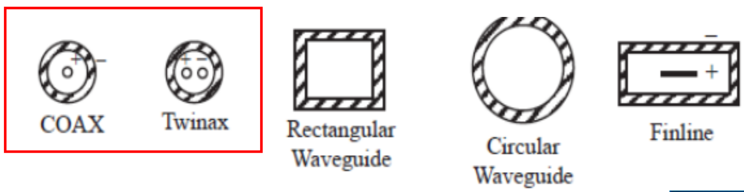
- At millimeter wave frequencies and above, **metal is not a good conductor, so metal waveguides can have increasing attenuation**. At these wavelengths dielectric waveguides can have lower losses than metal waveguides. Optical fiber is a form of dielectric waveguide used at optical wavelengths.
- One difference between dielectric and metal waveguides is that at **a metal surface the electromagnetic waves are tightly confined**; **at high frequencies the electric and magnetic fields penetrate a very short distance into the metal (smaller the skin depth).** In contrast, **the surface of the dielectric waveguide is an interface between two dielectrics, so the fields of the wave penetrate outside the dielectric in the form of an evanescent (non-propagating) wave.**

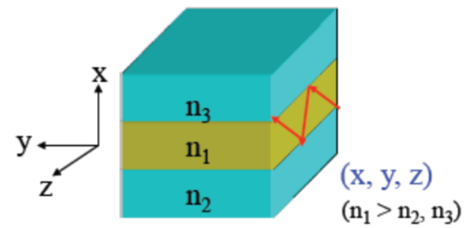
Field Confinement: Materials and Field Consideration

For <1GHz

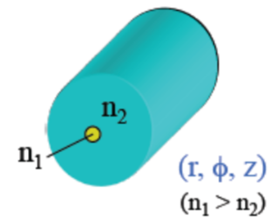
For >1GHz

For >100GHz

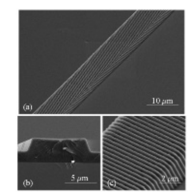




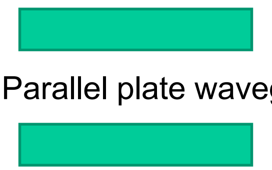
Planar (slab) waveguides for integrated photonics (e.g. laser chips)

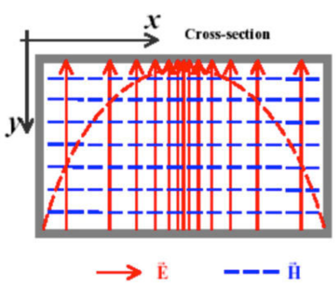


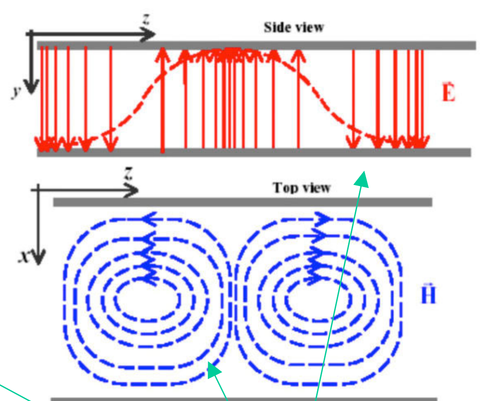
Cylindrical optical fibers

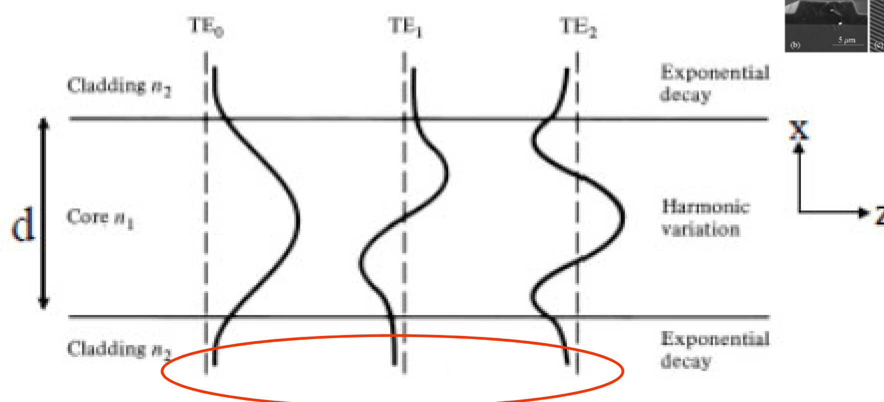


Parallel plate waveguide



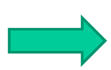






Tightly confined wave with small skin depth

Evanescent waves



Loss due to mismatch in real part of index (confinement)

Actual propagation direction

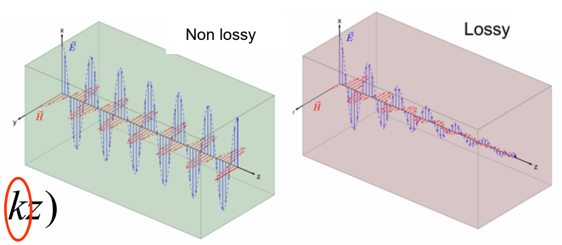
Loss due to guided material in propagation direction

Material consideration: Why metal not a good conductor

Look up real part of dielectric constant at higher frequency becoming negative and how it affects field in propagation direction.

Metal: For highly conducting medium in high frequency, the k constant is

$$k = k - ja = \omega\sqrt{\mu\varepsilon}(1-j\frac{\sigma}{\omega\varepsilon})^{1/2} \sim \omega\sqrt{\mu\varepsilon}(-j\frac{\sigma}{\omega\varepsilon})^{1/2} = \sqrt{\omega\mu(\frac{\sigma}{2})}(1-j)$$



negative plasmonic

Large and lossy

$$E(z,t) = \mathrm{Re}\{Ee^{j\omega t}\} = \hat{x}E_o\cos(\omega t - kz)$$

Dielectric: $k = k - ja = \omega\sqrt{\mu\varepsilon}(1\text{-}j\frac{\sigma}{\omega\varepsilon})^{1/2} \approx \omega\sqrt{\mu\varepsilon}(1\text{-}j\frac{\sigma}{2\omega\varepsilon})$

positive

Small and independent of wavelength

Material consideration: Why metal not a good conductor

All **these losses are in transverse** direction

Metal: **higher the frequency higher the attenuation**

For highly conducting medium, $\sigma/\omega\varepsilon >> 1$, the k constant can be simplify to

$$k = k - ja = \omega\sqrt{\mu\varepsilon}(1-j\frac{\sigma}{\omega\varepsilon})^{1/2} \sim \omega\sqrt{\mu\varepsilon}(-j\frac{\sigma}{\omega\varepsilon})^{1/2} = \sqrt{\omega\mu(\frac{\sigma}{2})}(1-j)$$

Simplification

$$\boxed{\alpha = \omega\mu\sigma/2}$$

Look up imaginary part of dielectric constant at higher frequency becoming more lossy in metal, but independent of wavelengths in dielectric material

Dielectric: **Independent of wavelength**

For slightly conducting media, where $\sigma/\omega\varepsilon << 1$, the constant k can be approximated by

$$k = k - ja = \omega\sqrt{\mu\varepsilon}(1-j\frac{\sigma}{\omega\varepsilon})^{1/2} \approx \omega\sqrt{\mu\varepsilon}(1-j\frac{\sigma}{2\omega\varepsilon})$$

Simplification

$$\boxed{\alpha = \frac{2}{\sigma}\sqrt{\frac{\varepsilon}{\mu}}}$$

Penetration Depth (dielectric and slight conductive)

According to Beer-Lambert lawType equation here., the intensity of an **electromagnetic wave inside a material falls off exponentially from the surface** as

$$I(z) = I_o e^{-\alpha z}$$

If δ_p denotes the penetration we have $\delta_p = 1/\alpha$ "Penetration depth" is one term that describes the decay of electromagnetic waves inside of a material. The above definition refers to the depth δ_p at which the intensity or power of the field decays to 1/e of its surface value. In many contexts one is concentrating on the field quantities themselves: the electric and magnetic fields in the case of electromagnetic waves. Since the power of a wave in a particular medium is proportional to the square of a field quantity, one may speak of a penetration depth at which the magnitude of the electric (or magnetic) field has decayed to 1/e of its surface value, and at which point the power of the wave has thereby decreased to 1/e or about 13% of its surface value:

$$\delta_e = \frac{1}{\alpha/2} = \frac{2}{\alpha} = 2\delta_p$$ (slightly conductive)

$$\delta_p = 1/\alpha = \sqrt{\left(\frac{2}{\omega\mu\sigma}\right)} = \delta$$ (highly conductive)

Note that δ is identical to the skin depth, the latter term usually applying to metals in reference to the decay of electrical currents or we only use penetration depth to descrie the media

Highly Conducting Media

For highly conducting medium, **σ/ωε >>1,** the k constant can be simplify to

$$k = k - ja = \omega\sqrt{\mu\varepsilon}(1 - j\frac{\sigma}{\omega\varepsilon})^{1/2} \sim \omega\sqrt{\mu\varepsilon}(-j\frac{\sigma}{\omega\varepsilon})^{1/2} = \sqrt{\omega\mu(\frac{\sigma}{2})}(1-j)$$

The penetration depth $\delta_p = 1/\alpha = \sqrt{(\frac{2}{\omega\mu\sigma})} = \delta$ (skin depth) only for highly conductive media.

higher the frequency higher the attenuation

$$\boxed{\alpha = \omega\mu\sigma/2}$$

Skin effect in conductor

We can derive a practical formula for skin depth :

$$\delta = \sqrt{\frac{2\rho}{(2\pi f)(\mu_o \mu_r)}}$$

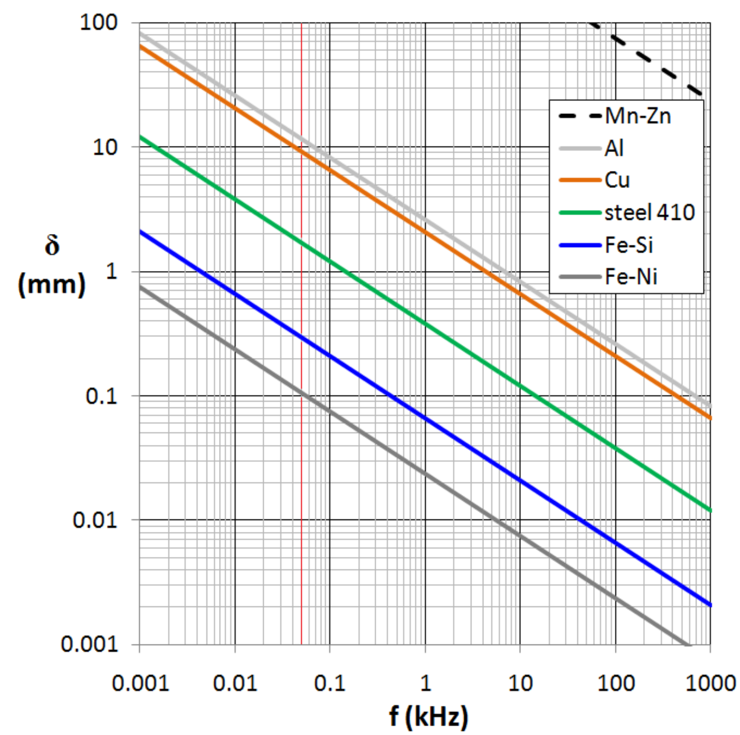
Where

δ = the skin depth in meters

μ_r = the relative permeability of the medium

ρ = the resistivity of the medium in Ω·m, also equal to the reciprocal of its conductivity: $\rho = 1/\sigma$ (for copper, $\rho = 1.68 \times 10^{-8}$ Ω·m)

f = the frequency of the current in Hz



Metal Conductivity (σ)

The common metals that have the highest resistivity (lowest conductivity) are:

1. Mercury
2. Stainless steel varieties
3. Titanium
4. Lead
5. Carbon
6. Carbon steel
7. Tungsten

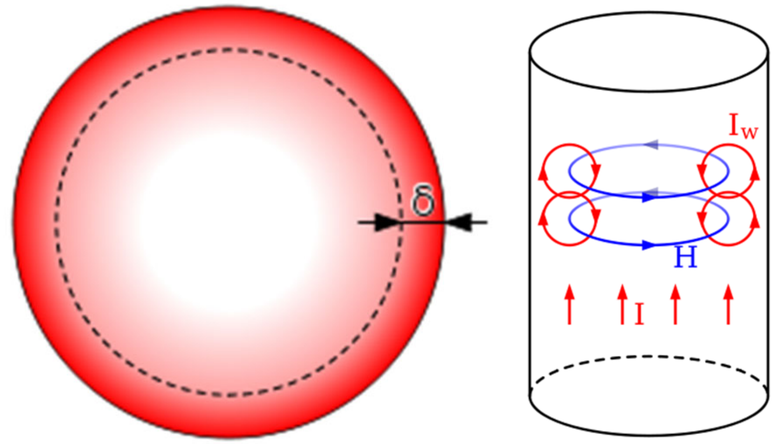
The common metals that have the lowest resistivity (highest conductivity) are:

1. Silver
2. Copper
3. Gold
4. Aluminum
5. Zinc
6. Brass
7. Nickel

Keep in mind, too, that purity of the metals effects conductivity and it inverse property, resistivity.

Skin effect

Skin effect is the tendency of an alternating electric current (AC) to become distributed within a conductor such that the current density is largest near the surface of the conductor, and decreases with greater depths in the conductor. The electric current flows mainly at the "skin" of the conductor, between the outer surface and a level called the skin depth. The skin effect causes the effective resistance of the conductor to increase at higher frequencies where the skin depth is smaller, thus reducing the effective cross-section of the conductor. The skin effect is due to opposing eddy currents induced by the changing magnetic field resulting from the alternating current. At 60 Hz in copper, the skin depth is about 8.5 mm. At high frequencies the skin depth becomes much smaller. Increased AC resistance due to the skin effect can be mitigated by using specially woven litz wire. Because the interior of a large conductor carries so little of the current, tubular conductors such as pipe can be used to save weight and cost.



Distribution of current flow in a cylindrical conductor, shown in cross section. For alternatin current, most (63%) of the electric current flows between the surface and the skin depth, δ, which depends on the frequency of the current and the electrical and magnetic properties of the conductor

For Slightly Conducting Media

For slightly conducting media, where <u>$\sigma/\omega\varepsilon \ll 1$,</u> the constant k can be approximated by

$$k = k - ja = \omega\sqrt{\mu\varepsilon}(1\text{-}j\,\frac{\sigma}{\omega\varepsilon})^{1/2} \approx \omega\sqrt{\mu\varepsilon}(1\text{-}j\,\frac{\sigma}{2\omega\varepsilon})$$

Independent of wavelength

$$\boxed{\alpha = \frac{2}{\sigma}\sqrt{\frac{\varepsilon}{\mu}}}$$

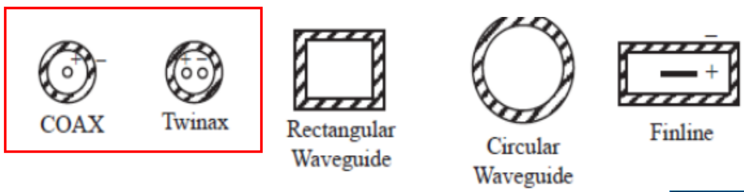
Penetration depth $\boxed{\delta_p = 1/\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}}$ (here we don't have skin depth, skin depth only refers to metal)

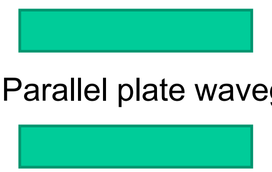
Second Distinction: Field Confinement

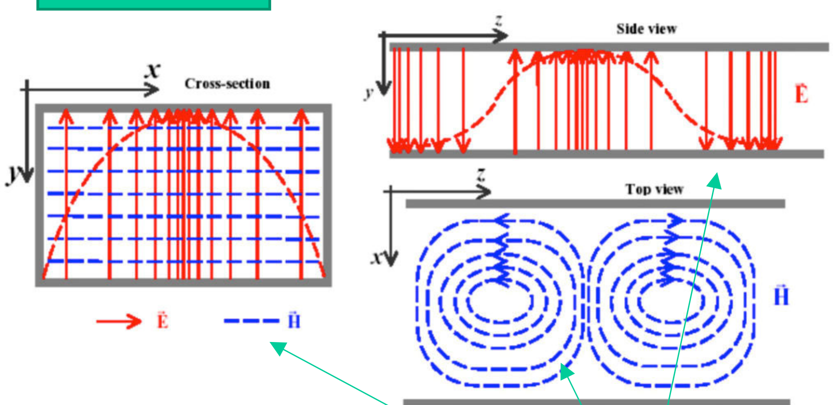
For <1GHz

For >1GHz

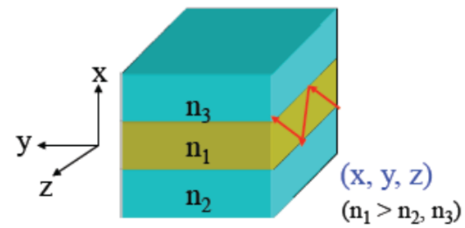
For >100GHz



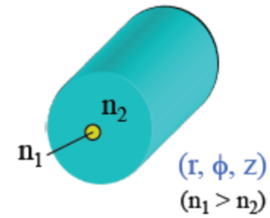




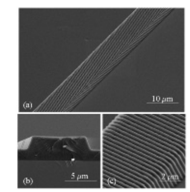
Tightly confined wave with small skin depth

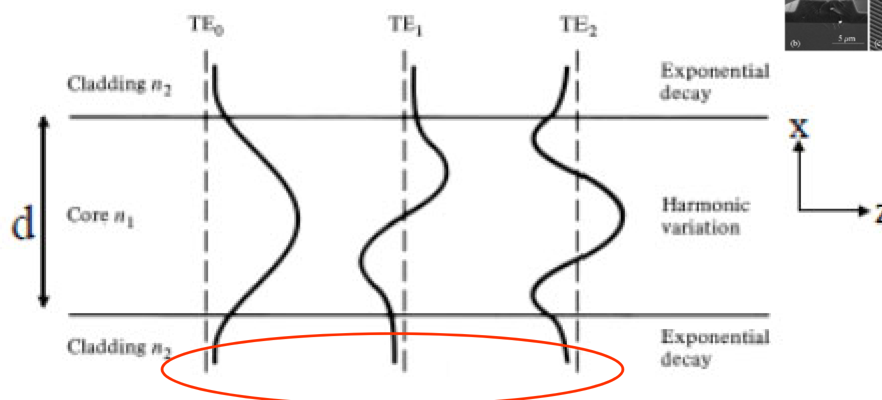


Planar (slab) waveguides for integrated photonics (e.g. laser chips)



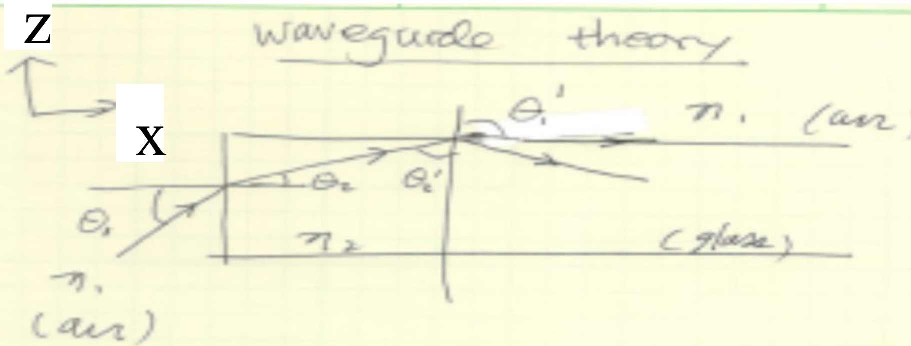
Cylindrical optical fibers





Evanescent waves

waveguide theory



Wave leakage occurs as dictated by the phase matching equation. The extent of leakage depends on the cladding material, which surrounds the core or guided material. In the case of metal, the leakage is rapidly absorbed, while in dielectric materials, it attenuates more gradually due to the matching of refractive indices

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$n_2 \sin\theta_2' = n_1 \sin\theta_1'$$

$\theta_1' = 90°$ ∴ total reflection occurs

then $\theta_2' = \theta_{critical}$

no wave travel outside the glass substrate, therefore glass substrate becomes a wave guide

However if $\theta_2' > \theta_{critical}$

Then

$$n_2 \sin\theta_2' = n_1 \underbrace{\sin\theta_1'}_{\substack{\Uparrow \\ >1}}$$

$$1 = \underbrace{\sin^2\theta_1'}_{>1} + \boxed{\underbrace{\cos^2\theta_1'}_{<0}}$$

since $k_1^2 = k_{1z}^2 + k_{1x}^2$

$$k_{1z}^2 = k_1^2 - k_{1x}^2$$

$$k_1^2 - (k_1 \sin\theta_1')^2 \Rightarrow k_{1z}^2 < 0$$

$$k_1^2 = k_1^2\sin^2\theta_1' + \boxed{k_1^2\cos^2\theta_1'}$$

k_{1z} = imaginary

Losses due to real part of ε at BC

Earlier attenuation is due to resistivity

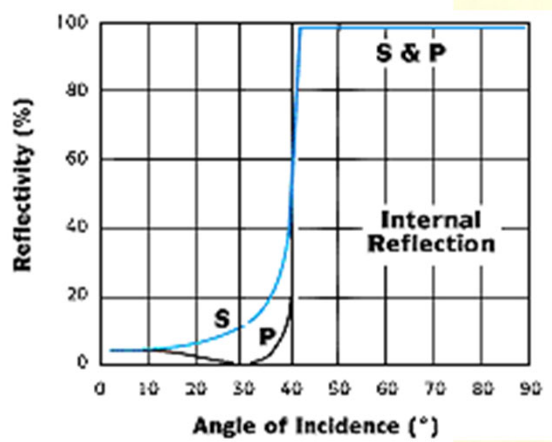
$E e^{j k_{1x} x + j k_{1z} z}$

$Re\{ (E e^{j k_{1x} X}) e^{j \overset{\text{imaginary}}{\overset{\parallel}{k_{1z}}} z} \}$

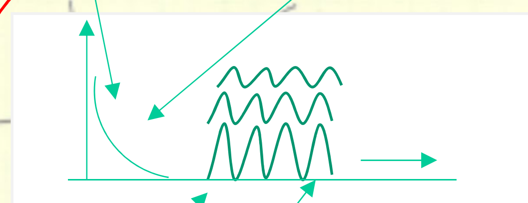
$= E_0 \cos k_{1x} X \; e^{-j\alpha_z z}$

where α_z = i~~maginary~~ k_{1z} = imaginary

Wave leakage occurs as dictated by the phase matching equation. The extent of leakage depends on the cladding material, which surrounds the core or guided material. In the case of metal, the leakage is rapidly absorbed, while in dielectric materials, it attenuates more gradually due to the matching of refractive indices



$e^{-j\alpha_z z}$



$\cos k_{1x} X$

Evanescent wave

use for sensing and wave coupling

$\cos k_{1x}$x at different z

Comparison of current free space and guide wave telecommunication

Difference between Fiber and Microwave

Specifications	Optical Fiber line	Microwave
Capacity	>100Gb/sec (depending on freq)	in Gb/sec
Cost	Costs as per feet or meter	cost per link, it is incremental based on distance
Deployment time	It increases with distance and vary linearly	Fast
Terrain	The system will become more costlier in difficult terrain region	Suitable for any terrain region, Need to have Line of Sight between two points(transmit and receiver)
Re-use	Fiber once deployed can not be re-located in most of the cases	Microwave equipments can be removed and re-located in other regions if required
Climate effect	Normally fiber is not influenced except in the flood conditions	Microwave link is influenced by climate, Adaptive modulation coding (AMC) is used as solution in changing channel environment
Regulation	Needs right of ways and proper infrastructure	requires spectrum regulation

RF wireless world

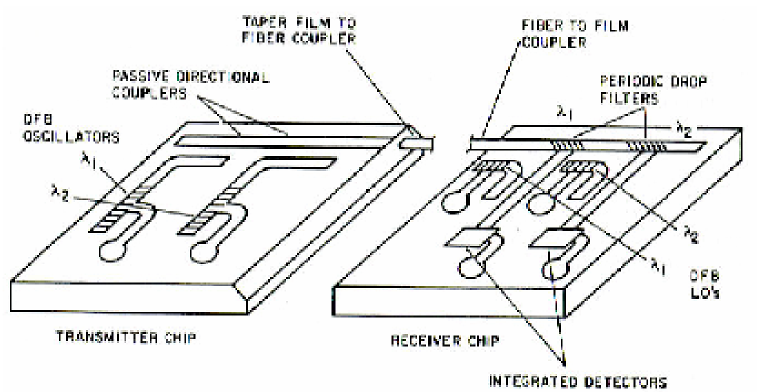
Optical System

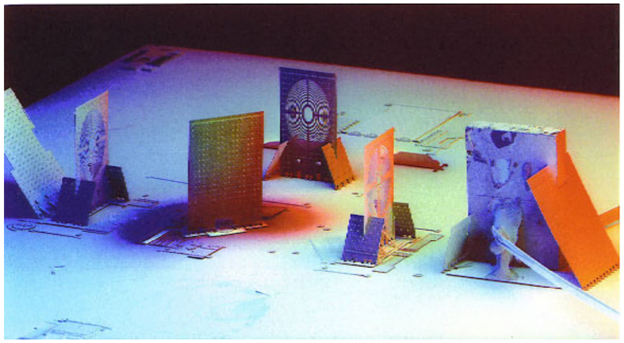
- Free space Optics
- Guided Optics

Optical MEMS (MOEM) and Waveguide Integrated Optics

Waveguide Integrated Optics
(what's known as integrated optics in earlier day)

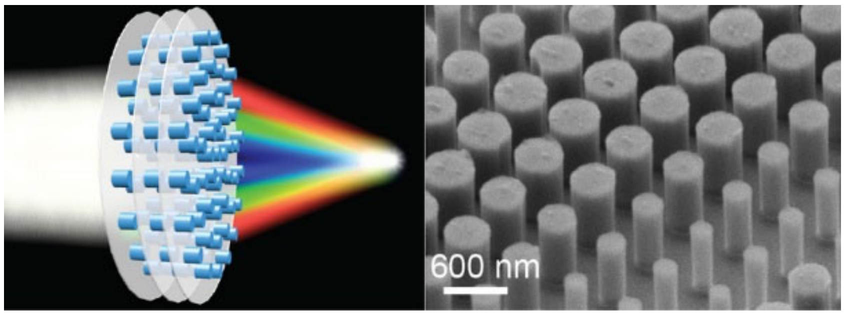
Optical MEMS
(can be free space or waveguide)



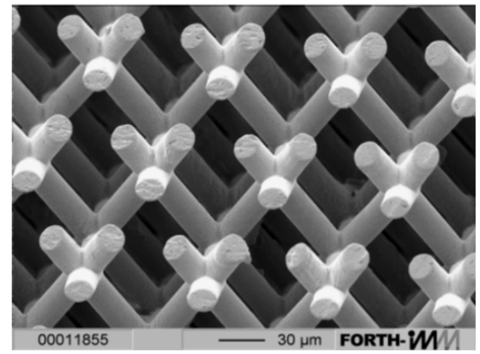


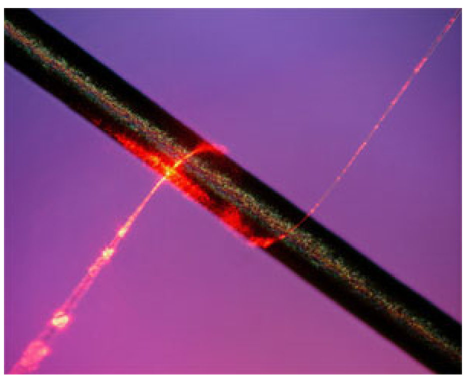
Photonic integrated circuit include optical MEMS and waveguide integrated optics

Nano optics

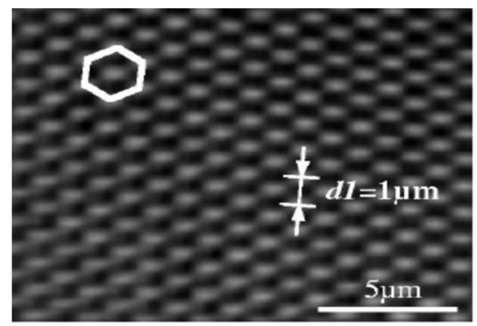


Diffraction lens





subwavelength-diameter optical



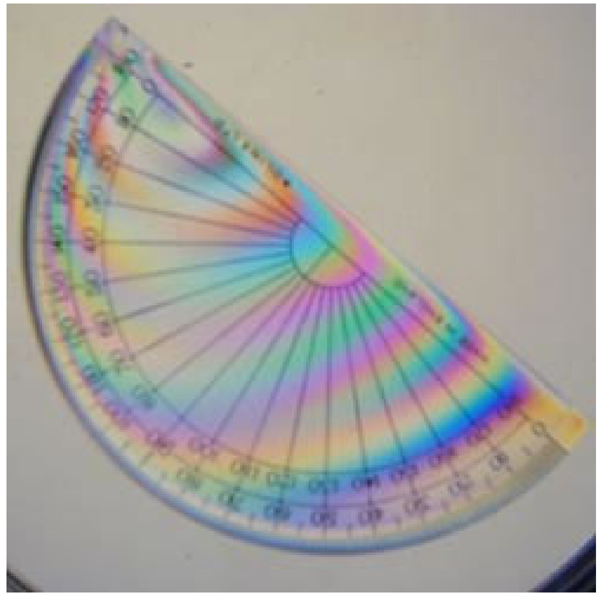
Photonic bandgap Crystal

Photonics Opportunities

•Medicine-biomedical (laser surgery and in noninvasive diagnostic tools)
•Environmental (measure the pollutants in air and water)
•Energy (harness solar energy)
•Transportation (provides guidance, collision avoidance, and continuous tuning of engines based on driving conditions)
•Defense (weapon guidance, remote sensing, image processing, and high-energy laser operation)
•Computers and Communication and information technology (gathering, manipulating, storing, routing, and displaying information)
•Manufacturing with photonics and test and analysis (industrial lasers that cut, weld, trim, drill holes, and heat-treat products. inspection is performed using spectroscopy, interferometry, machine vision, and image processing)
•Consumer electronics (camera, cell phone, TV, monitor etc.)

Free Space Optics Application

Photoelasticity



Department of Materials Science and Metallurgy
University of Cambridge

The effect that an isotropic material can become birefringent (anisotropic), when placed under stress is called photoelasticity.

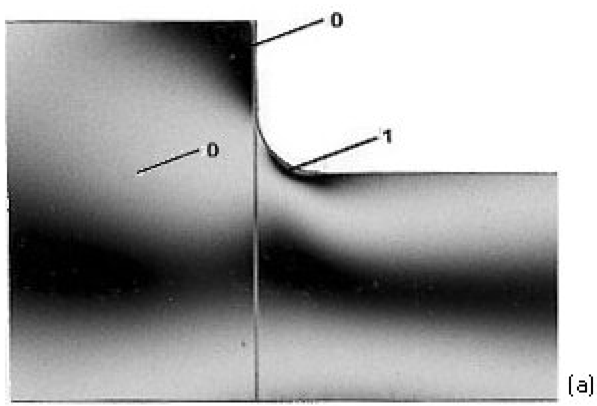
Under compression $\rightarrow$ negative uniaxial crystal.

Under tension $\rightarrow$ positive uniaxial crystal.

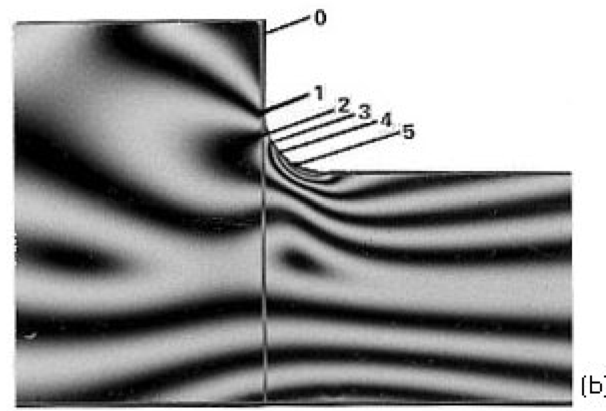
Induced birefringence is proportional to the stress.

can be used to study stress patterns in complex objects (e.g. bridges) by building a transparent scale model of the device.

Photoelasticity effect



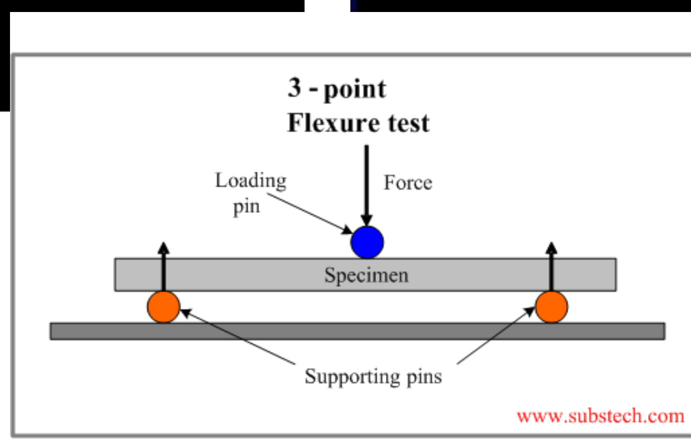
Low load



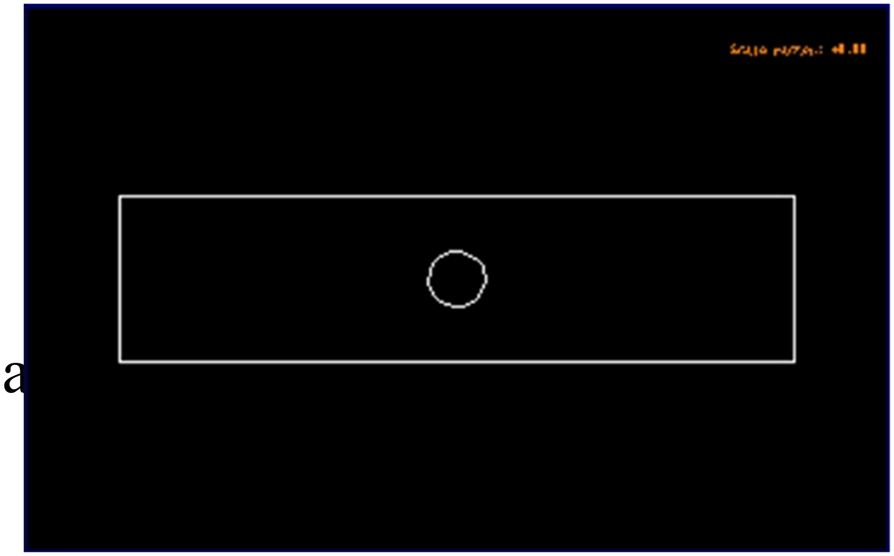
High load

University of Maryland

As the load is increased and new fringes appear, the earlier fringes are pushed toward the areas of lower stress. With further loading, additional fringes are generated in the highly stressed regions and move toward regions of zero or low stress until the maximum load is reached. The fringes can be assigned ordinal numbers (first, second, third, etc.)



Annealed bar undergoing 3-point bending under a circular polariscope



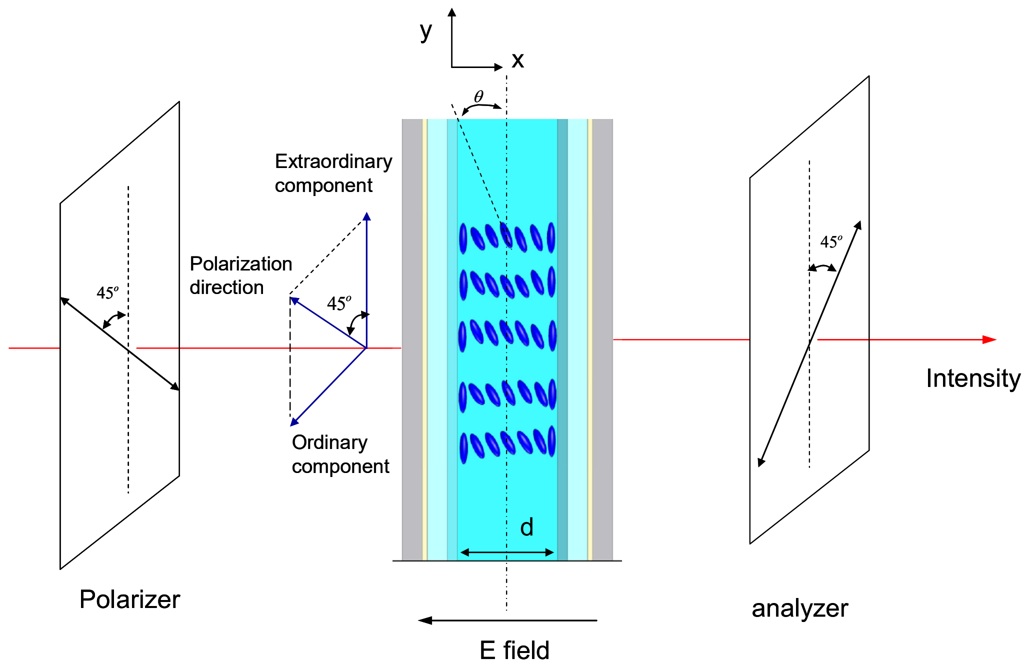
Annealed bar undergoing 3-point bending under a circular polariscope

Phase shifter using Birefringent Material

$$T=\begin{pmatrix}\frac{1}{2} & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{2}\end{pmatrix}\begin{pmatrix}\exp(-j\Delta\phi) & 0\\ 0 & 1\end{pmatrix}\begin{pmatrix}\frac{1}{2} & -\frac{1}{2}\\ -\frac{1}{2} & \frac{1}{2}\end{pmatrix}=\frac{1}{4}(e^{(-j\Delta\phi)}-1)\begin{pmatrix}1 & -1\\ 1 & -1\end{pmatrix}$$

$$I=I_o\left|T\right|^2=I_o\sin^2(\tfrac{\Delta\phi}{2})$$
$$=\frac{I_0}{2}(1-\cos\Delta\phi)$$

$\Delta\phi$ phase difference between the ordinary and extraordinary components



Moire

Moiré effect is the mechanical interference of light by superimposed network of lines.

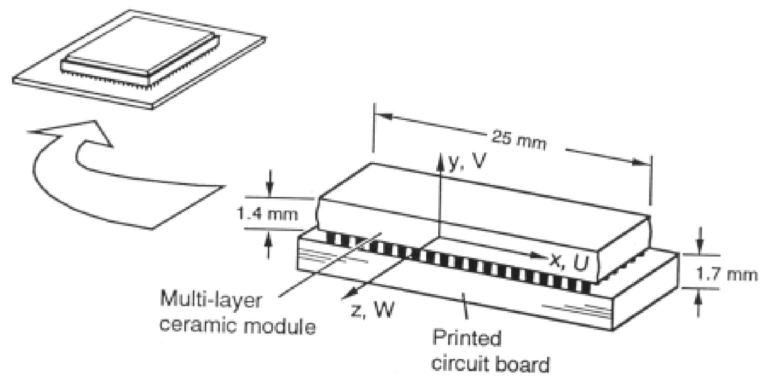
The pattern of broad dark lines that is observed is called a moiré pattern.

Superimposed Gratings

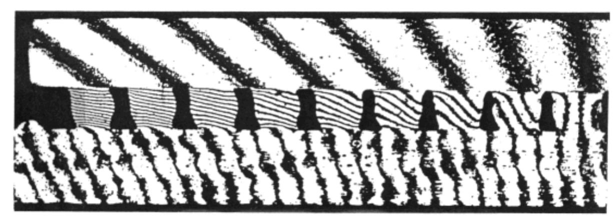
Moire

In most basic form, Moiré methods are used to measure

Displacement fields; either



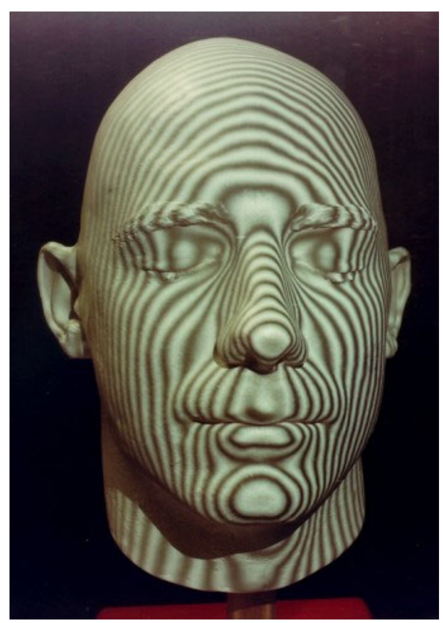
- in plane displacement



U (or u_x) displacement field of a ceramic ball grid array package assembly, subjected to an isothermal loading of ΔT = -60°C (cooling the specimen from 82°C to room temperature).

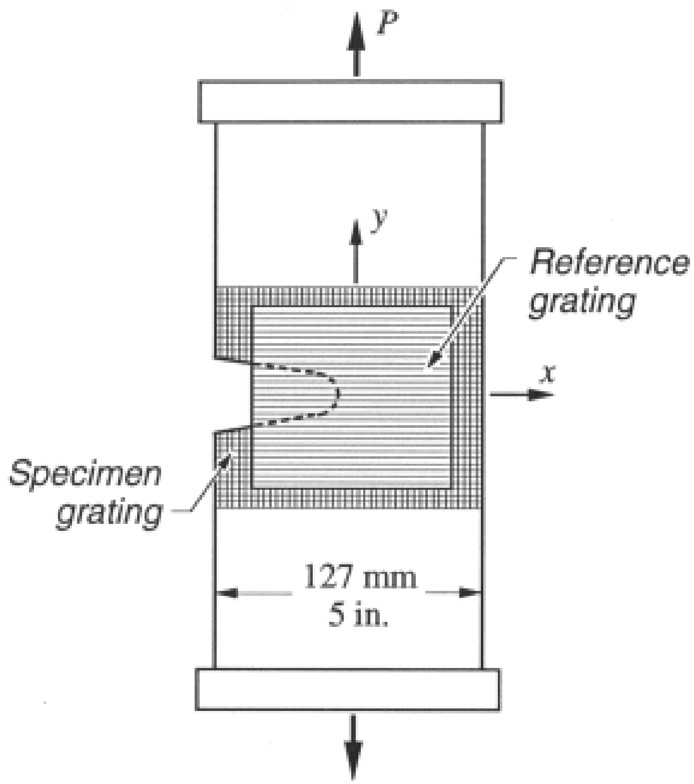
Univ. of Maryland

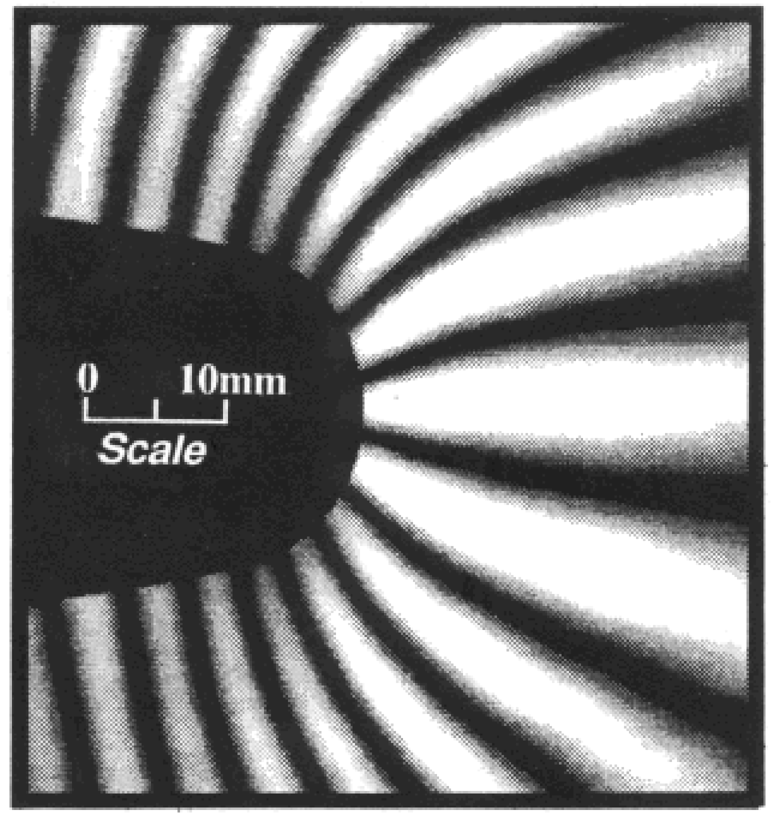
- out of plane



Moiré Fringes Superimposed on Model Head

Geometric Moiré





V (or u_y) displacement field of a deeply notched tensile specimen. The contour interval is 50 μm per fringe.

In-Plane

Geometric moiré

Sensitivity (No. of fringes per unit displacement)	Less than 100 lines/mm
Contour Interval (displacement/fringe order)	Greater than 10 micrometers
Field of View	Large

Moiré Interferometery

Sensitivity (No. of fringes per unit displacement)	2.4 lines/micrometer
Contour Interval (displacement/fringe order)	.417 micrometers
Field of View	Small (typically 5mm to 50mm)

Microscopic Moiré Interferometry

Sensitivity (No. of fringes per unit displacement)	4.8 lines/micrometer
Contour Interval (displacement/fringe order)	208 to 20.8 nanometers
Field of View	Microscopic (typically 50 micromters to 1 mm)

Out of Plane

Shadow Moiré/Projection Moiré

Sensitivity (No. of fringes per unit displacement)	less than 100 lines/mm
Contour Interval (displacement/fringe order)	greater than 10 micrometers
Field of View	Large (up to 100 mm)

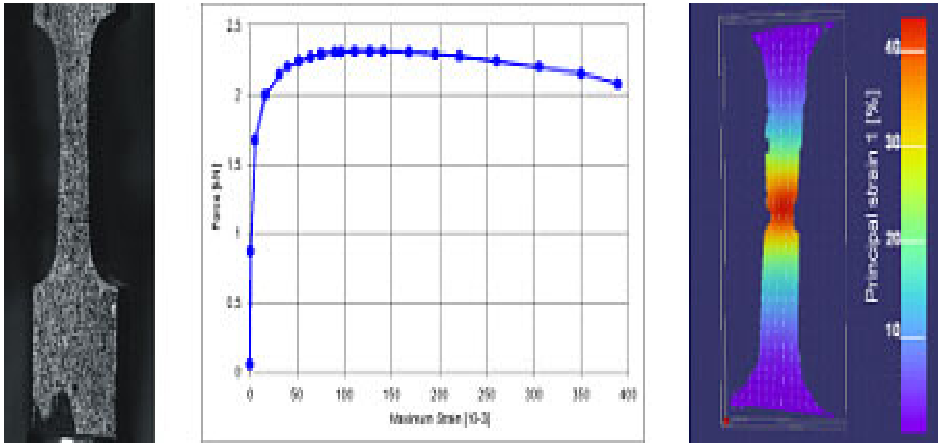
INFRARED FIZEAU INTERFEROMETRY

Sensitivity (No. of fringes per unit displacement)	200 to 400 lines/mm
Contour Interval (displacement/fringe order)	2.5 to 5 micrometers
Field of View	Medium (5 to 45 mm)

Digital Image Correlation

Material properties

DIC offers characterization of material parameters far into the range of plastic deformation. Its powerful data analysis tools allow the determination of the location and amplitude of maximum strain, which are important functions in material testing.



Live image (left), maximum principal strain (middle), principal strain (right)

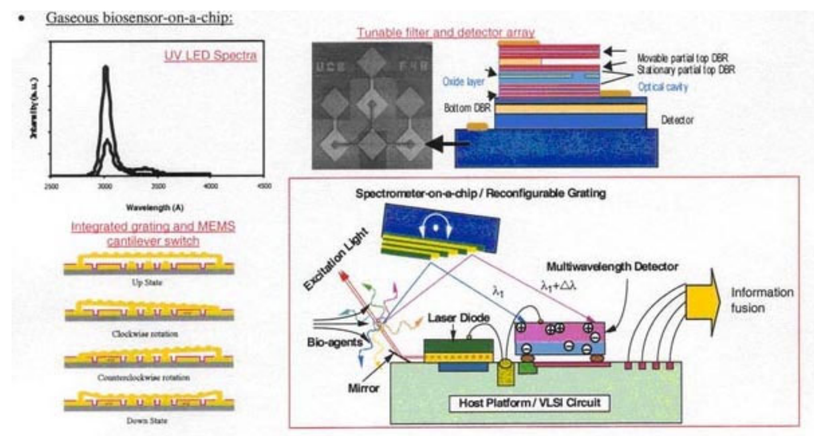
Guide Wave and Devices

Fiber optic and Waveguide Sensors

Fiber optic Sensors

Ocenan optics

waveguide based optoelectronic bio-sensor systems



Center for Bio-Optoelectronic Sensor SystemsUniversity of Illinois ,

Fiberoptic Sensors

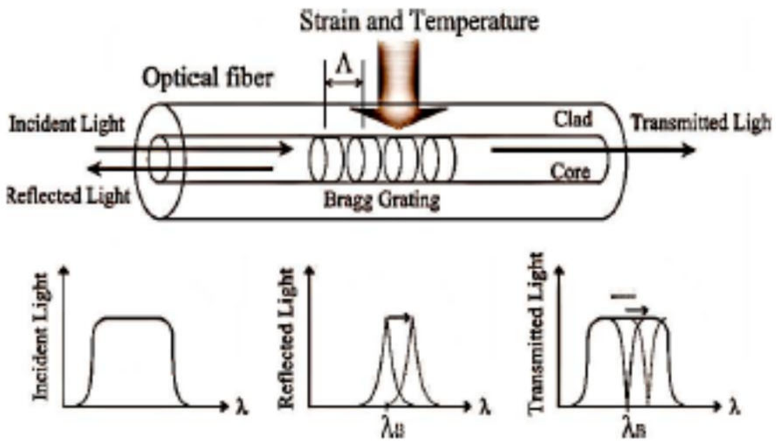


Figure 3. Fiber Bragg grating.

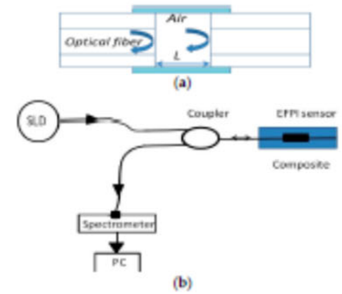


Figure 9. (**a**) One of the typical EFPI sensor design; and (**b**) schematic experimental arrangement for the EPFI sensor [70].

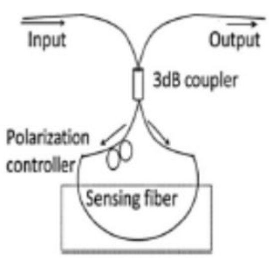


Figure 11. Schematic of the sensor based on a Sagnac interferometer.

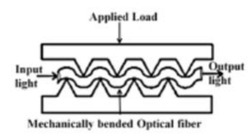


Figure 14. Micro bend sensor concept [40].

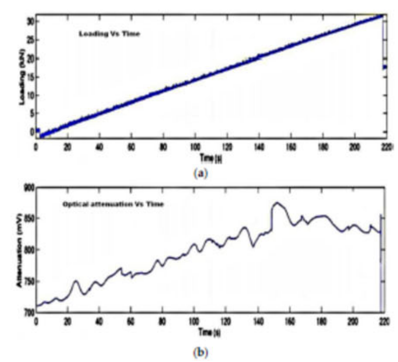
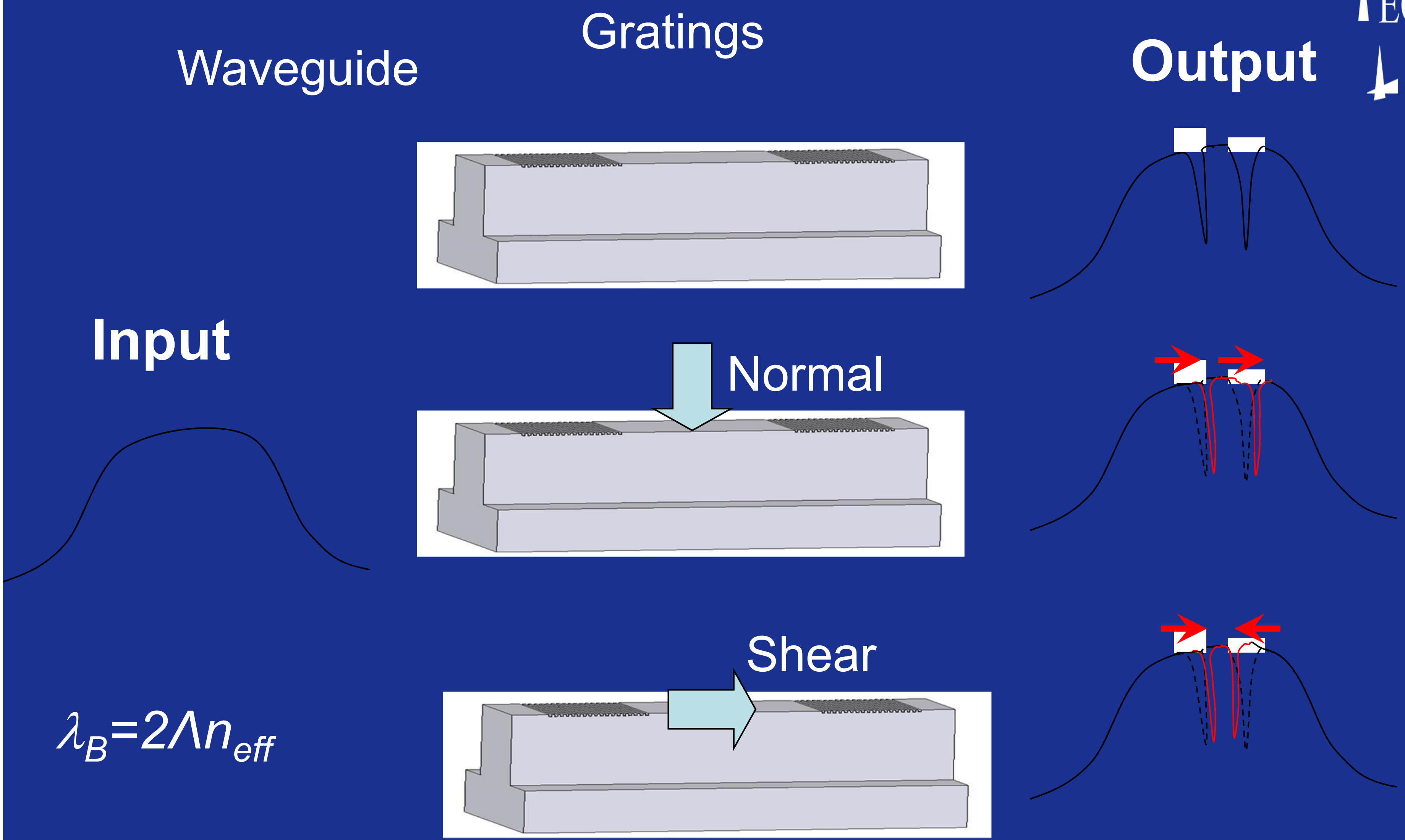


Figure 15. The temporal profiles corresponding to loading (**a**) and optical signal attenuation (**b**) [40].

M. Ramakrishnan, Sensor, 2016

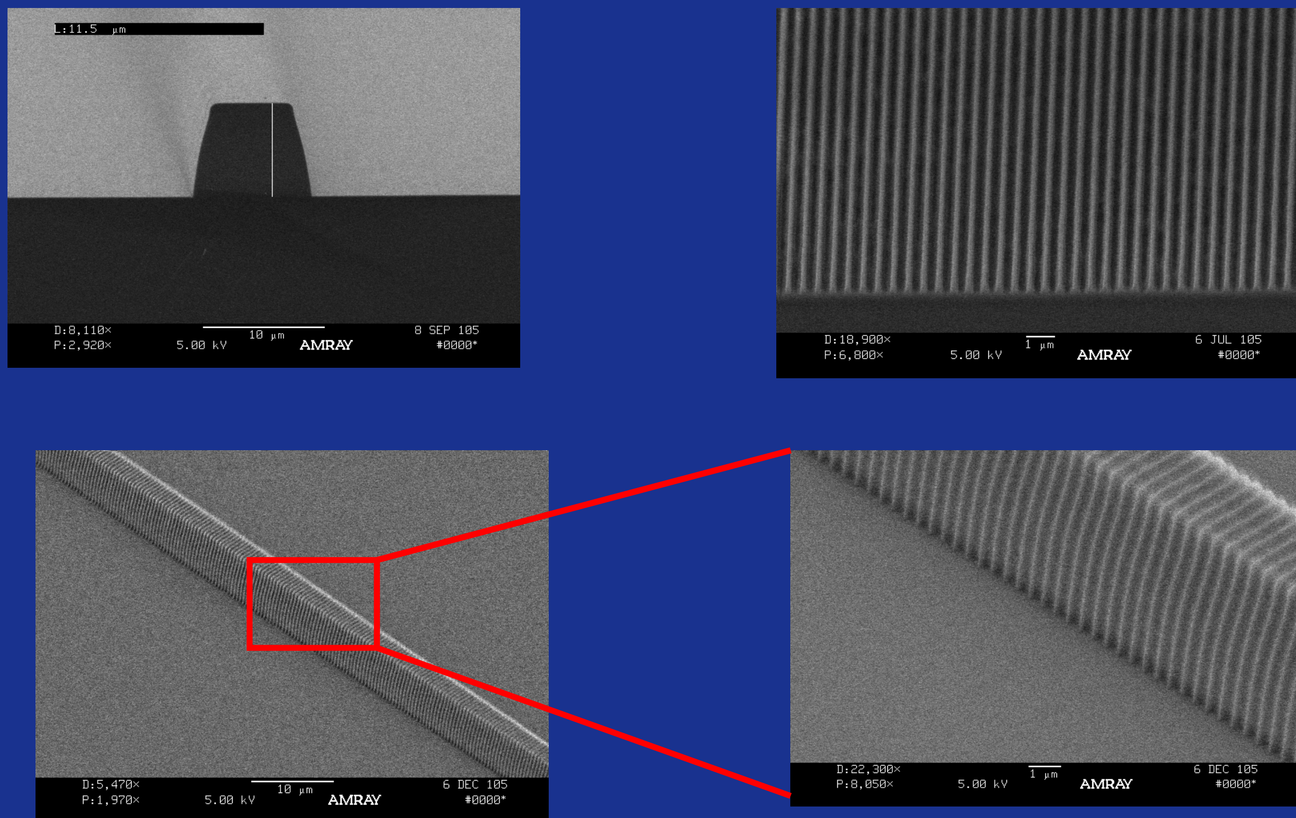
W. Wang

Concept of Detecting



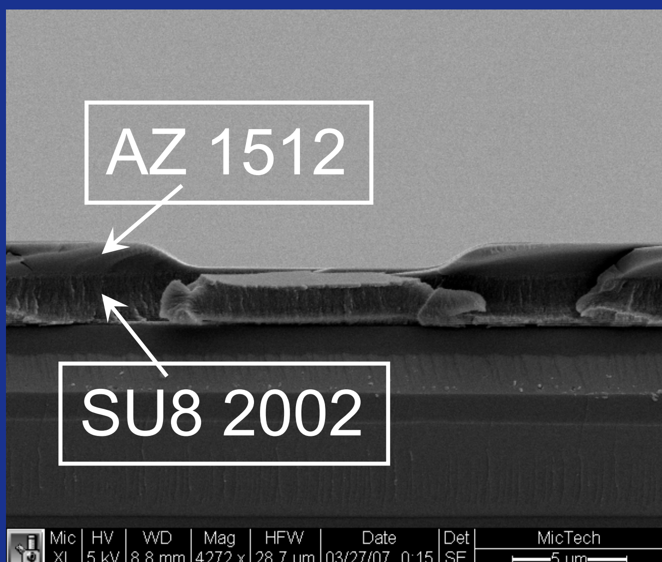
$\lambda_B = 2\Lambda n_{eff}$

Bragg grating Sensor



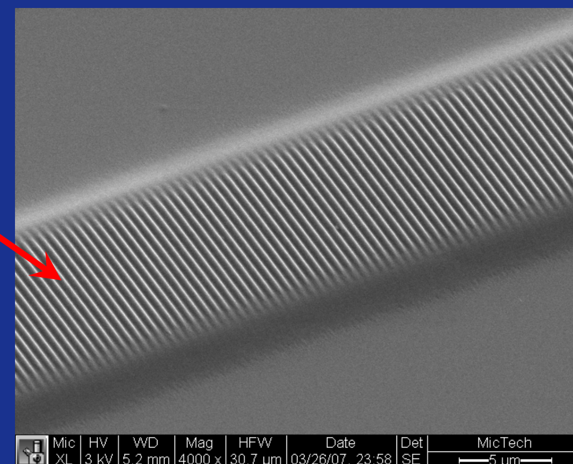
Devices for light technology and sensing

Elastomeric Bragg grating device

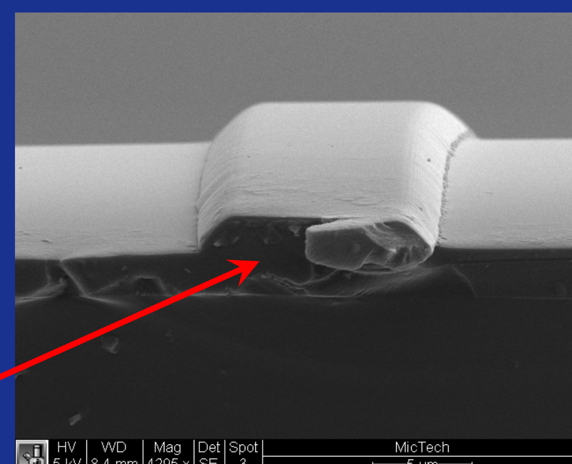


AZ1512 Mold

hPDMS grating



hPDMS cross-section



Optical performance of grating sensor

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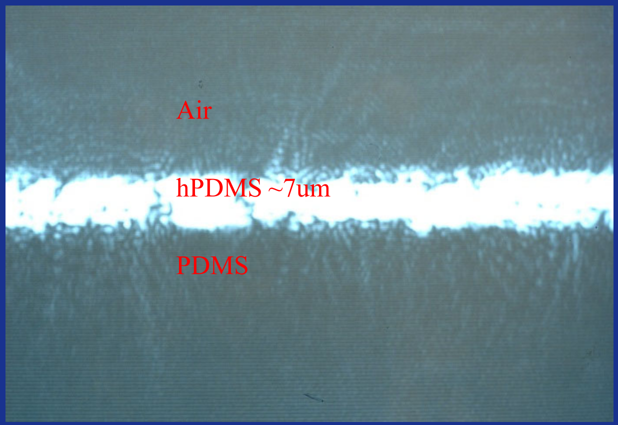
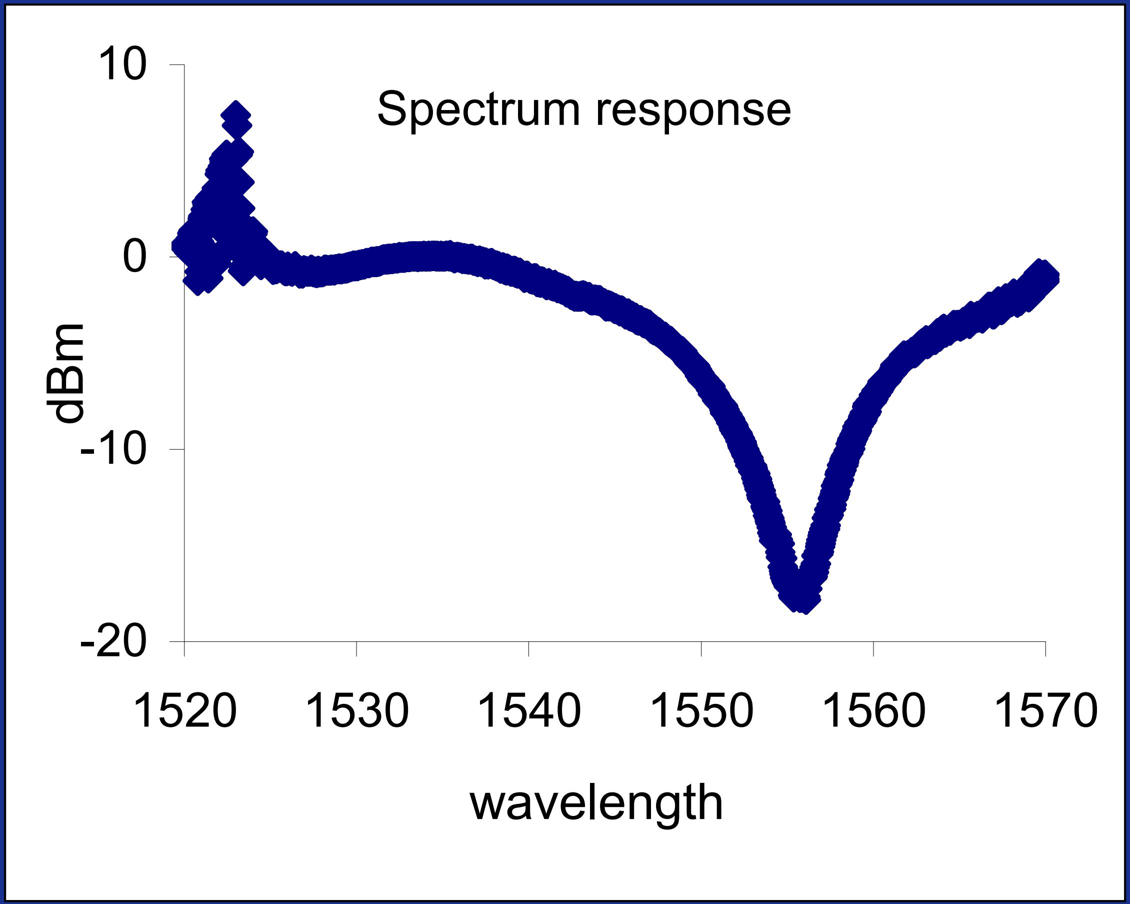


Waveguide output intensity profile



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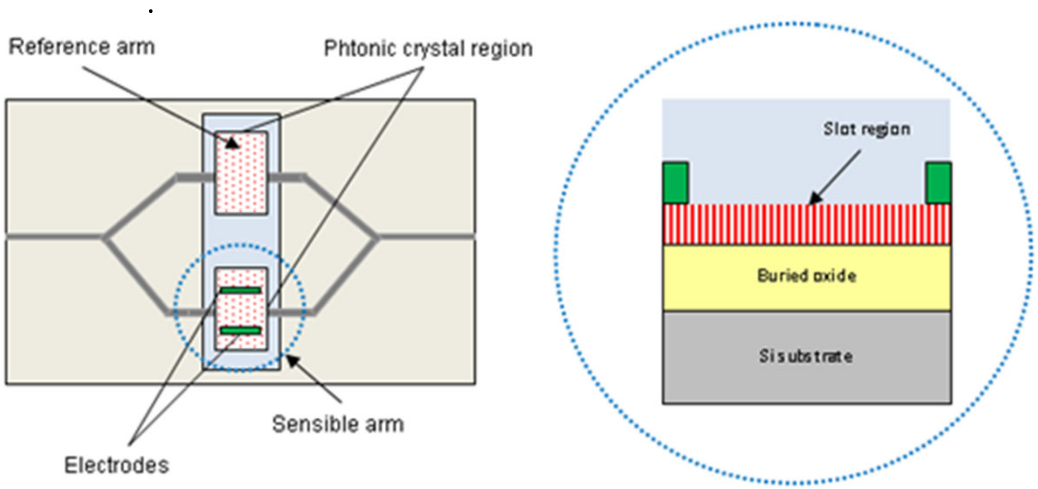
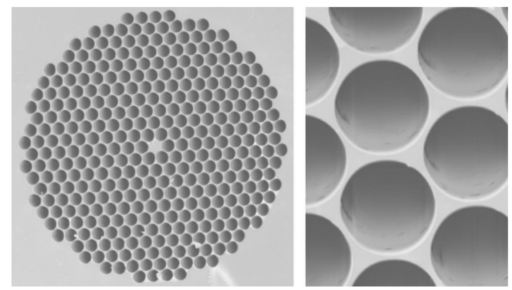
Periodic Structure Sensors



(a) **(b)**



MZI with slot Photonic Crystal waveguide (a) and cross-sectional view (b). A, Troia

SEM micrographs of a photonic-crystal fiber produced at US Naval Research Laboratory. (left) The diameter of the solid core at the center of the fiber is 5 μm, while (right) the diameter of the holes is 4 μm

Technology based on low-loss periodic dielectric materials.

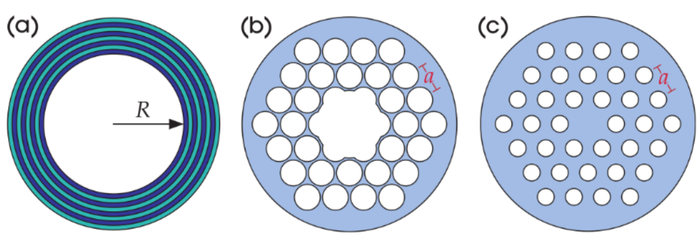


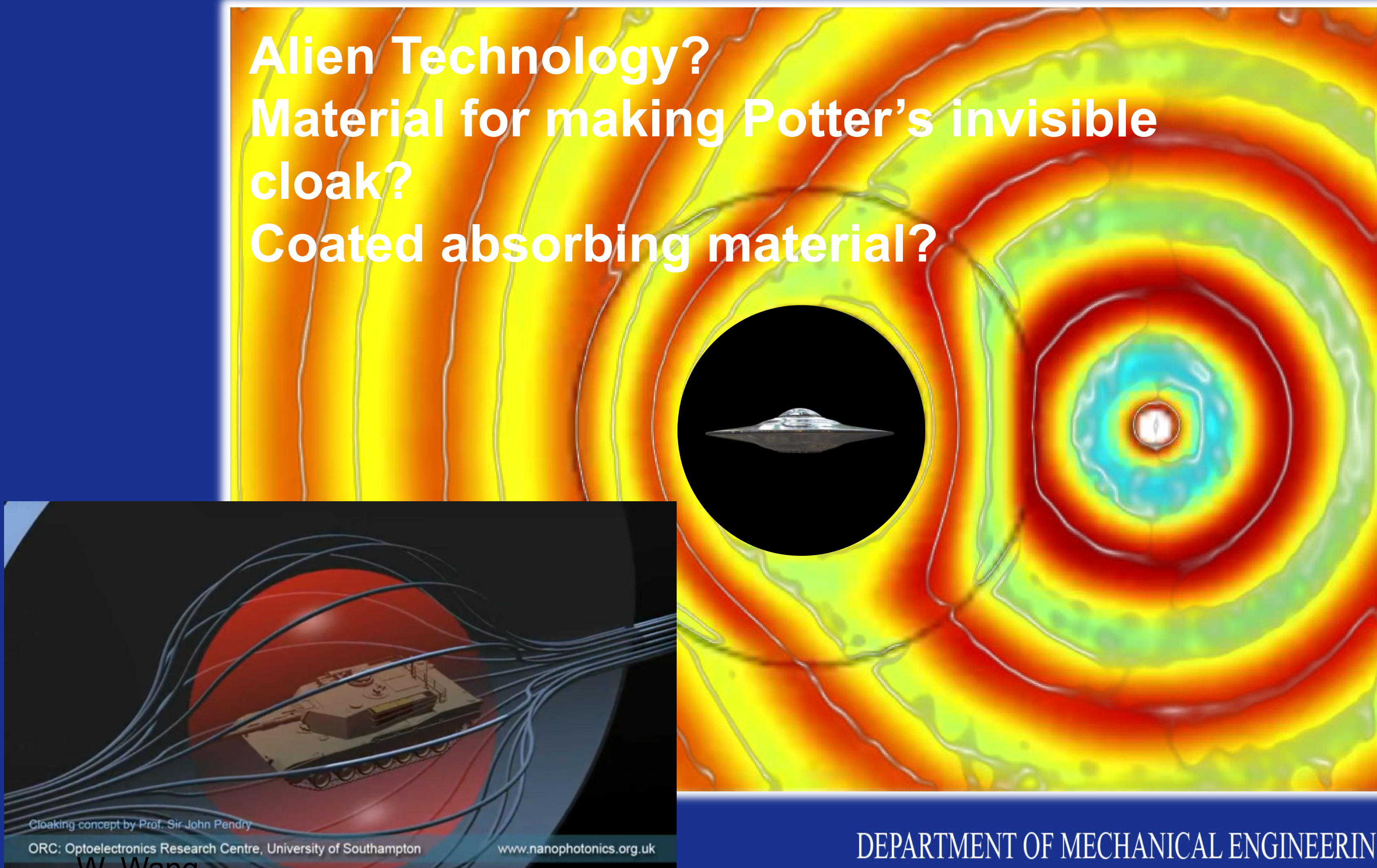
Figure 1: Three examples of photonic-crystal fibers. (a) Bragg fiber, with a one-dimensionally periodic cladding of concentric layers. (b) Two-dimensionally periodic structure (a triangular lattice of air holes, or "holey fiber"), confining light in a hollow core by a band gap. (c) Holey fiber that confines light in a solid core by index guiding.

Everything is possible!

- Cloaking
- Super lens (n=1)
- Lossless
- Zero refractive index
- UFO?

Stealth?

Alien Technology?
Material for making Potter's invisible cloak?
Coated absorbing material?



What's a Metamaterial?

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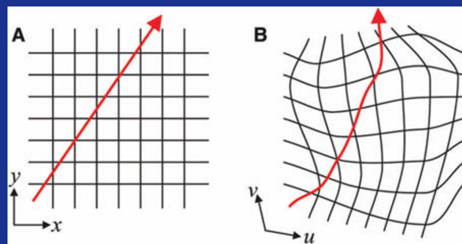
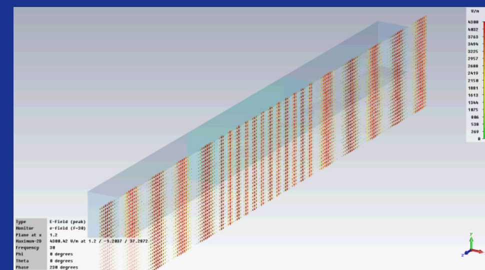
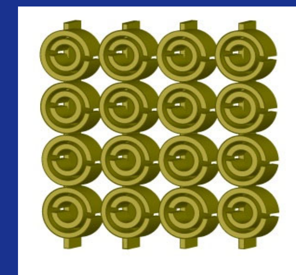
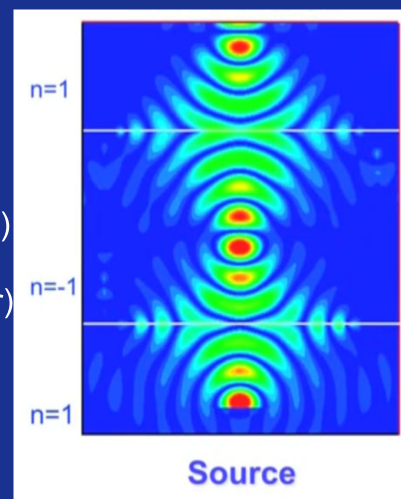
- Meta-?
  – Alloy?
  – Concrete?

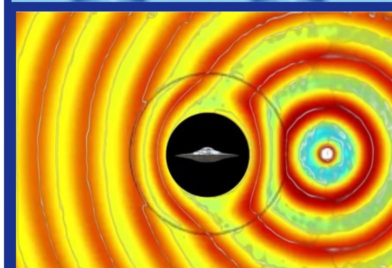






- Artificially engineered materials
  – V.G.Veselago (1968) Theory
  – J.B.Pendry (1996, 1998) –ε (thin wires)
  – J.B.Pendry (1999) –μ (split-ring resonator)
  – D.R.Smith (2000) LHM (combo)



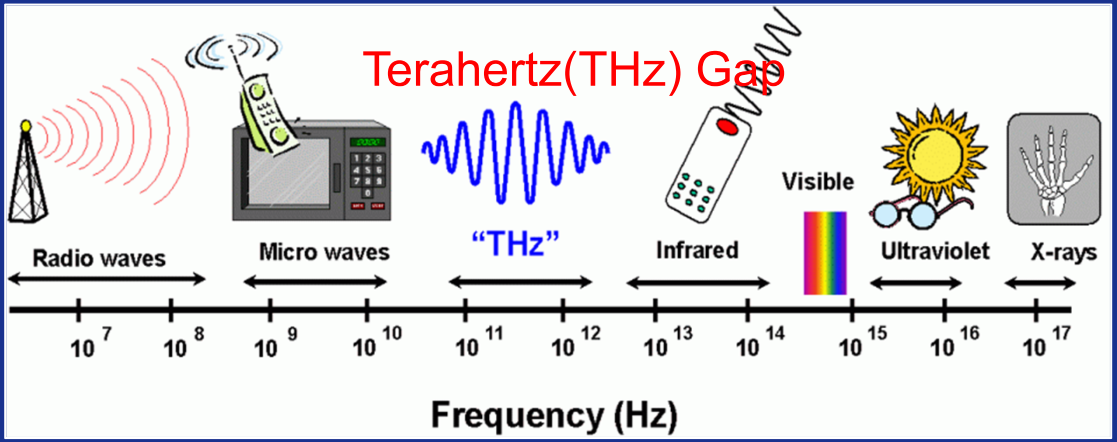


- Applications
  – Wave manipulation: -n, cloaking, superlens, transformation optics

Ortwin Hess, Optics: arewell to Flatland
Nature 455, 299 (2008).

- How to categorize it?

What is TeraHertz?

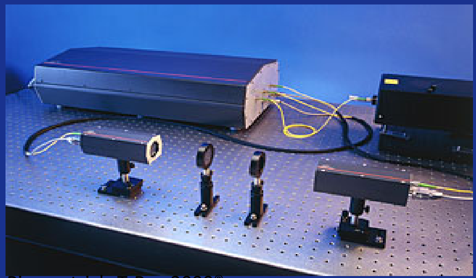


- **THz radiation (EM wave)**
  - THz wave (0.1 THz to 10THz, 30μm to 3mm)
  - Non-ionizing & non-destructive (frequency is low)
  - Penetrate most of dielectric material (fabric, plastic or tissue)
  - Several absorption lines for water
  - Rotational & vibrational frequencies of most molecules

THz Applications

- **Nondestructive detection (NDT)**
  - THz image system
    - Explosive detection
    - Concealed weapons detection
    - Moisture content
    - Composite, structural defect, Coating thickness
- **Spectrum measurement**
  - THz finger print
  - Chemical analysis
  - Skin cancer detection
  - Molecular rotation, protein folding

L-3 Communications Security & Detection Systems, Inc.



Picometrix's T-Ray 2000®

Picometrix's T-Ray 4000®

Constraints:

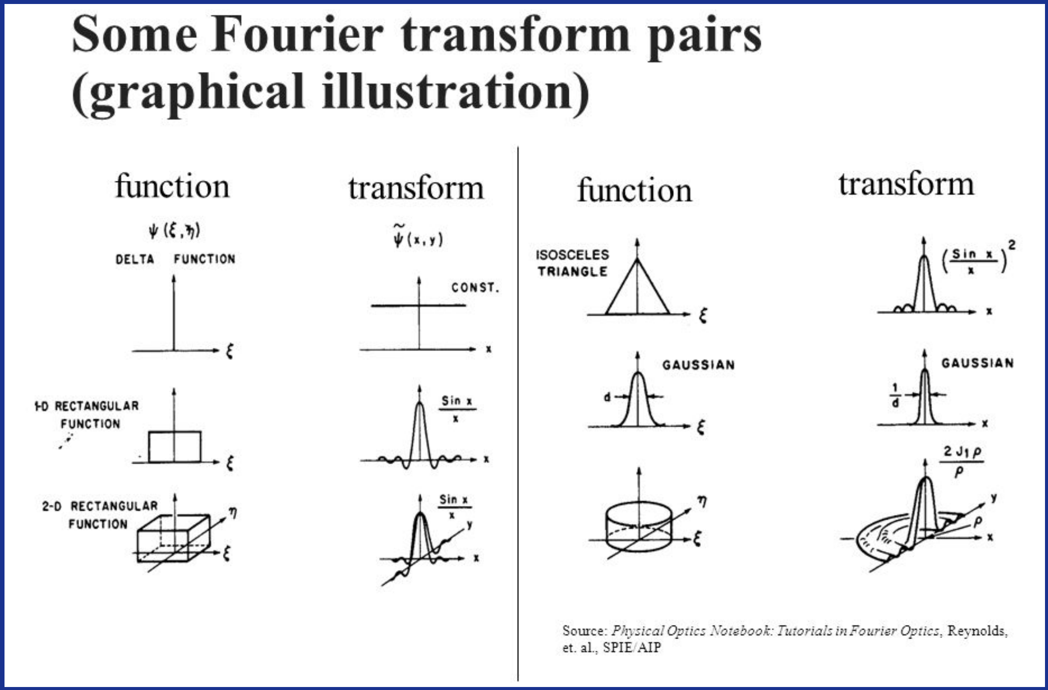
Short working distance -low power
Resolution limited by pixel No. of bolometer based camera
mechanical stage for beam steering

61

Fourier Optics

Study of classical **optics** using **Fourier** transforms, in which the wave is regarded as a superposition of plane waves that are not related to any identifiable sources; instead they are the natural modes of the propagation medium itself.

- Spatial transformation



My Research (Back to the Future)

My Research (Hasta la vista)

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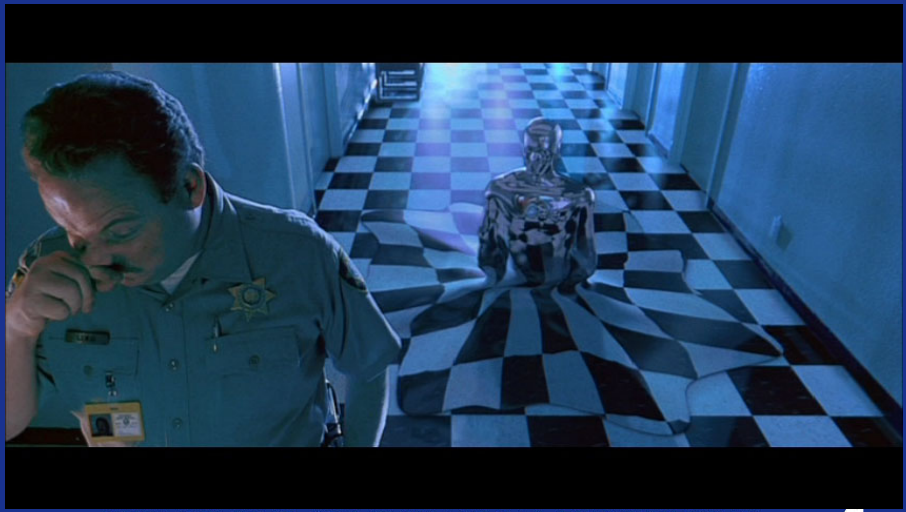

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Four dimension material

- Transformation Optics
- Exploring axis (axes) transformation and computing
- Temporal spatial materials and structures
- Analog computing, electronic and structure
- Meta structure and materials development

Demonstration of free space and intrinsic sensors

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- Plexiglas (guide wave)
- Water bag (lensing, refraction)

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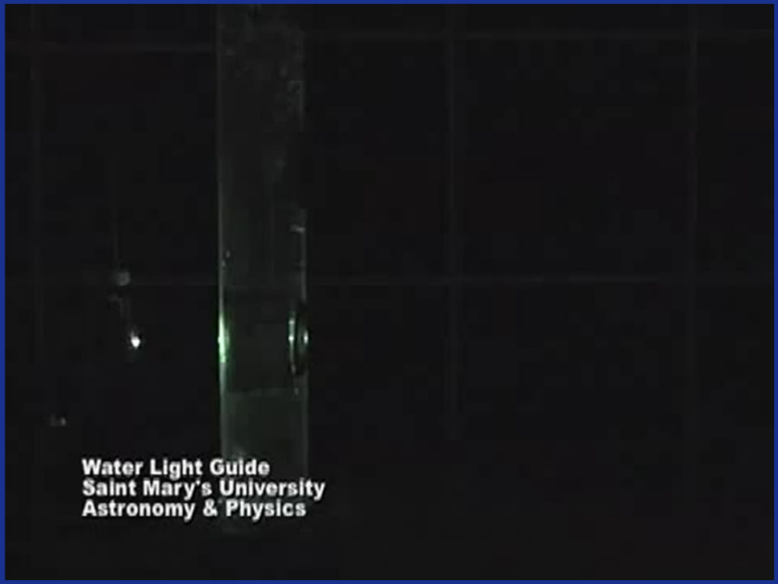
History of Total Internal Reflection



- First demonstration of light remained confined to a falling stream of water (TIR) in 1841 by Daniel Colladon in Geneva.
- Demonstrated internal reflection to follow a specific path to John Tyndall (1870 experiment in London).

Take home Design Exercise (extra credit)

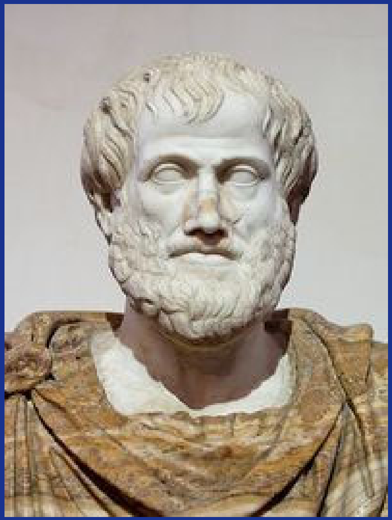
- I want you to use something you have at home to make it into some kind of optical device.

(only if you turn in your design by next Monday!!)

My way of teaching

- Experiential learning- learning through reflection on doing
- Tactile learning!
- How many people remember stuff you learn from Physics class?
- How many people actually apply what you learn from your class to your everyday life outside school?

Experimental Learning



"For the things we have to learn before we can do them, we learn by doing them."

— Aristotle, The Nicomachean Ethics

Experimental Learning

The general concept of learning through experience is ancient. Around 350 BC, Aristotle wrote in the *Nichomachean Ethics* "for the things we have to learn before we can do them, we learn by doing them". But as an articulated educational approach, experiential learning is of much more recent vintage. Beginning in the 1970s, David A. Kolb helped to develop the modern theory of experiential learning, drawing heavily on the work of John Dewey, Kurt Lewin, and Jean Piaget.

wikipedia

The test of all knowledge is experiment.
Experiment is the sole judge of scientific “truth”…. There are *theoretical* physicists who imagine, deduce, and guess at new laws, but do not experiment; and then there are *experimental physicists* who experiment, imagine, deduce and guess.

- Richard Feynman, *Feynman lectures on Physics (Nobel Laureate)*

Show Videos of Previous Projects

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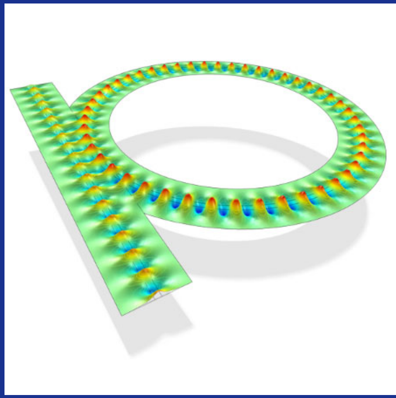
- Previous year final project
- My lab: http://depts.washington.edu/mictech/
- Fun Engineering Design Projects from classes I taught in the past: https://www.youtube.com/results?search_query=gnoba08

https://www.youtube.com/user/UWengr100

Software for ray and wave optics

- Ray: COMSOL ray optics module, QIOPTIQ, OSLO trace pro, Zemax
- Wave: COMSOL, Rsoft, Optiwave (mostly FEM, FTDT)



COMSOL

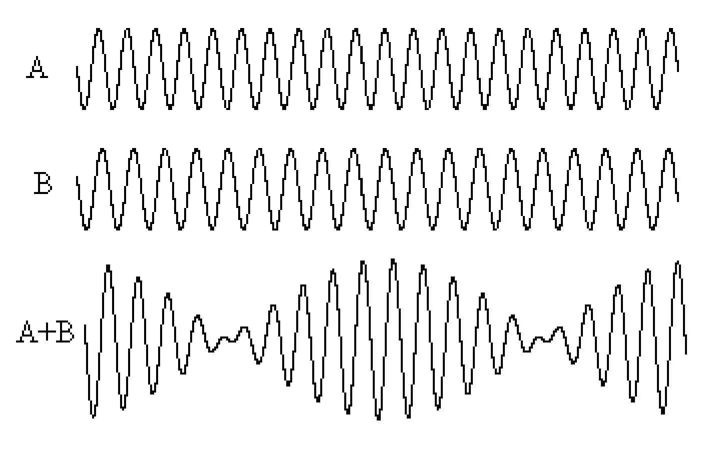
Example of an optical waveguide, a wave propagates around a ring and interferes with a wave propagating in a straight waveguide

Our way

- Solidworks + Ray Optics (Why?)
  - RP our design!
  - Edible Optics !

Absolute and Relative Measurement

- Nothing is absolute in real life (everything is measured on some reference and even the reference is based on another reference or the reference is usually not stable). so why bother to measure absolute value. So best way to measure anything is look at the relative change instead of absolute change.



$$\sin A+\sin B = 2\sin(A+B)/2 * \cos(A-B)/2$$

Let $A = k_1x+\omega_1t+\phi_1 \qquad k_1 = 2\pi n_1/\lambda$

$B = k_2x+\omega_2t+\phi_2 \qquad k_2 = 2\pi n_2/\lambda$

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DEPARTMENT OF MECHANICAL ENGINEERING
UNIVERSITY OF WASHINGTON

Historical Perspective on Light

What is Light?

- Behave like mechanical particle (reflection, refraction)
- Behave like wave (interfere, diffract, partly electric, partly magnetic)
- Energy transfer between light and matter (behave like particle with wavelike behavior)

What is Light?

All the fifty years of conscious brooding have brought me no closer to the answer to the question, “what are light quanta?” Of course today every rascal thinks he knows the answer, but he is deluding (fooling) himself.

- Albert Einstein, 1951

Early days, a light beam was thought to consist of particles. Later, the phenomena of interference and diffraction were demonstrated which could be explained only by assuming a wave model of light. Much later, it was shown that phenomena such as photoelectric effect and Compton effect could be explained on if we assume a particle model of light.

* Ajoy Ghatak, Optics, Macgraw Hill, 2010

* The photoelectric effect is the observation that many metals emit electrons when light shines upon them. Electrons emitted in this manner can be called photoelectrons. The phenomenon is commonly studied in electronic physics, as well as in fields of chemistry, such as quantum chemistry or electrochemistry (Wikipedia).
* *Compton scattering is the inelastic scattering of a photon by a charged particle, usually an electron. It results in a decrease in energy (increase in wavelength) of the photon (which may be an X ray or gamma ray photon), called the Compton effect. (Wikipedia)

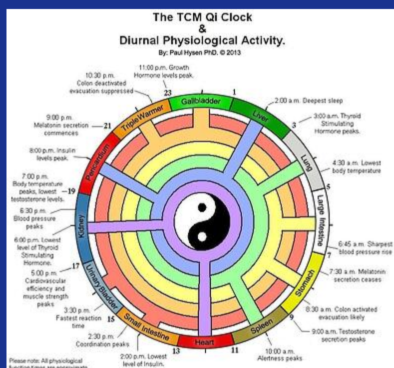
Animism

Animism (from Latin: anima, 'breath, spirit, life') is the belief that <u>objects, places, and creatures all possess a distinct spiritual essence</u>. Potentially, animism perceives all things—animals, plants, rocks, rivers, weather systems, human handiwork, and perhaps even words—as animated and alive. Animism is used in the anthropology of religion as a term for the belief system of many indigenous peoples, especially in contrast to the relatively more recent development of organized religions.

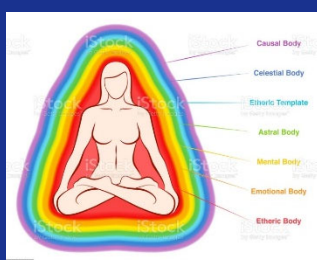
Evolution: from region to science (due to our curiosity)

Translation

Yin	Yang
Structure	Function
Night	Day
Cold	Hot
Earth	Sky
Moon	Sun
Slow	Fast
Humidity	Dryness
Ascends energy	Descends energy
Fluids	Energy
Calm	Expressive
Death	Birth

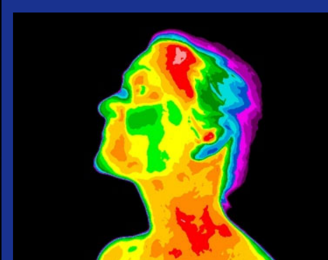


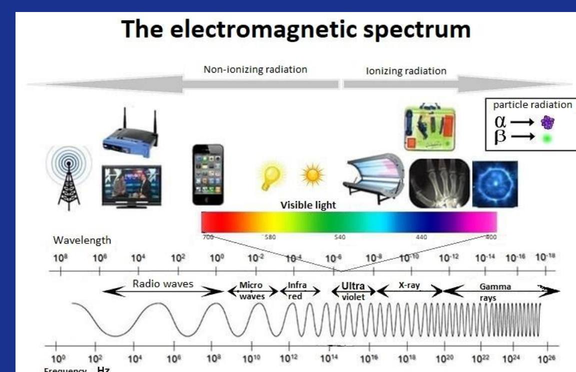
If prediction of future going too far then we call it science fiction but when technology catch up later, then we start calling it visionary science.
W.C. Wang



Aura ad Chi

coronal discharges
Kirlian Photography





EM Radiation

What fails: Not an exact science
Alchemy, secrete formula

My interpretation of Science

We explain things and phenomena based on what our language and background knowledge can explain – normally in ways that might look strange to others but make senses to us at the time based on what we can understand. So Scientific explanation to me is another man-made "game or tool", language or imagination that we created to help us explain things so we can use to explain or communicate with each other for things we want to quickly use or explain or understand. The things we so called science are All based on our observation and creative mind but ideas are not absolute and defined.

W.C. Wang

Particle Model

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- Newton- argued that the geometric nature of the laws of reflection and refraction could only be explained if light was made of particles, which he referred to as **corpuscles**, as waves do not tend to travel in straight lines.

Corpuscular model:

Most important experimental facts which led to the early belief in the corpuscular model of light were:

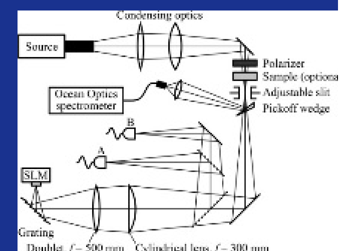
1. Rectilinear propagation of light which results in the formation of sharp shadows (no diffraction)

- Assume light travel in straight lines

2. light could propagates in vacuum (sound can't)

Corpuscular Model

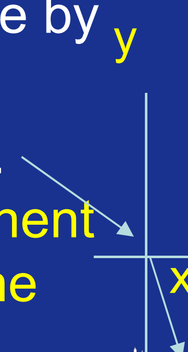
The corpuscular model is the simplest model of light. According to the theory, a **luminous body emits stream of particles in all direction**. Issac Newton, in this book Opticks also wrote,” Are not the ray of light every small bodies emitted from shinning substance?”



Based on this, light is assumed to be **consisted of very small particles so that when two light beams overlap**, a **collision between the two particles rarely occurs**. Using this model, one can explain the laws of reflection and refraction (Snell’s law):

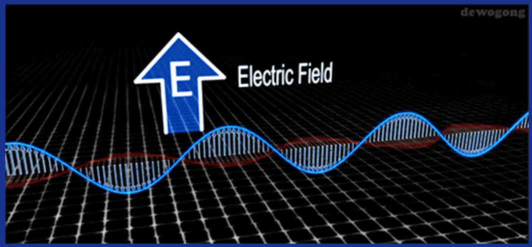
- -Reflection law follows considering the elastic reflection of a particle by plane surface.

- - Refraction law assume that the motion is confined to the xy plane. The trajectory of particle is determined by conservation of x component momentum (component parallel to the interface, perpendicular to the direction of propagation)



Wave Model

- ***From 17th century on many scientists and physics and mathematician were able to use <u>mathematical model to explain a lot of these wave phenomena of light with its expression.</u>***

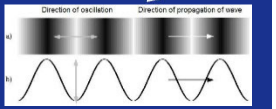


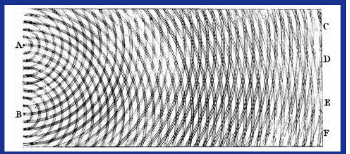
- ***Most accurate model so far to explain many things we see in light. However, one <u>misconception of all these are these expression can explain everything about light.</u> The fact is that these wave equations are just <u>abstract Mathmatical model that helps explain when light behave like wave or how it propagate in space or how it interact with each other.</u>***

Wave Model

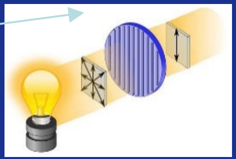
Historic progress of discovery of light and explained in terms of waves.

- A wave has a wavelength, a speed and a frequency.
- Grimaldi- observe diffraction of white light through small aperture quote, ”light is a fluid that exhibits wave-like motion.” (1665)
- Huygen- propose first wave model explaining reflection and refraction(1678)
- Young- perform first interference experiment could only be explained by wave. (1801)
- Malus- observed polarization of light. (1802)
- Fresnel- gives satisfactory explanation of refraction and equation for calculating diffraction from various types of aperture (1816)
- Oersted- discover of current (1820)
- Faraday- magnetic field induces electromotive force (1830)
- Maxwell- Maxwell equation, wave equation, speed of EM wave (1830)
- Hertz- carried out experiment which produce and detect EM wave of frequencies smaller than those of light and law of reflection which can create a standing wave.

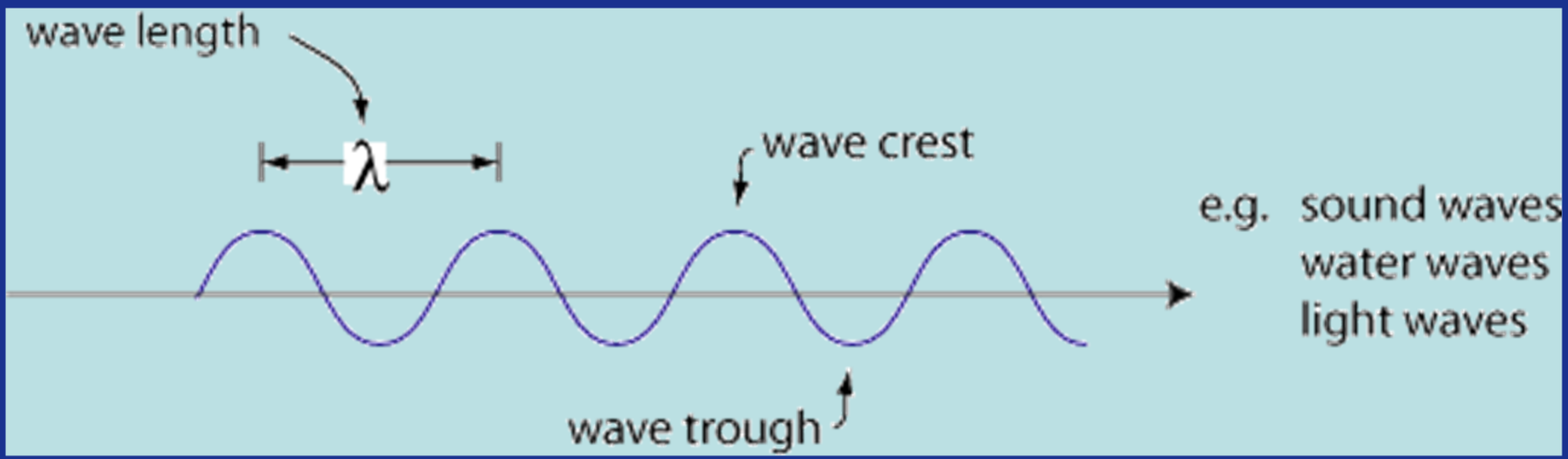




Ripple tank interference



Wave-like Behavior of Light



In the 1600s Christiaan Huygens, a Dutch physicist, showed that light behaves like a wave.

Examples of Wave behavior

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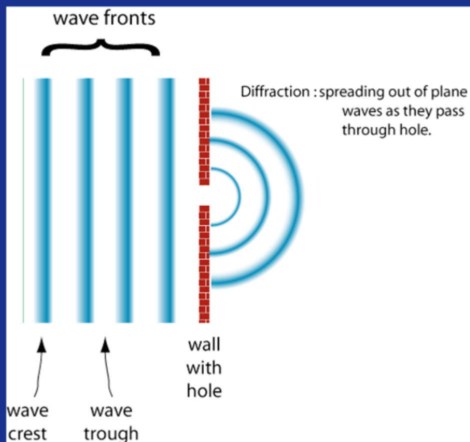
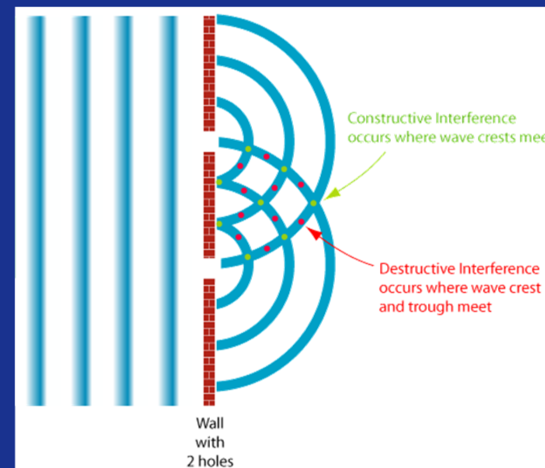
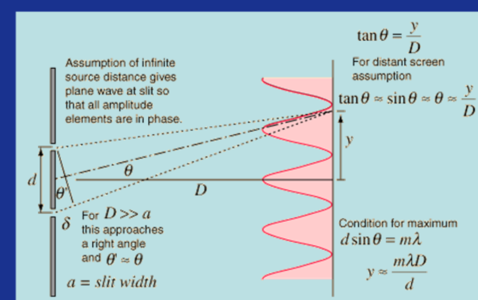
Diffraction



As the width of the slit becomes larger than the wavelength the wave is diffracted less.

Interference





- **Young- perform first interference experiment that could only be explained by wave. (1801)**
- **Huygen in diffraction of light**

Wave-like Behavior of Light

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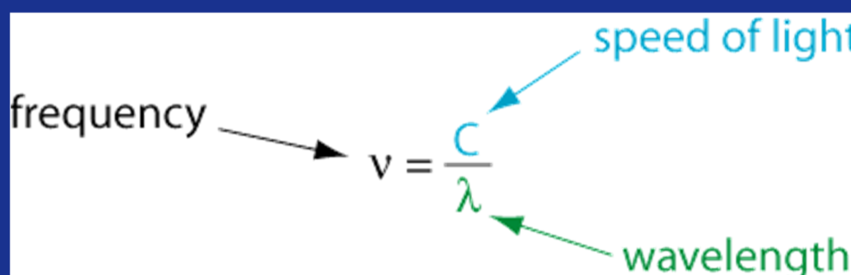
It was James Clerk Maxwell who showed in the 1800s that light is an electromagnetic wave that travels through space at the speed of light. The frequency of light is related to its wavelength according to



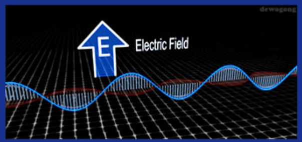
Well, more exactly is his work was based on Faraday (next page)

Light as EM Wave (Historic perspective)

Faraday thought EM wave was longitudinal wave

In 1845, Michael Faraday discovered that the plane of polarization of linearly polarized light is rotated when the light rays travel along the magnetic field direction in the presence of a transparent dielectric, an effect now known as Faraday rotation. T**his was the first evidence that light was related to electromagnetism. In 1846 he speculated that light might be some form of disturbance propagating along magnetic field lines. Faraday proposed in 1847 that light was a high-frequency electromagnetic vibration, which could propagate even in the absence of a medium such as the ether.**

Faraday's work inspired James Clerk Maxwell to study electromagnetic radiation and light. Maxwell discovered that self-propagating electromagnetic waves would travel through space at a constant speed, which happened to be equal to the previously measured speed of light. From this, Maxwell concluded that light was a form of electromagnetic radiation: he first stated this result in 1862 in On Physical Lines of Force. In 1873, he published A Treatise on Electricity and Magnetism, which contained a full mathematical description of the behavior of electric and magnetic fields, still known as **Maxwell's equations**.

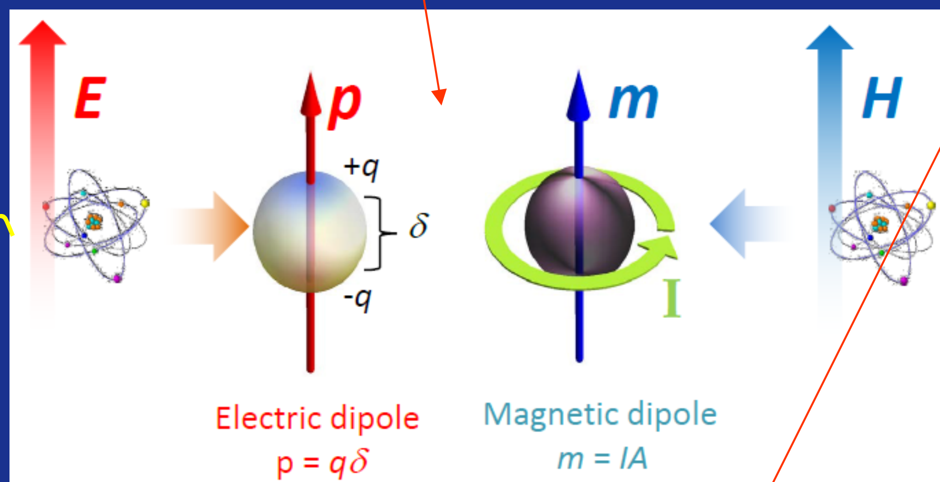


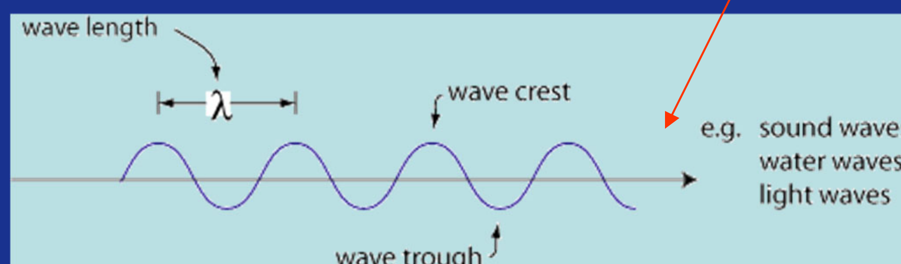
In the quantum theory, photons are seen as **wave packets of the waves described in the classical theory of Maxwell**. The quantum theory was needed to explain effects even with visual light that Maxwell's classical theory could not (such as **spectral lines**).

Electromagnetic Wave

From the electromagnetic point-of-view, an atom is just an electric or magnetic, *polarizable* dipole. Light is just an electromagnetic wave

Wave theory assumption of an atom

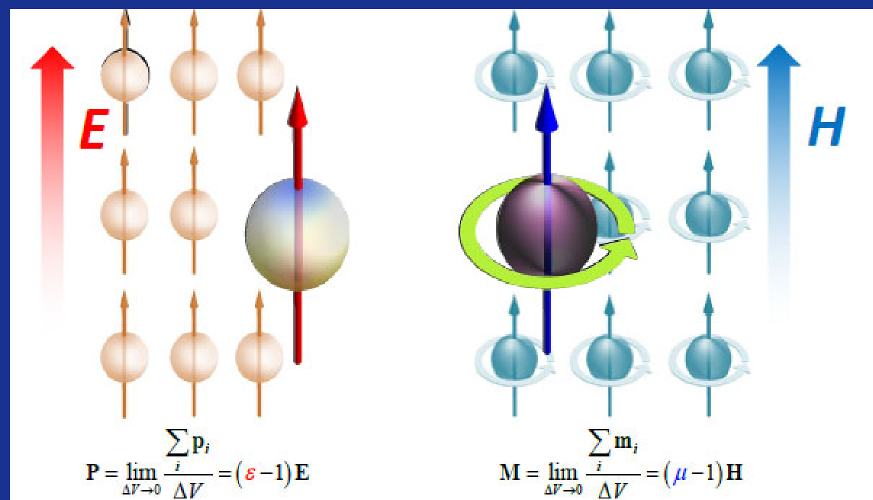


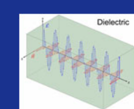


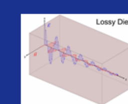
Electromagnetic Wave

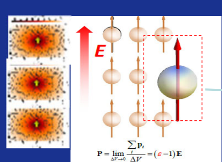
A material is a collection of electric and magnetic dipoles. Homogenization allows this collection to be *continuous*.

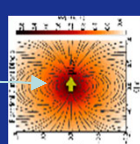
Wave theory assumption of a material

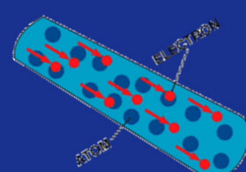


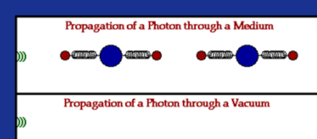


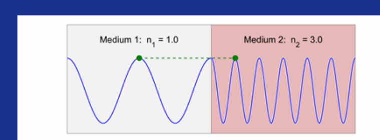












Wave Model

Is wave theory sufficient to explain everything? No

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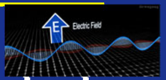
There **is not a satisfactory explanation that makes it easier to understand how light, and electromagnetic waves propagate without a medium.**

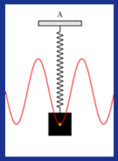
The mathematical concept of fields explains how the forces involved in electrodynamics extend into empty space, however, it does not really explain why it is the way that it is. If you want to understand how electromagnetic fields work, I would say the best way to get this understanding is to study electrodynamics and solve problems until you get a feel for how things work. You have to get used to the idea of invisible field lines extending out from charged objects and wrapping around moving currents.

To answer this question, we need to address a number of **assumptions within it**:

Some of the wave definition of how EM radiation and photon generated and prorogate

1. EM (electromagnetic) waves **are self reinforcing**. A changing electric field induces a magnetic field. Meanwhile, a changing magnetic field induces an electric field. When the electric field "falls", it creates a magnetic field. At the E field's zeroth point, the magnetic field is at its peak. Similarly for the magnetic field. That is, each one creates the other, like a see-saw. In addition, the fields induced are at right angles to their changes, the result of which is a beam of EM radiation traveling along through the universe (until it intersects with something). **So, EM waves don't need any other particle to support them, a wave feeds off itself.**

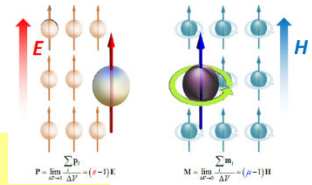


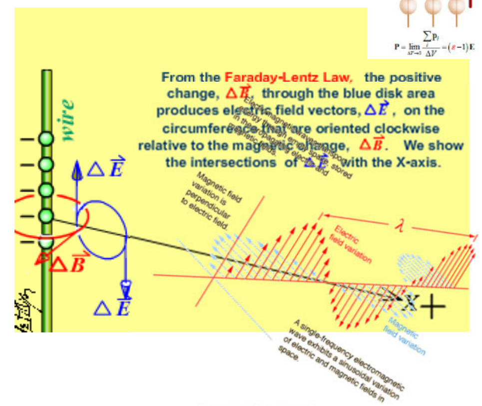


2. An electron carries a negative charge (the proton carries a positive charge). **These particles have an electric charge property. Because of this, they can interact with EM waves and affect them. However, their presence is not required for the wave to propagate** (see first assumption).
3. What we call a vacuum (as in vacuum of space) **is really not a vacuum**. It is filled with a **sea of particles, real and virtual.** Furthermore, it is **layered with many fields; think of these fields as fluids (that's what I'll call them). There's the electric fluid, the magnetic fluid, the weak fluid, the strong, the quark fluid, and space-time.**
4. **A photon (or EM wave) is actually a wave moving through the electromagnetic fluid - just like a water wave moves through the ocean (water fluid).** So, the vacuum is not empty, **EM waves are self reinforcing**, and **electrons carry electric charge but are not the creators of the electric field (although the bend it and the *photons are the force carriers of that field*).**

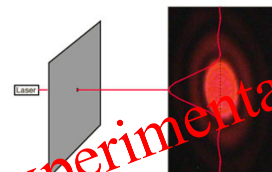
So, what is light? What are photons? What are radio waves? Regardless of the total energy transported, all are special aspects of electromagnetic waves that differ in their wavelength. They remain invisible unless they interact with matter.

Wave and Quantum





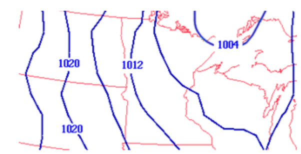
- When you think of light as a wave of E and B fields, I advice you to not think of them as photons or particles. But just visualize circular waves in water. This is similar except for the fact that it is in three dimensions and that the waves are not through any medium but these 'fields'. So all you need is to disturb a field, and you'll always get a light wave just like when u disturb water.



$$I = I_o \frac{\sin^2(\frac{\pi a \sin\theta}{\lambda})}{(\frac{\pi a \sin\theta}{\lambda})^2}$$

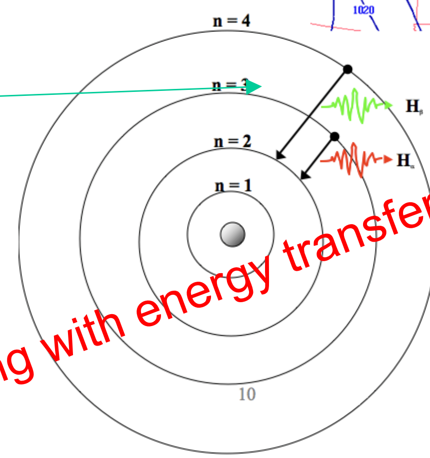
All based on experimental observations

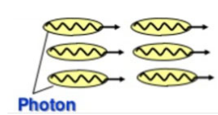
- It's interesting that field lines are just a mathematical convenience or abstract model invented by Faraday, and are no more real in the physical sense than isobars on weather maps or contour lines on maps



Frequency gets higher and matter gets smaller, we go quantum

- On the other hand. When you think of **light as a photon**, you **go quantum**. Here everything goes crazy. You no longer use fields. But purely use **concept of energy, and discrete orbits** (Bohrs model) When electron falls from one discrete orbit to another (and this is not a classical fall, fall, but a quantum fall, where it disappears from one orbit and appears in another). And this leads to generation of photons, or light!





$$E_{n'} - E_{n''} = h\nu$$

When dealing with energy transferring

Light as photon (quantum)

recap what quantum theory say about photon and EM wave.

A photon is another way of looking at an electromagnetic wave and it is a quantum-mechanical particle. However, **the photon has zero mass** and **no electric charge.**

The relationship between the photon momentum, wavelength and frequency is:

$p = h/\lambda$, where p is the momentum and h being the Planck constant.

The momentum of a photon (zero mass) is given by $p = E/c$, and the wavelength by $\lambda = c/f$, where c is the speed of light in vacuum.

--> $E = hc/\lambda = hf$ Just a discontinuity about frequency (sometimes f in $\lambda = c/f$ and sometimes v in $E = hv$).

Difference between Light as photon and EM Wave

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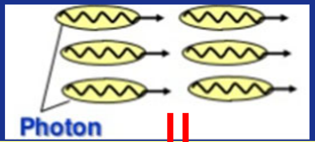
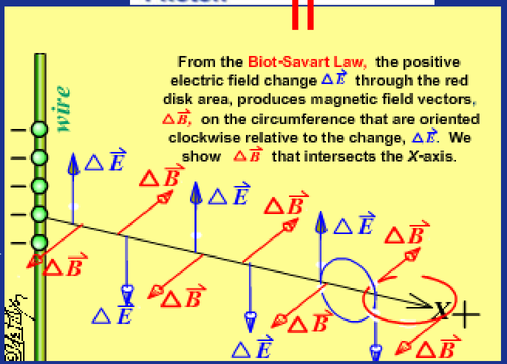
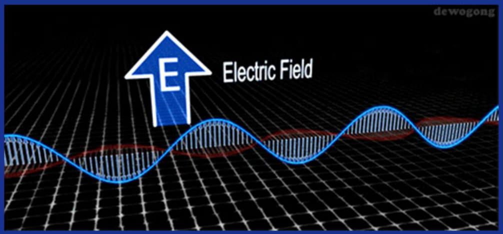
Electromagnetic radiation can be described in terms of a stream of **mass-less charge-less particles,** called photons, each traveling in a wave-like pattern at the speed of light. Each photon contains a certain amount of energy. The different types of radiation are defined by the amount of energy found in the photons ($E_g= hC/\lambda$). (quantum)





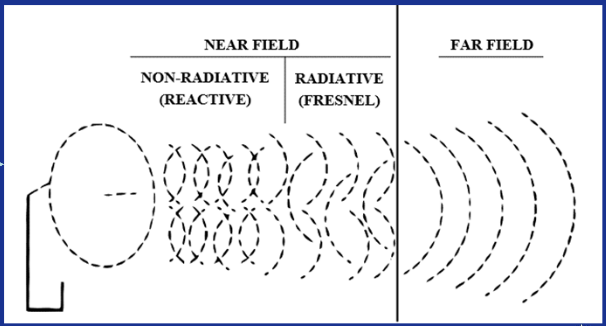
Photons are particles **forming the electromagnetic field and they are also waves.** Their de Broglie wavelength is the same that the one associated to their wavelength of the electromagnetic field. And **the electromagnetic wave propagates at the velocity of light as the photons contained within the electromagnetic field.** The movement produces oscillating electric and magnetic fields, which travel at right angles to each other in a bundle of light energy called a photon



In **homogeneous, isotropic media**, the oscillations of the two fields (E and B) are perpendicular to each other and perpendicular to the direction of energy and wave propagation, forming a **transverse wave**.

$$E(z,t) = \hat{x}E_o \cos(\omega t - kz)$$

Electromagnetic radiation is associated with those EM waves that are free to propagate themselves ("radiate") **without** the continuing influence of the moving charges that produced them, because they have achieved sufficient distance from those charges. Thus, EMR is sometimes referred to as the **far field**



Quantum Theory of Light

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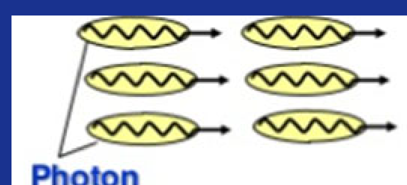
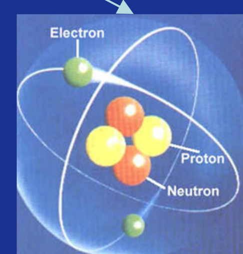
Wave-Particle Duality of Light

Quantum theory tells us that both **light and matter consists of tiny *particles* *which have* *wavelike properties*** associated with them. **Light** is composed of particles called photons, and **matter** is composed of particles called electrons, protons, neutrons. It's only when the mass of a particle gets small enough that its wavelike properties show up.

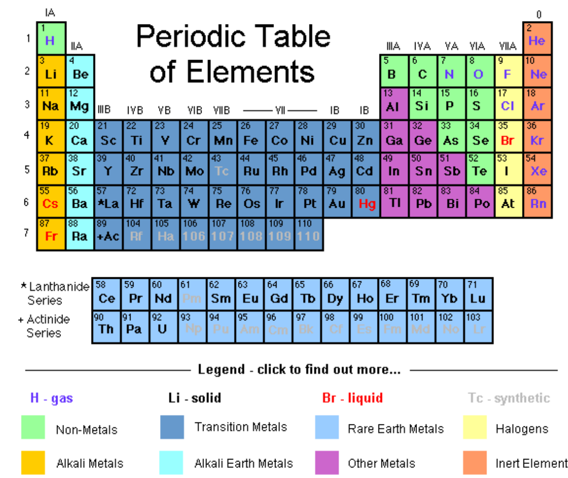
Generally speaking, at lower frequency it behaves like a wave, while at higher frequency it acts like a particles. Page 97



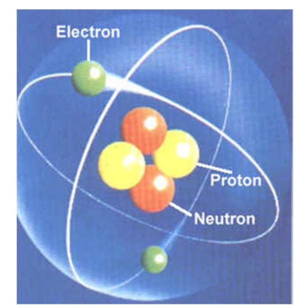


For example, at large quantum number the physics becomes to classical. Page 97

Where does light come from? (Quantum)

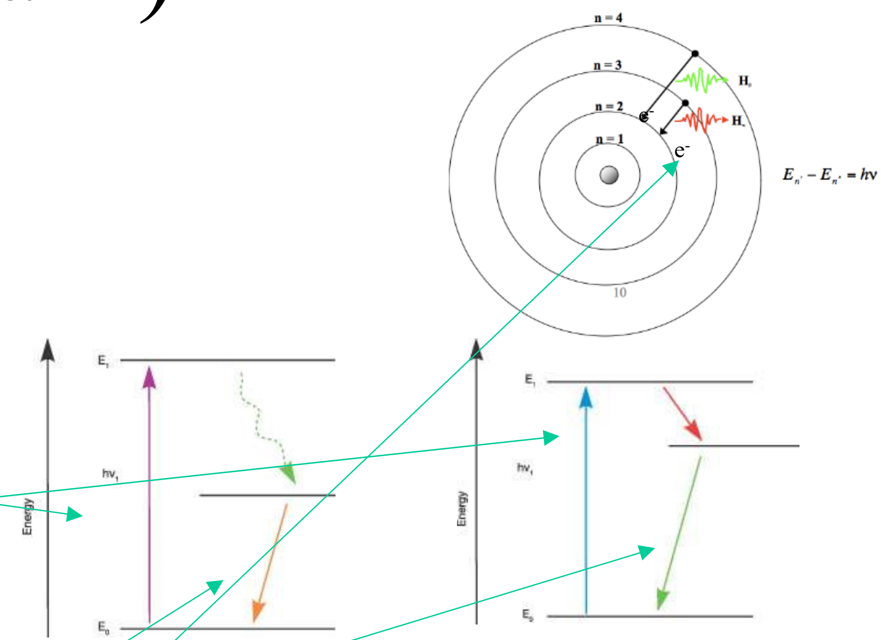


- The Sun, stars, etc..
- But how do they make light?
- It all starts with ATOMS
- A nucleus surrounded by electrons that orbit.
- Like the planets in the solar system, electrons stay in the same orbit, unless…



Where does light come from? (Quantum)

- Electrons get kicked into a different orbit
- This doesn't happen very often in solar systems, but it does in atoms
- If you add energy to an atom (absorbs), the electrons will jump to higher energy orbits.
- When atom cools (energy releases), electrons jump back to original orbits.
- As they jump back, they emit light, a form of energy

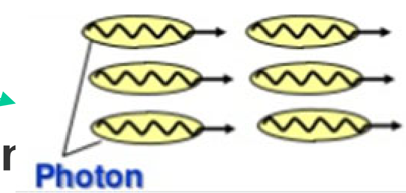


Examples of Light absorption of a photon to raise atom or molecule to higher energy levels and emitting a photon and instead relaxes to the lower energy state

Photons

According to quantum physics, the **energy of an electromagnetic wave is quantized**, i.e. it can only exist in **discrete** amount. The basic unit of energy for an electromagnetic wave is called a **photon**. The energy E of a photon is proportional to the wave frequency f,

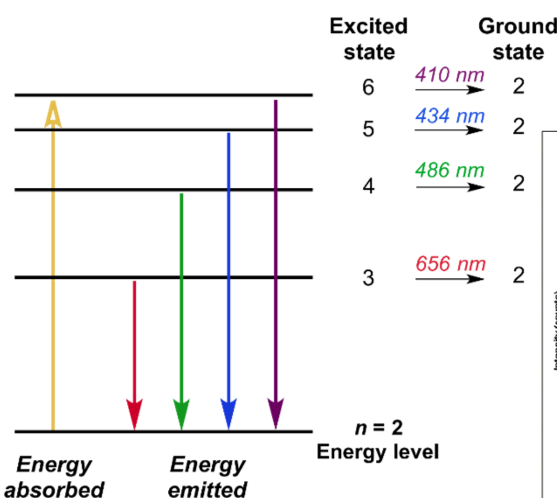
$$E = h \times f = h \times C_o / \lambda$$

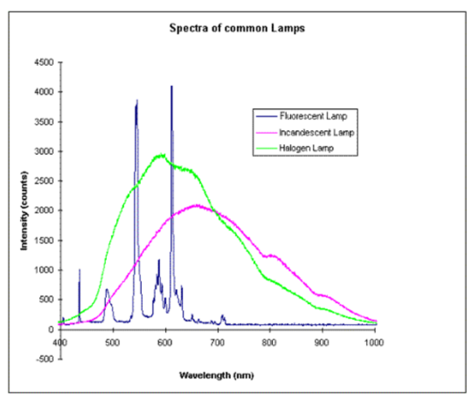


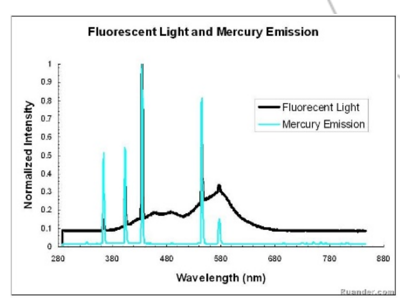
where the constant of proportionality h is the **Planck's Constant**,

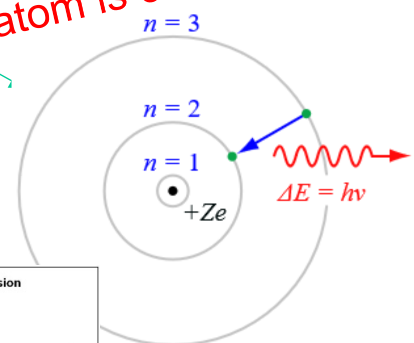
$$h = 6.626 \times 10^{-34} \text{ J s.}$$

Light generated from an atom is call photons

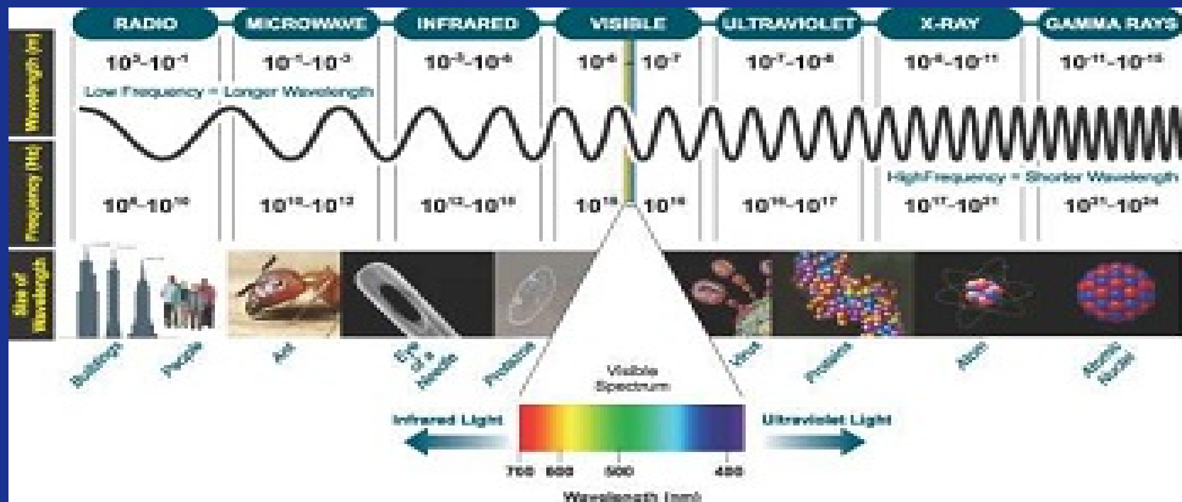




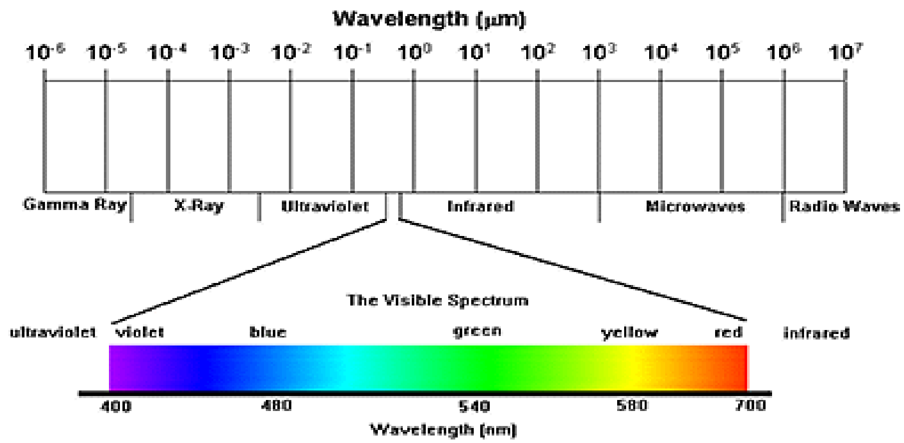




Light Spectrum



- Light is part of the electromagnetic spectrum
- Visible light is not inherently different from the other parts of the electromagnetic spectrum with the exception that the human eye can detect visible waves
- A stream of photons which are massless particles each travelling with wavelike properties at the speed of light



The electromagnetic spectrum can be divided into several wavelength (frequency) regions, among which only a narrow band from about 400 to 700 nm is visible to the human eyes. Note that there is no sharp boundary between these regions. The boundaries shown in the above figures are approximate and there are overlaps between two adjacent regions.

•**Radio Waves**: 10 cm to 10 km wavelength.
•**Microwaves**: 1 mm to 1 m wavelength. The microwaves are further divided into different frequency (wavelength) bands: **(1 GHz = 10^9 Hz)**

- **P band**: 0.3 - 1 GHz (30 - 100 cm)
- **L band**: 1 - 2 GHz (15 - 30 cm)
- **S band**: 2 - 4 GHz (7.5 - 15 cm)
- **C band**: 4 - 8 GHz (3.8 - 7.5 cm)
- **X band**: 8 - 12.5 GHz (2.4 - 3.8 cm)
- **Ku band**: 12.5 - 18 GHz (1.7 - 2.4 cm)
- **K band**: 18 - 26.5 GHz (1.1 - 1.7 cm)
- **Ka band**: 26.5 - 40 GHz (0.75 - 1.1 cm)

•**Visible Light**: This narrow band of electromagnetic radiation extends from about 400 nm (violet) to about 700 nm (red). The various color components of the visible spectrum fall roughly within the following wavelength regions:

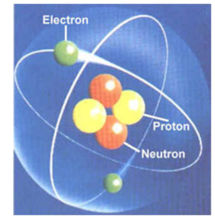
- **Red**: 610 - 700 nm
- **Orange**: 590 - 610 nm
- **Yellow**: 570 - 590 nm
- **Green**: 500 - 570 nm
- **Blue**: 450 - 500 nm
- **Indigo**: 430 - 450 nm
- **Violet**: 400 - 430 nm

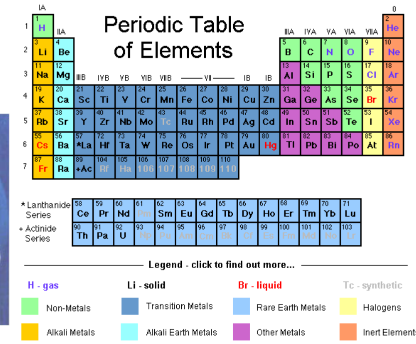
•**Ultraviolet**: 3 to 400 nm
•X-Rays:
•10^{-8} to 10^{-11} m
•**Gamma Rays**: 10^{-11} m (10 pm) or lower

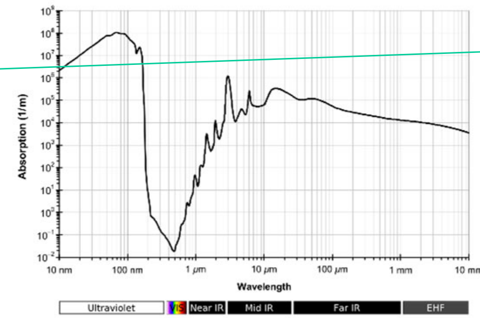
Light matter interaction

Matter is composed of atoms, ions or molecules and it is light's interaction with matter which gives rise to the various phenomena which can help us understand the nature of matter.

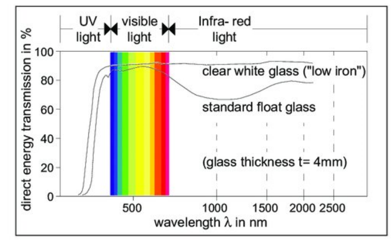
The atoms, ions or molecules have defined energy levels usually associated with energy levels that electrons in the matter can hold. Light can be generated by the matter or a photon of light can interact with the energy levels in a number of ways.



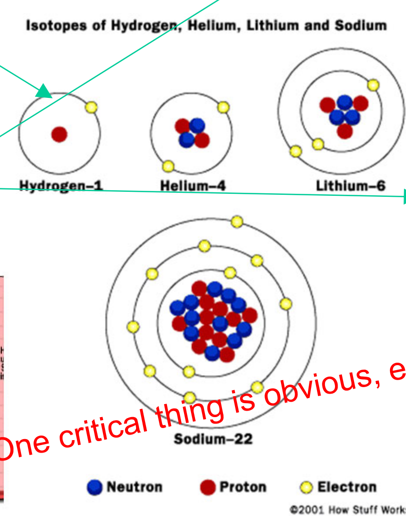


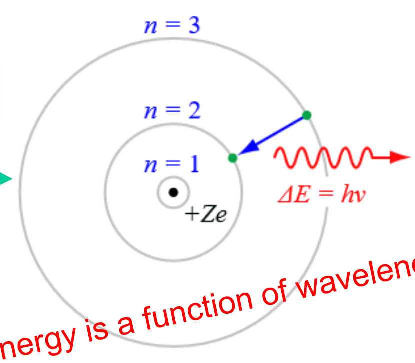


Example of light absorption spectrum in water



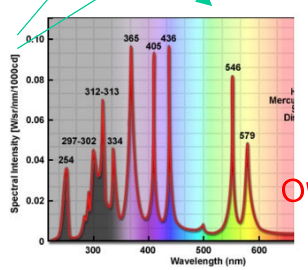
Example of light transmission spectrum in Low iron glass





One critical thing is obvious, energy is a function of wavelength

Bohr's model of Hydrogen atom



Mercury emission spectrum

w.wang

W.Wang

Third Rock from the Sun

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- Light (energy) and Matter

Quantum Theory

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The theory of special relativity was the first part of the revolution in 20th century physics. It was also found that Classical mechanics is not correct when particles move at high speed (<speed of light).

The second part of the revolution was the formulation of the theory of quantum mechanics. Its origin lay in the study of the **radiation emitted by hot bodies** and in the **photoelectric effect**.

In the following section we will discuss blackbody radiation, photoelectric effect, Compton's scattering, Bohr's "hydrogen" model, to wave-particle duality and correspondence principle

Radiation emits or absorbs by hot bodies

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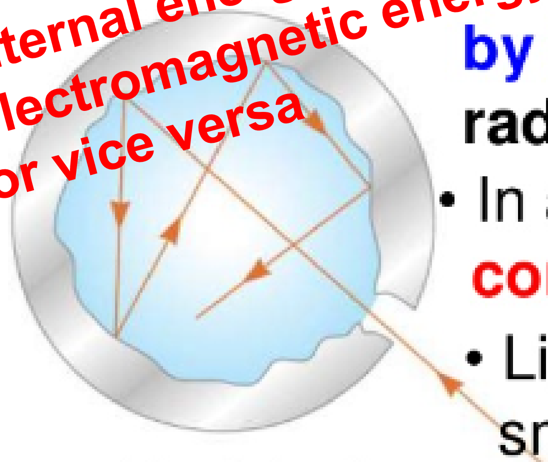
- The foundation of the Planck's quantum theory is a **theory of black body radiation.**
- Black body is defined as **an ideal system or object that absorbs and emits all the em radiations that is incident on it.**

conversion of a body's internal energy into electromagnetic energy or vice versa



black body

- The **electromagnetic radiation emitted by the black body** is called **black body radiation**.
- In an **ideal** black body, incident light is **completely absorbed**.
- Light that enters the cavity through the small hole is reflected multiple times from the interior walls until it is completely absorbed.

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Why is the distribution of thermal radiation among the various wavelengths the same for all bodies at a given temperature?

Function of Frequency and temperature

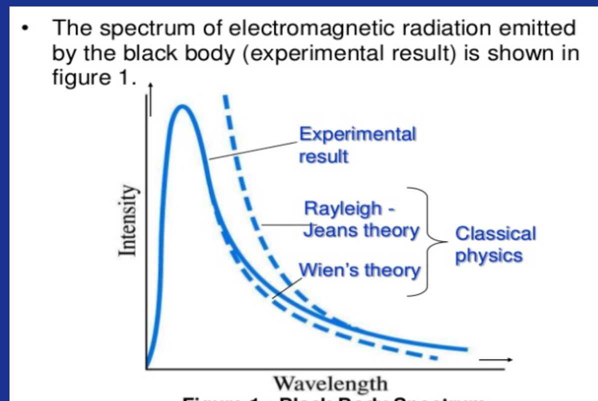
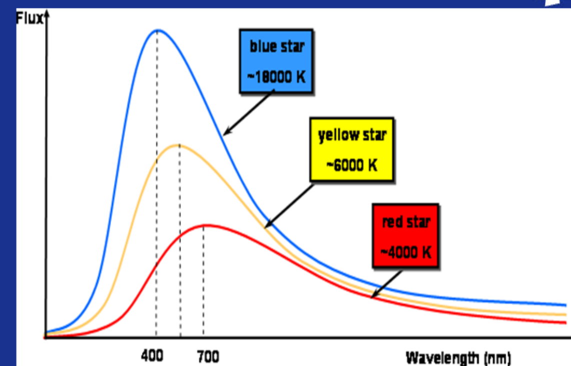


Figure 1 : Black Body Spectrum



Blackbody radiation is a theoretical concept in quantum mechanics in which a **material or substance completely absorbs all frequencies of light** or represents a conversion of **a body's internal energy into electromagnetic energy.** Because of the laws of thermodynamics, this ideal body must also re-emit as much light as it absorbs. Although there is no material that can truly be a blackbody, some have come close. **Carbon in its graphite form** is about 96% efficient in its absorption of light.

Blackbody Radiation

Wein's displacement law: wavelength at which the energy density is a maximum is related to the temperature.

$$\lambda_{max} T = 0.00298m * K$$

Wein's radiation law: valid at short wavelengths.

$$u_\lambda = \frac{A\lambda^{-5}}{e^{B/\lambda T}}$$

Rayleigh-Jeans law: valid at long wavelengths.

$$u_\lambda = C\lambda^{-4}$$

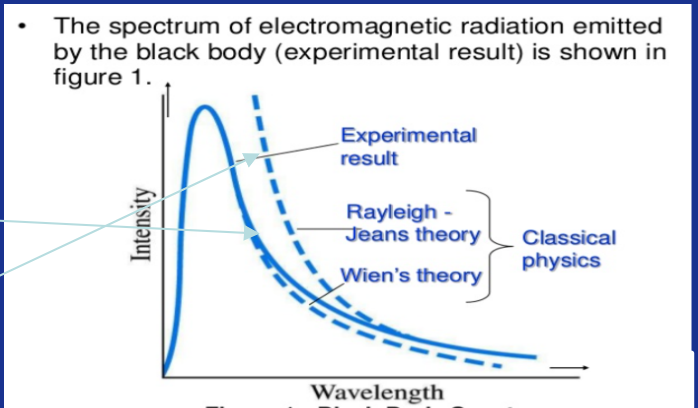
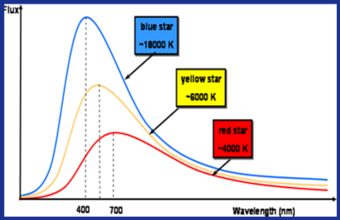


Figure 1 : Black Body Spectrum

Planck simply combined these two conditions into one and obtained a new radiation formula:

$$u_\lambda = \frac{A\lambda^{-5}}{e^{B/\lambda T} - 1}$$



Example

The peak in the radiation from the sun occurs at about 500 nm. What is the sun's surface temperature, assuming that it radiates as a blackbody.

Solution:

From Wein's law

$$T = 0.00298 / 500 \times 10^{-9} = 5800\,\mathrm{K}$$

In comparison, the temperature of the filament of an incandescent bulb is about 2000 K.

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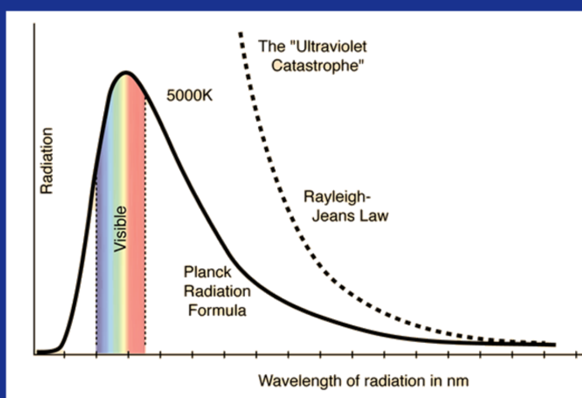
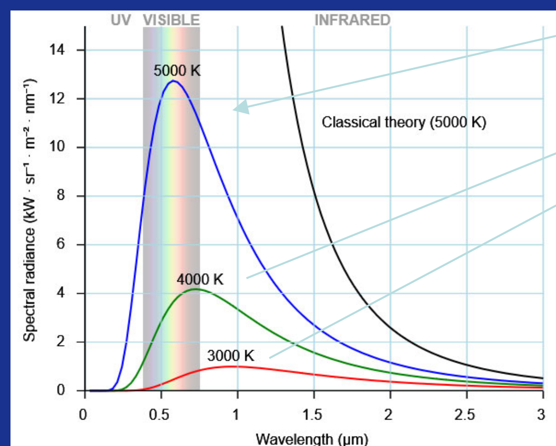
Blackbody Radiation

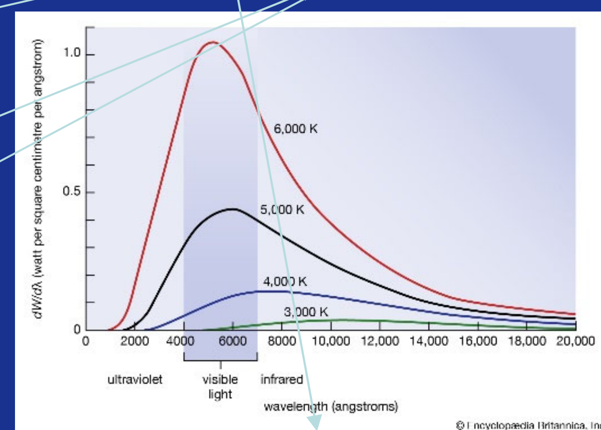
As the temperature decreases, the peak of the black-body radiation curve moves to lower intensities and longer wavelengths. The black-body radiation graph is also compared with the classical model of Rayleigh and Jeans

The type of light produced by an object will depend on its temperature, so let's digress slightly to investigate what ``temperature'' is. **Temperature** is a measure of the random motion (or energy) of a group of particles. Higher temperature (T) means more random motion (or energy).

$$u_\lambda = \frac{8\pi h\lambda^{-5}}{e^{hc/\lambda kT} - 1}$$



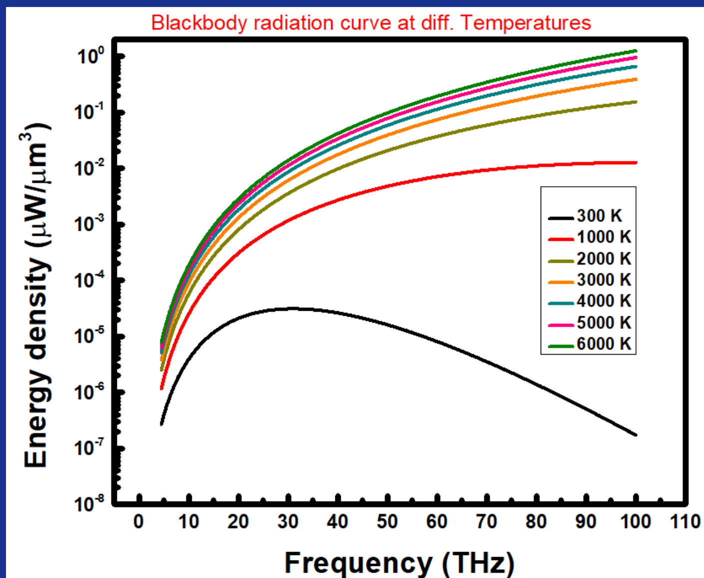


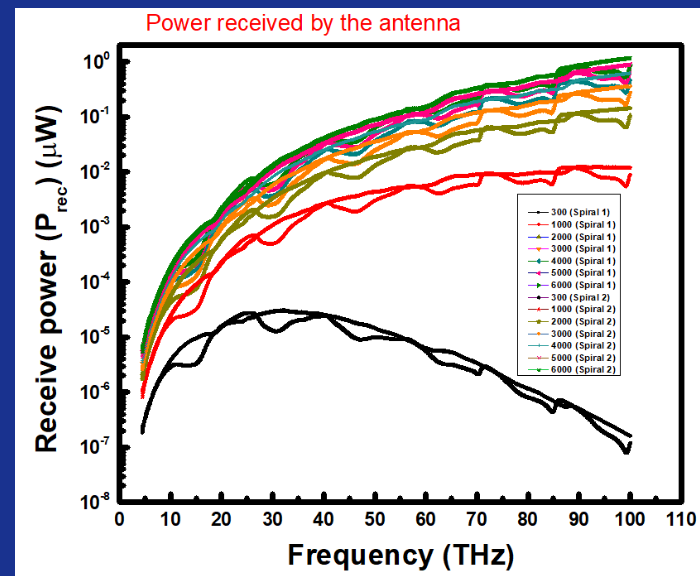


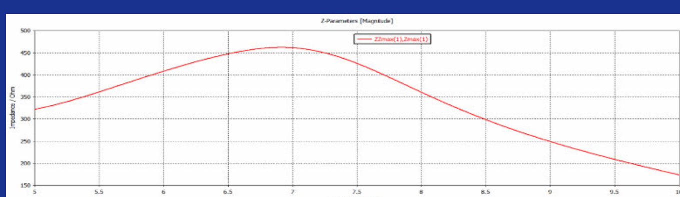
Blackbody radiation at a constant input temperature

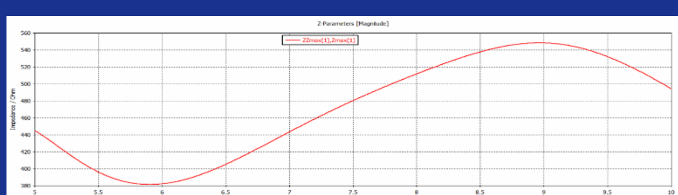
As the temperature decreases, the peak of the black-body radiation curve moves to lower intensities and longer wavelengths.

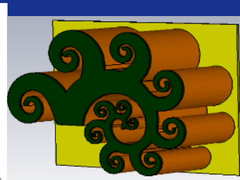
blackbody radiation from sun detected by a spiral antenna











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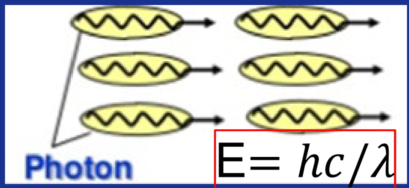
UNIVERSITY OF WASHINGTON

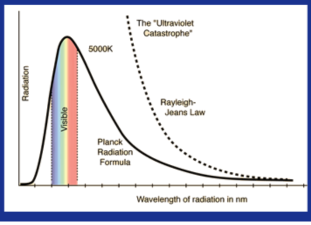
Plank's Radiation

Planck determine the number of ways a given total energy could be distributed among a fixed number of the oscillators in the black body to calculate the entropy of the system. Total energy of the oscillators is divided into energy "elements" of size E. He found that he can obtain the same expression provided he set E=hf, where f is the frequency and h is a constant. The earlier raditation equation becomes:

$$u_\lambda = \frac{A\lambda^{-5}}{e^{B/\lambda T} - 1} \Rightarrow u_\lambda = \frac{8\pi h\lambda^{-5}}{e^{hc/\lambda kT} - 1}$$



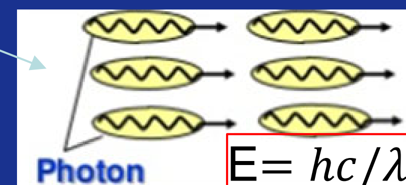
Where it yields the complete spectrum of the cavity radiation.



Einstein's Quantum Hypothesis

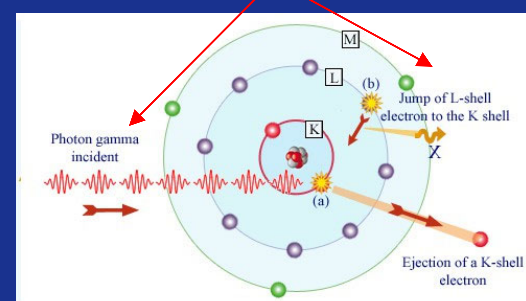
Einstein proved that Planck's radiation law could be derived if the energy of each individual oscillator is **quantized in steps of hf**. In the nth "level", the energy is

$$E = nh \times f = nh \times C_o / \lambda$$



Einstein's hypothesis implies that an oscillator can emit or absorb radiation only in multiples of hf.

Planck discovered in 1900, is that energy is not continuous but quantized—meaning that it can only be transferred in individual "packets" (or particles) of the size hf. Each of these energy packets is known as a quantum (plural: quanta).



Although Planck had introduced the constant h, the idea that **the energy of an oscillator is really quantized came from Einstein.**

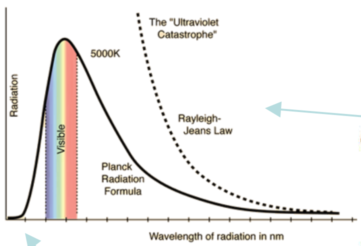
Example

A block of mass 0.2 kg oscillates at the end of a spring (k=5 N/m) with an amplitude of 10 cm. what is its "quantum number" n?

Generally speaking, at lower frequency it behaves like a wave, while at higher frequency it acts like a particles.

Solution:

$$E = \tfrac{1}{2} kA^2 = 0.025\ \mathrm{J}$$



$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = 0.8\ \mathrm{Hz}$$

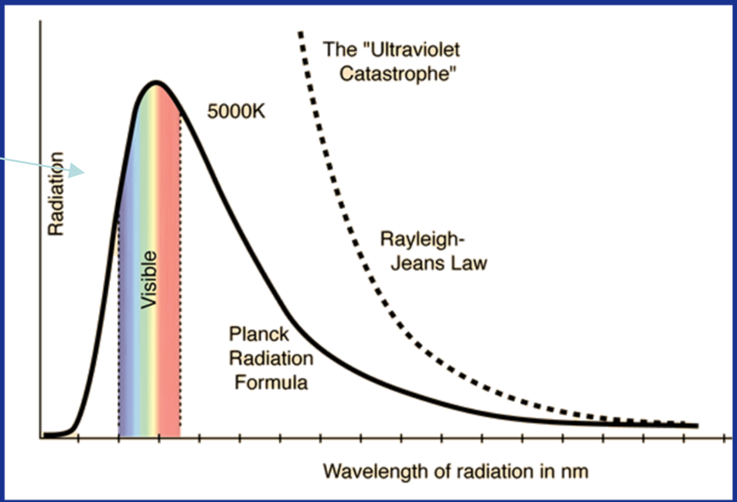
For example, at large quantum number the physics becomes classical.

$$n = E_n / hf = 10^{32} \quad \text{(extremely large)}$$

Under what condition the quantum effect becomes significant? *E* small, *f* high.

Einstein's Quantum Hypothesis

- Einstein hypothesis was more or less ignored since few scientists were concerned with the problem of cavity radiation. However, by 1908 most physicists had become aware of the drastic disagreement between the prediction of classical physics and the radiation curve at short wavelength.

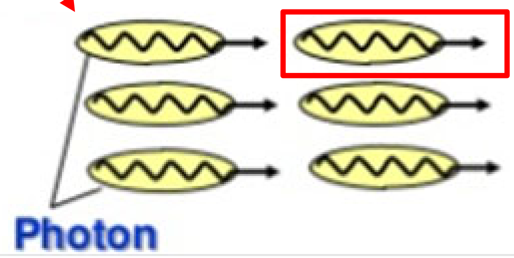


In summary

Photons

- In 1905, Albert Einstein proposed that light comes in bundle of energy (light is transmitted as tiny particles), called **photons**.
- Photon is defined as **a particle with zero mass consisting of a quantum of electromagnetic radiation where its energy is concentrated.**

Quantum means "fixed amount" (quantize)



$E = hc/\lambda$

- According to this assumptions, the quantum E of the energy for radiation of frequency f is given by

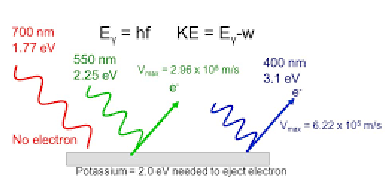
$$\boxed{E = hf} \qquad c = f\lambda$$

where h : Planck constant $= 6.63 \times 10^{-34}\ J\,s$

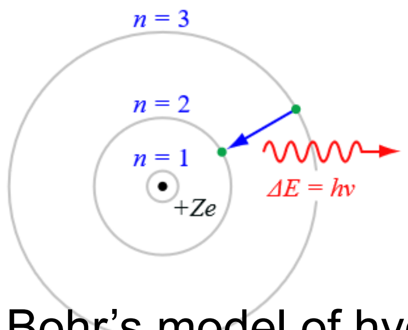
$$\boxed{E = \frac{hc}{\lambda}}$$

Planck's quantum theory

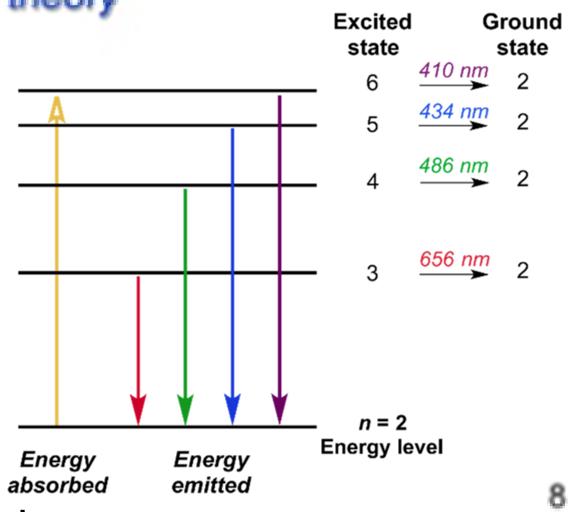
But the detail of what the electrons were doing was odd. They could be made to carry more energy simply by changing the colour of light. In particular, the electrons released from a metal bathed in violet light carried more energy than electrons released by a metal bathed in red light.



Photoelectric effect



Bohr's model of hydrogen



In summary

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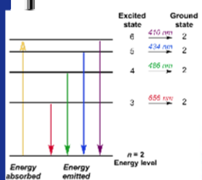
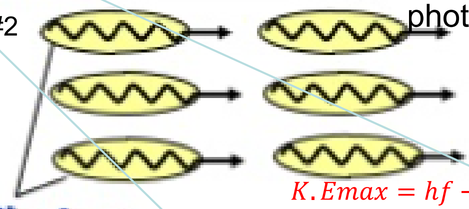
Planck's Quantum Theory	Classical theory EM Theory
Energy of the e.m radiation is **quantised**. (**discrete**)	Energy of the e.m radiation is **continously**.
Energy of e.m radiation ***depends*** on its **frequency or wavelength**	Energy of e.m radiation ***does not*** depend on its **frequency or wavelength** (**depends on Intensity**) $I \propto A^2$

Energy level of matter and photon

Key points #2





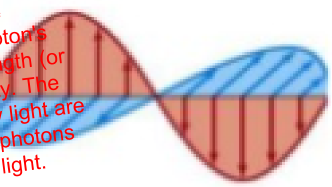
Photon

$K.Emax = hf - Wo$

$eV = hf - Wo$

$\#\ of\ e \times V_s = (\#\ of\ hf - Wo)$

KE of electron not a function of intensity but frequency. a brighter object is more intense but not necessarily more energetic. Remember that a photon's energy depends on the wavelength (or frequency) only, not the intensity. The photons in a dim beam of X-ray light are much more energetic than the photons in an intense beam of infrared light.



$$\boxed{E = hf}$$

Key points #1

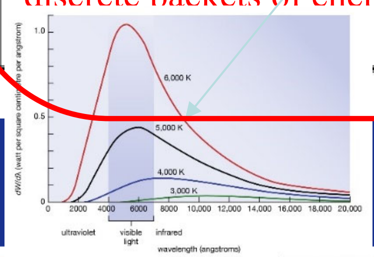
The EM radiation emitted by the black body is discrete packets of energy known as quanta..

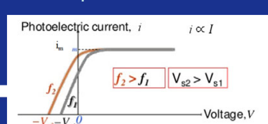
$$\boxed{E_{classical} = k_B T}$$

k_B = Boltzman's constant

T = temperature

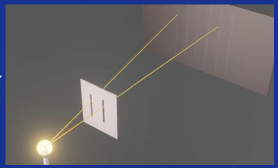
Applies to all mechanical and classical equations

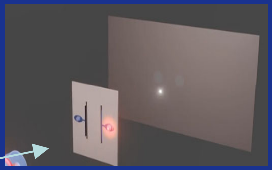




Photoelectric Effect

The work of **Young and Fresnel** had converted the scientists from the corpuscular theory to the wave theory of light. Maxwell predicted that light is an electromagnetic wave and Hertz' experiment was its crowning glory.

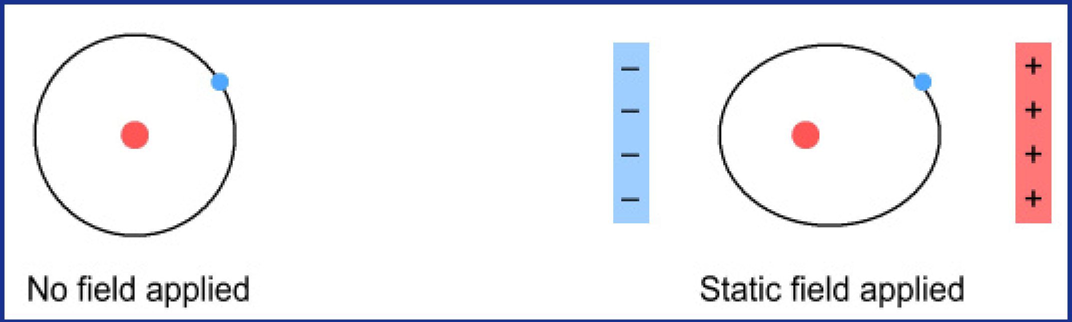




Ironically, **the very experiment that demonstrates that light is an electromagnetic wave also produced the first evidence of its corpuscular nature.**

Hallwachs found that when a zinc plate is illuminated with ultraviolet light, it becomes positively charged, and in some alkali metals, visible light can produce this emission of electrons, now called the photoelectric effect.

Orbital frequencies of electrons

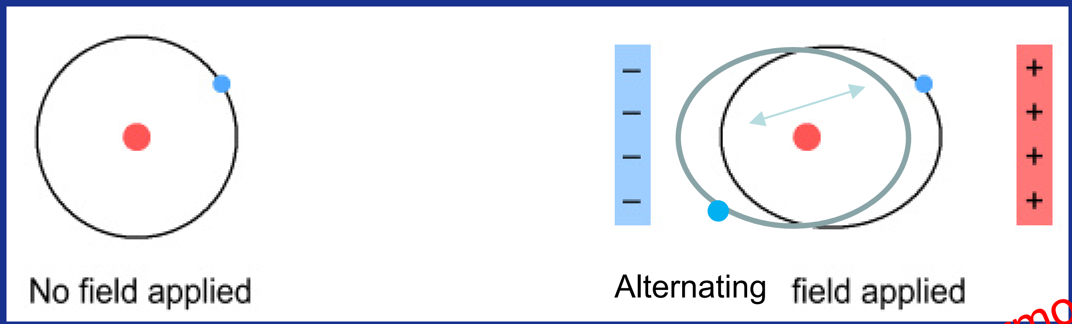


Let’s first consider the impact of a fixed electric field. That is, light of zero frequency. What will happen? If the field is weak, the atom’s (positive) nucleus will be pulled in one direction, and it’s electrons in the other. The result would be an electron in an elliptic orbit.

Now consider the impact of a low-frequency wave, i.e. light with frequency lower than the electron’s orbital frequency. As it impacts, the wave pushes the electron ‘to-and-fro’: sometimes in the same direction as what the electron is moving, and other times in the opposing direction. Or if you prefer, sometimes pushing it toward the nucleus and other times away.

As a result the orbit is shaken up but not enough for it to break. Once the incoming wave stops, the orbit returns to its previous circular path.

Orbital frequencies of electrons



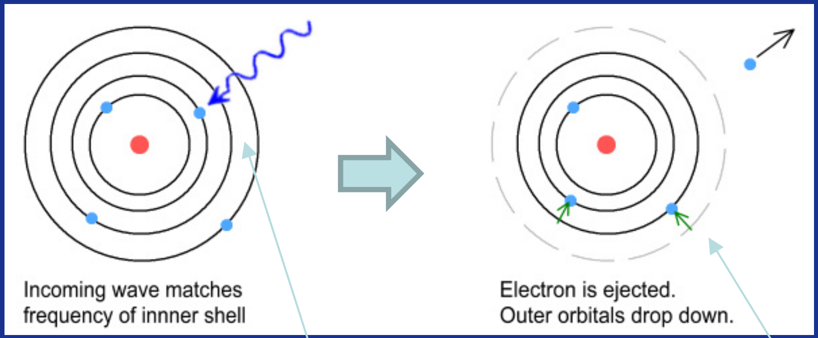
Now consider the situation where the input frequency matches the orbital frequency. Initially there might be some 'confusion' as the incoming wave pushes and pulls in the opposing direction to what the electron is moving. But after a short while the orbit will 'phase lock' or synchronize with the incoming wave.

The situation is now somewhat different from the preceding example. Here the incoming wave pulls on the electron in an outward direction (away from the nucleus) during both 'halves' of the orbit.

The incoming wave in now at the 'natural frequency' of the electron's motion. Just like the wine glass, the orbit experiences so much force it is 'shattered'. Or rather, the electron is pulled away from the nucleus where the attractive force is reduced.

At this point the existing speed of the electron takes over and enables the electron to fly fully away from the nucleus. The atom is ionized. From an observer's viewpoint it might appear that the incoming wave ionized the atom because the light's frequency had the right 'energy'. But in reality it did so because it matched the frequency of the orbit.

Orbital frequencies of electrons



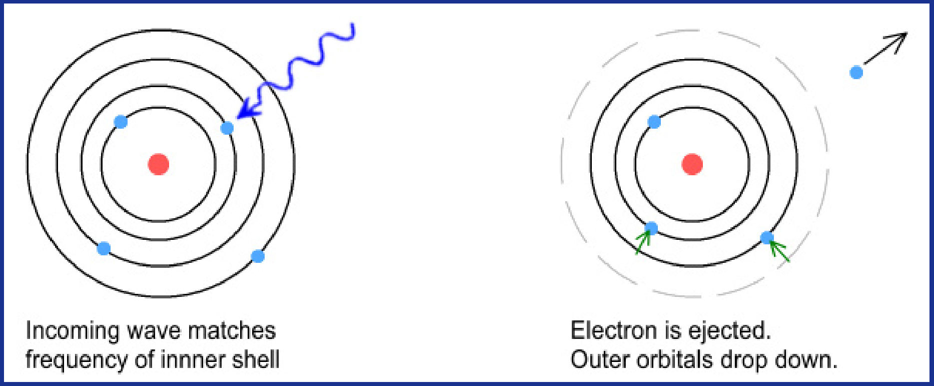
The above describes a situation in which an electron can be ejected when the incoming wave matches the frequency of the orbit. So what about the situation where the wave frequency is higher than the orbit frequency? In this case the wave is no longer at the natural frequency and thus shouldn't be able to eject the electron. Except we know it does.

To understand why, look again at the Solar System. Here we have many planets orbiting at different distances from the Sun. The inner planets have the highest frequency and the outer planets the lowest. An atom is a similar situation. There are inner electrons orbiting at high frequency and outer ones at low frequency. When a high frequency wave comes in, the outer electrons might not be very affected. But the inner ones will be. When an incoming wave matches the natural frequency of an inner electron, that electron will be ejected.

So how does this affect the outer electrons? There are two possibilities to consider. The first is that, in the process of being ejected, the inner electrons would pass through the orbits of the outer electrons. This might destabilize their orbits to the point that they also got ejected.

The second and more likely scenario is that once the inner electrons got ejected, the outer electrons would drop in altitude to fill the newly vacated orbit space. Once there, the electron that took up the old orbit would have the frequency of the incoming wave. It too would be ejected and another electron would drop to fill that vacancy. This process would continue until electrons of (originally) lower frequency (and higher altitude) were also ejected.

Orbital frequencies of electrons



Basically not Larger the energy, the better it is. It is based on if incoming photon has the correct wavelength, then specific band of electrons will be ejected

Thus a high frequency corresponding to an electron in an inner orbit ends up ejecting all the electrons from the outer orbits.

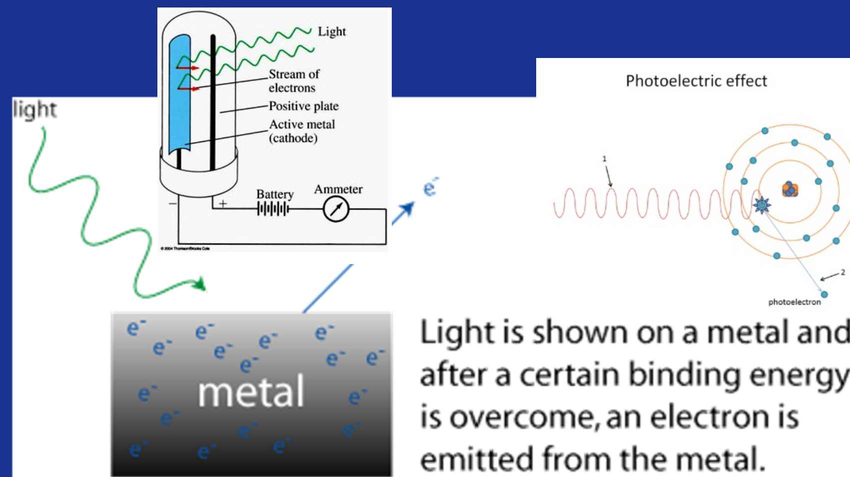
Another thing that should be apparent is that **higher frequencies correspond to higher orbit speeds**. And so an electron ejected from a higher orbit speed should be expected to leave the material surface faster. **This is consistent with the observation that higher frequencies eject electrons at higher energies (speeds).**

It should also be noted that this ejection process would be quite fast. I.e. ***it should happen within a few cycles of the incoming wave. So to us it would appear instantaneous***. This is consistent with the observation that the photoelectric effect is immediate and **doesn't require a build-up of energy before an electron is ejected.**

Particle with Wave-Like Behavior

Photoelectric Effect

At this point you may think that it's pretty obvious that light behaves like a wave. So, why how do we know that light is really composed of particles called photons? Support for this idea comes from an experiment that is called the photoelectric effect.



An important feature of this experiment is that the electron is emitted from the metal with a specific kinetic energy (i.e. a specific max speed<= speed of light). (if velocity doesn't change, kinetic energy doesn't change)- it will not speed up

When light intensity increased (brighter light- more photons), the kinetic energy of the emitted electron did not change, however, it varies with wavelength! The numbers of electrons instead increased! So energy conserved!

Photoelectric Exp

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(2)

$E = W_o + K.E_{max}$

$hf = W_0 + \frac{1}{2} mv_{max}^2$

Einstein's photoelectric equation

(a) $E = hf = h\frac{c}{\lambda}$ = photon energy

f = frequency of em radiation /incoming light

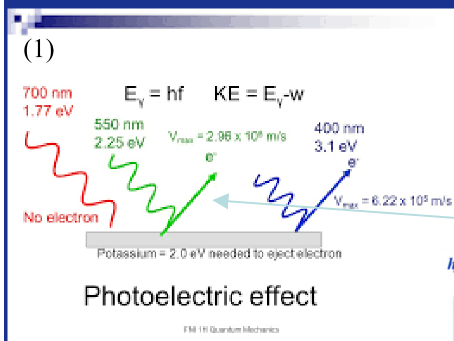
$K.E_{max} = \frac{1}{2} mv_{max}^2$ = maximum kinetic energy of ejected electron.

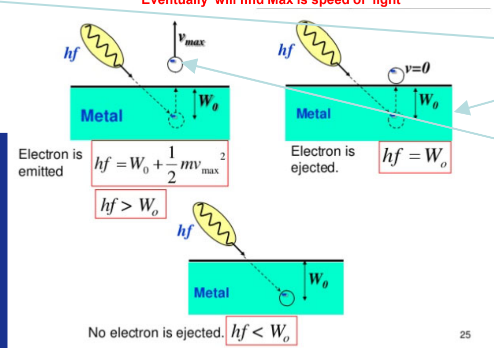
v_{max} = maximum speed of the photoelectron

NOT speed of light

Eventually will find Max is speed of light

(1)



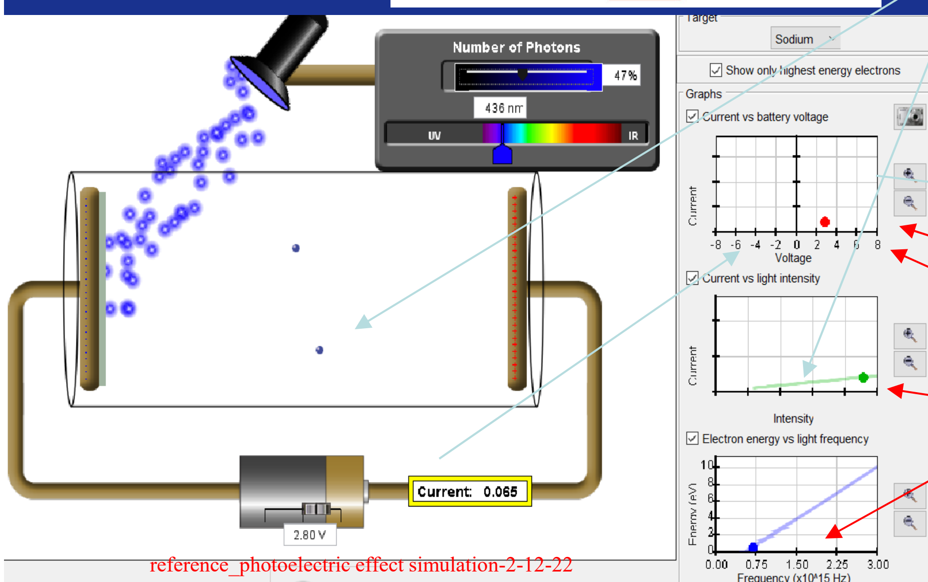


Photoelectron is proportional to the light intensity

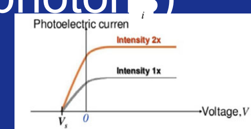
maximum kinetic energy of the electrons depends on the light source wavelength and the plate material, but not intensity of the source.

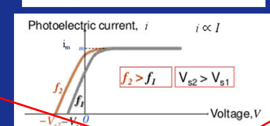
Light can strike the surface of some metals causing an electron to be ejected

- No matter how brightly the light shines, electron are ejected only if the light has sufficient energy (w_o) (sufficiently short wavelength)
- After the necessary energy (w_o) is reached, the current (# of electrons emitted per second) increases as the intensity brightness) of the light increases (# of photons)



reference_photoelectric effect simulation-2-12-22





PE needed to stop electron from moving in Exp

$e = (hf - W_o)/(V+V_s)$

Electron's KE

Where $W_o = h\nu_o$

$K.E_{max} = hf - W_o$

$\boxed{eV_s = hf - W_o}$

(a) $V_s = \frac{h}{e} f - \frac{W_o}{e}$

maximum kinetic energy of the electrons depends on the light source wavelength and the plate material, but not intensity of the source.

$V + V_s = (hf - W_o)/e$

V_s = min voltage needed to stop electron from moving

$K.Emax = hf - Wo$

$eV = hf - Wo$

(b)

$\boxed{\# \text{ of } e \times V_s = (\# \text{ of } hf - Wo)}$

Photoelectron is proportional to the light intensity

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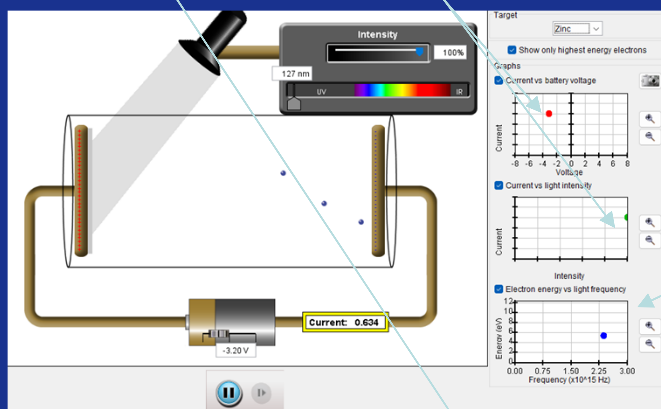
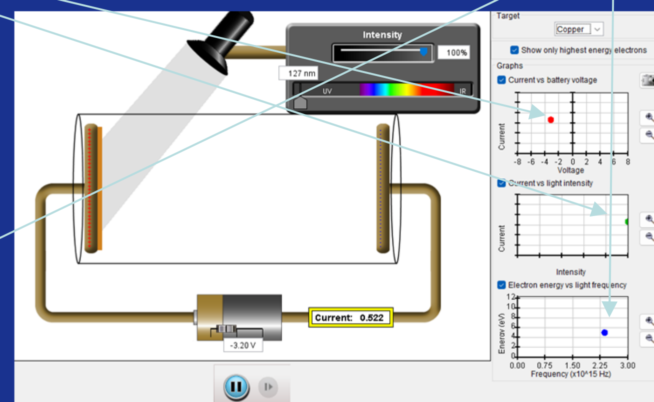
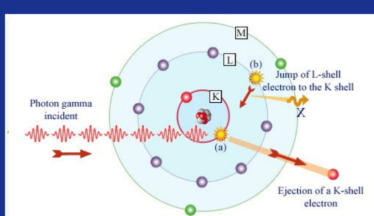
Photoelectric Exp

1. Different materials (intensity and wavelengths are fixed):
- Current varies with materials Wo
- Energy (speed of electron goes up so are -> eV) – KE







$$\boxed{K.E_{max}} = hf - W_o$$

$$\boxed{eV_s} = hf - W_o$$

$$V_s = \frac{h}{e} f - \frac{W_o}{e}$$

For single electron

Quantization of energy (photon in pocket of oscillating energy or quantization of energy level in matter).

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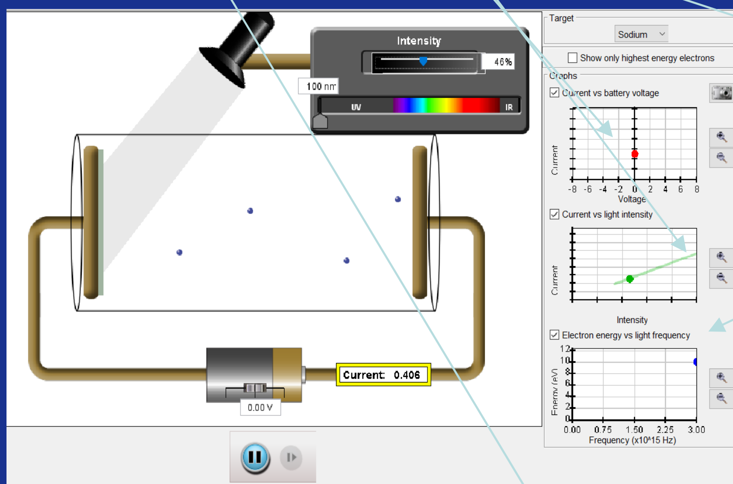
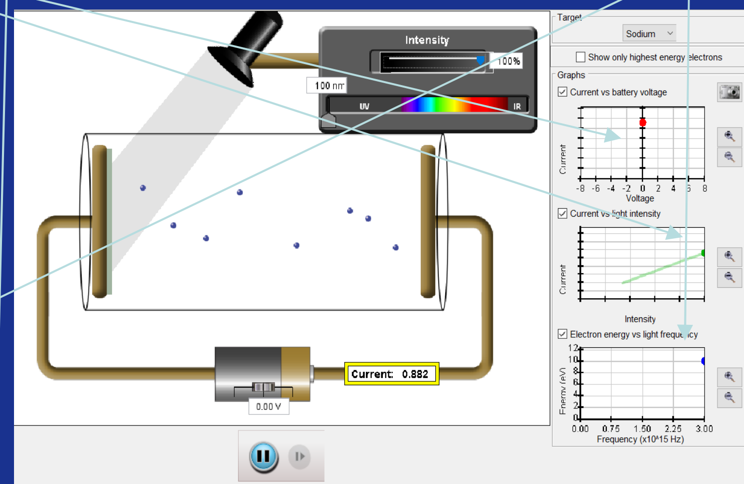
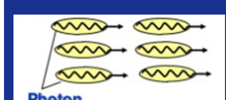
University of Washington

Photoelectric Exp

2. Different intensity (fixed material and wavelength):
- Current increases with intensity
- Energy (speed of electron stay the same so are -> eV) – KE







$$K.Emax = hf - Wo$$
$$eV = hf - Wo$$
$$\boxed{\#\ of\ e\ x\ V_s} = (\#\ of\ hf\text{-}Wo)$$

For single electron

For all electrons generated by photons

E= hc/λ

e- e- e- e-

Increase intensity only increase number of electrons or photons with same energy pocket or level.

Photoelectric Exp

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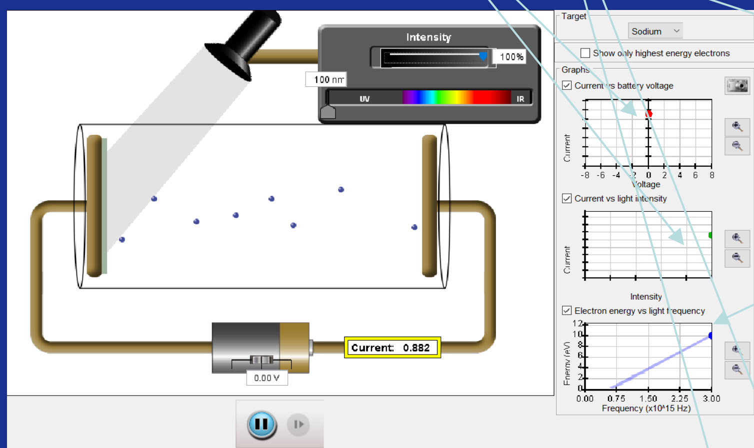
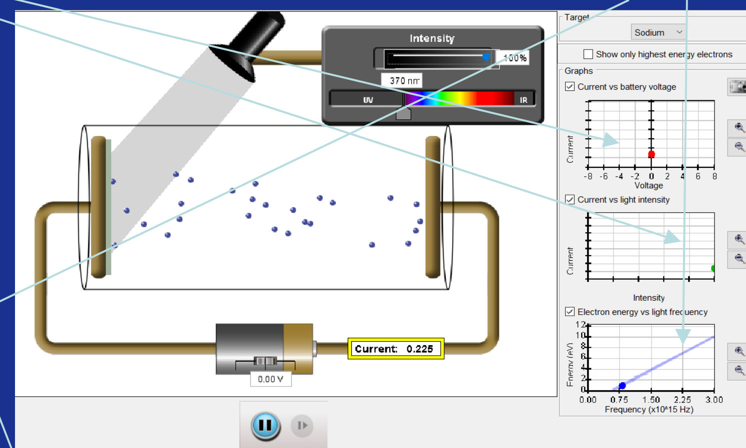
3. Different wavelengths (fixed material and intensity):
- Current decreases with increases in wavelength
- Energy (speed of electron also varies so are -> eV) – KE





$E = hc/\lambda$

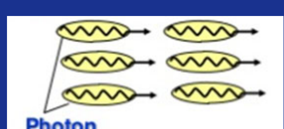
$\boxed{K.Emax} = hf - Wo$

$\boxed{eV} = hf - Wo$

$\boxed{\#\ of\ e\ x\ V_s} = (\#\ of\ hf\text{-}Wo)$

For single electron

For all electrons generated by photons



Energy is a Function of <u>frequency</u> in both matter and photon

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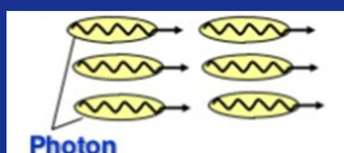
From Photoelectric Exp

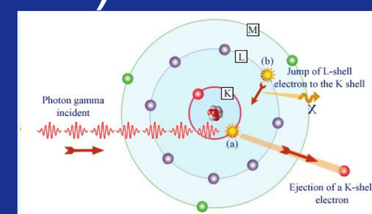
- Energy is a Function of <u>frequency</u> in both matter and photon.

$$\boxed{E = hc/\lambda}$$

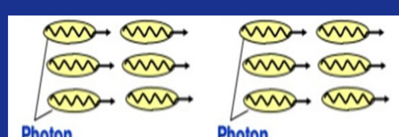
- <u>Quatization of energy</u> (photon in pocket of oscillating energy or quantization of energy level in matter).

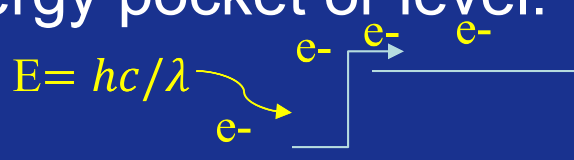


$$E = \text{nh} \times f = \text{nh} \times C_o / \lambda$$



- <u>Increase intensity only increase number of electrons or photons</u> with same energy pocket or level.





Photoelectric Exp

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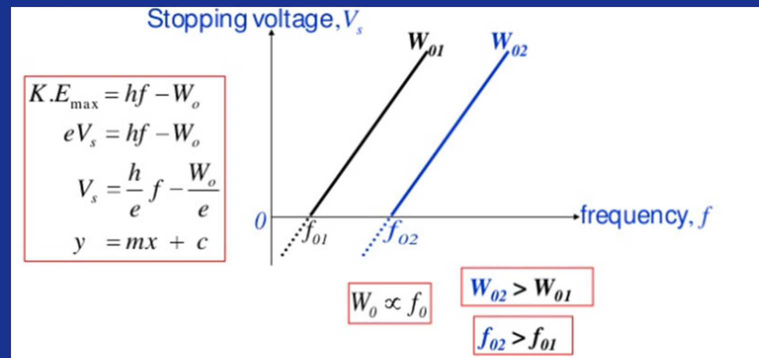
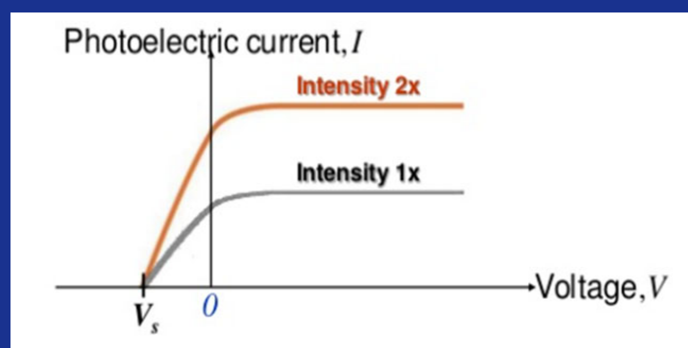
K.E. is a Function of frequency,



Photon must have sufficient energy to allow electron to overcome its thread hold energy (wo) to break free, here it shows each wavelength of light has its own curve to excite its energy band of electrons in matter so electron will have different KE

Energy band is discrete and generated photon and excited electrons has fix energy



$$K.E_{max} = hf - W_o$$

$$\boxed{eV_s = hf - W_o}$$

$$V_s = \frac{h}{e} f - \frac{W_o}{e}$$

maximum kinetic energy of the electrons depends on the light source wavelength and the plate material, but not intensity of the source.

$K.Emax = hf - Wo$

$eV = hf - Wo$

$\boxed{\#\ of\ e \times V_s = (\#\ of\ hf - Wo)}$

Photoelectron is proportional to the light intensity

When intensity of the specific wavelength of photon increases, we don't see the speed of electron increases. Instead, the numbers of electrons with same energy ($1/2mv^2$) actually are created and thus induced current increases (# of electrons /sec)

From magazine

Physicists had discovered that a chunk of metal becomes positively charged when it is bathed in visible or ultraviolet light. They called this the "photoelectric effect".

The explanation was that atoms in the metal were losing negatively-charged electrons. Apparently, the light delivered enough energy to the metal to shake some of them loose.

But the detail of what the electrons were doing was odd. They could be made to carry more energy simply by changing the colou of light. In particular, the electrons released from a metal bathed in violet light carried more energy than electrons released by a metal bathed in red light.

This doesn't make much sense if light is simply a wave.

You usually change the amount of energy in a wave by making it taller – think of the destructive power of a tall tsunami – rather than by making the wave itself longer or shorter. Each quantum packs a discrete energy punch

By extension, the best way to increase the energy that light transfers to the electrons should be by making the light waves taller: that is, making the light brighter. Changing the wavelength, and thus the color, shouldn't make as much of a difference.
Einstein realized that the photoelectric effect was easier to understand by thinking of light in terms of Planck's quanta.

He suggested that light is carried in tiny quantum packets. Each quantum packs a discrete energy punch that relates to the wavelength: the shorter the wavelength, the denser the energy punch. This would explain why violet light packets, with a relatively short wavelength, carried more energy than red light packets, with a relatively longer one.

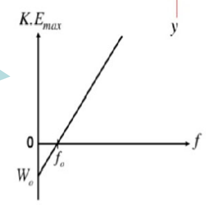
- Generally, Einstein's photoelectric equation;

$$\boxed{E = W_o + K.E_{\max}}$$

$$K.E_{\max} = E - W_o$$

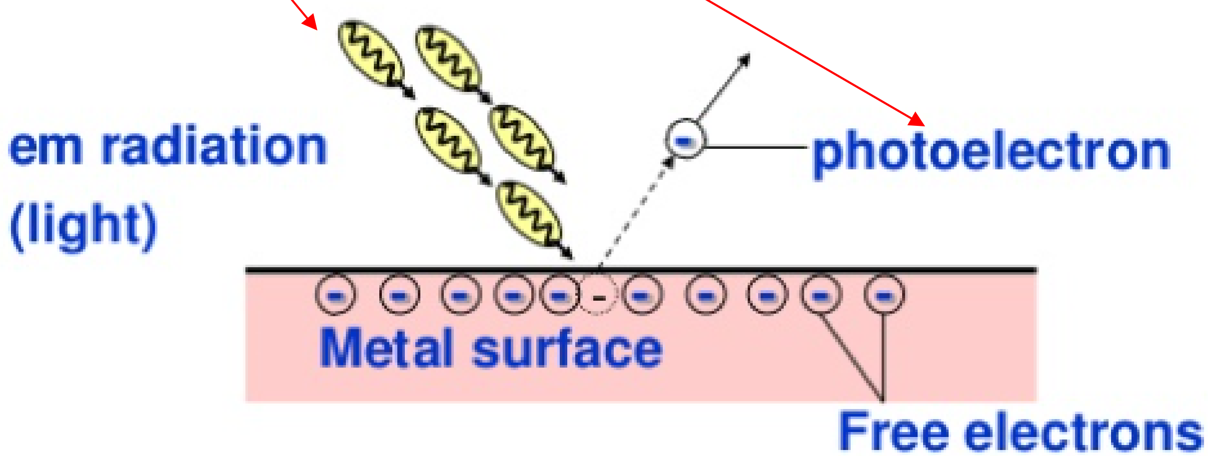
$$K.E_{\max} = hf - W_o$$

$$y = mx + c$$

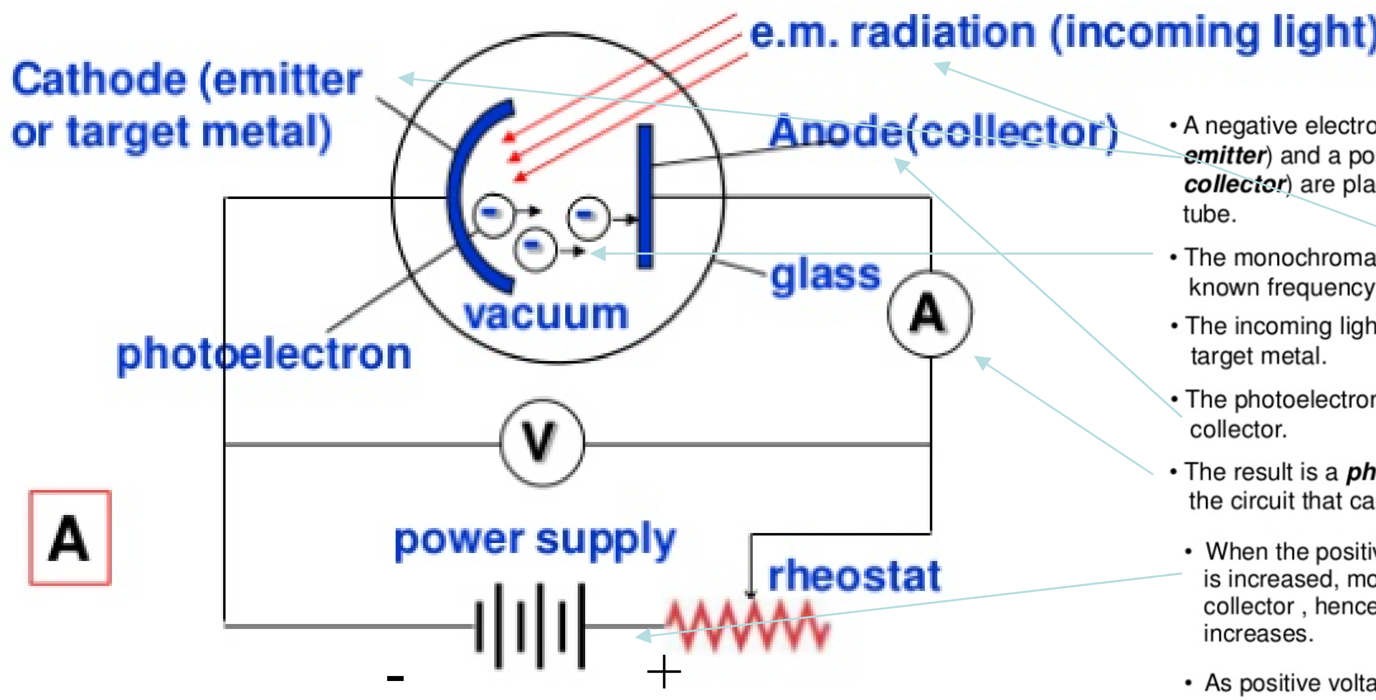


$$\boxed{f \uparrow K.E_{max} \uparrow}$$

- The photoelectric effect is ***the emission of electrons from the metal surface when electromagnetic radiation of enough frequency falls/strikes/ incidents /shines on it.***
- **A photoelectron** is **an electron ejected due to photoelectric effect** (an electron emitted from the surface of the metal when light strikes its surface).



- The photoelectric effect can be measured using a device like that pictured in figure below.



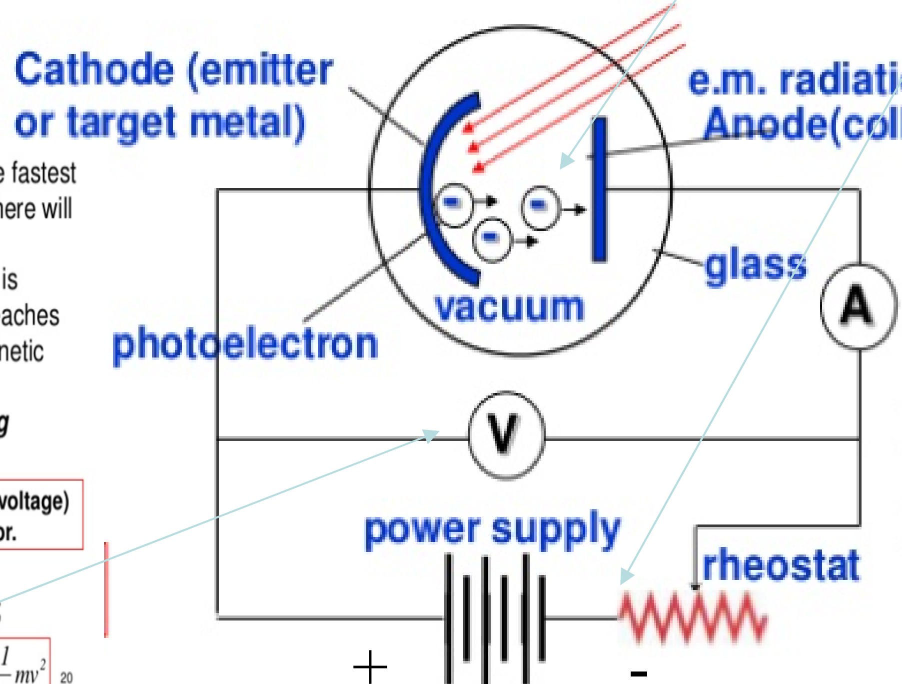
The photoelectric effect's experiment

- A negative electrode (**cathode** or **target metal** or ***emitter***) and a positive electrode (**anode** or ***collector***) are placed inside an evacuated glass tube.
- The monochromatic light (UV- incoming light) of known frequency is incident on the target metal.
- The incoming light ejects photoelectrons from a target metal.
- The photoelectrons are then attracted to the collector.
- The result is a ***photoelectric current*** flows in the circuit that can be measured with an ammeter.
- When the positive voltage (potential difference) is increased, more photoelectrons reach the collector , hence the photoelectric current also increases.
- As positive voltage becomes sufficiently large, the photoelectric current reaches a maximum constant value I_m, called **saturation current**.

Saturation current is defined as **the maximum constant value of photocurrent in which when all the photoelectrons have reached the anode.**

18

- When the voltage is made negative by reversing the power supply terminal as shown in figure below, the photoelectric current decreases since most photoelectrons are repelled by the collector which is now negative electric potential.



Reversing power supply terminal

(to determine the stopping potential)

- If this reverse voltage is small enough, the fastest electrons will still reach the collector and there will be the photoelectric current in the circuit.
- If the reverse voltage is increased, a point is reached where the photoelectric current reaches zero – no photoelectrons have sufficient kinetic energy to reach the collector.
- This reverse voltage is called the ***stopping potential***, V_s.

V_s is defined as **the minimum reverse potential (voltage) needed for electrons from reaching the collector.**

- By using conservation of energy :
  (loss of KE of photoelectron = gain in PE) ;

$$K.E_{max} = eV_s$$

$$\boxed{eV_s = \frac{1}{2}mv^2}$$

20

Einstein's theory of Photoelectric Effect

- According to Einstein's theory, an electron is ejected/emitted from the target metal by a collision with a single photon.

- In this process, all the photon energy is transferred to the electron on the surface of metal target.

- Since electrons are held in the metal by attractive forces, some minimum energy, W_o (***work function***, which is on the order of a few electron volts for most metal) is required just enough to get an electron out through the surface.

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- If the frequency f of the incoming light is so low that is $hf < W_o$, then the photon will not have enough energy to eject any electron at all.

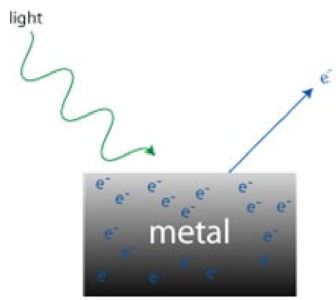
Different metal has different Wo

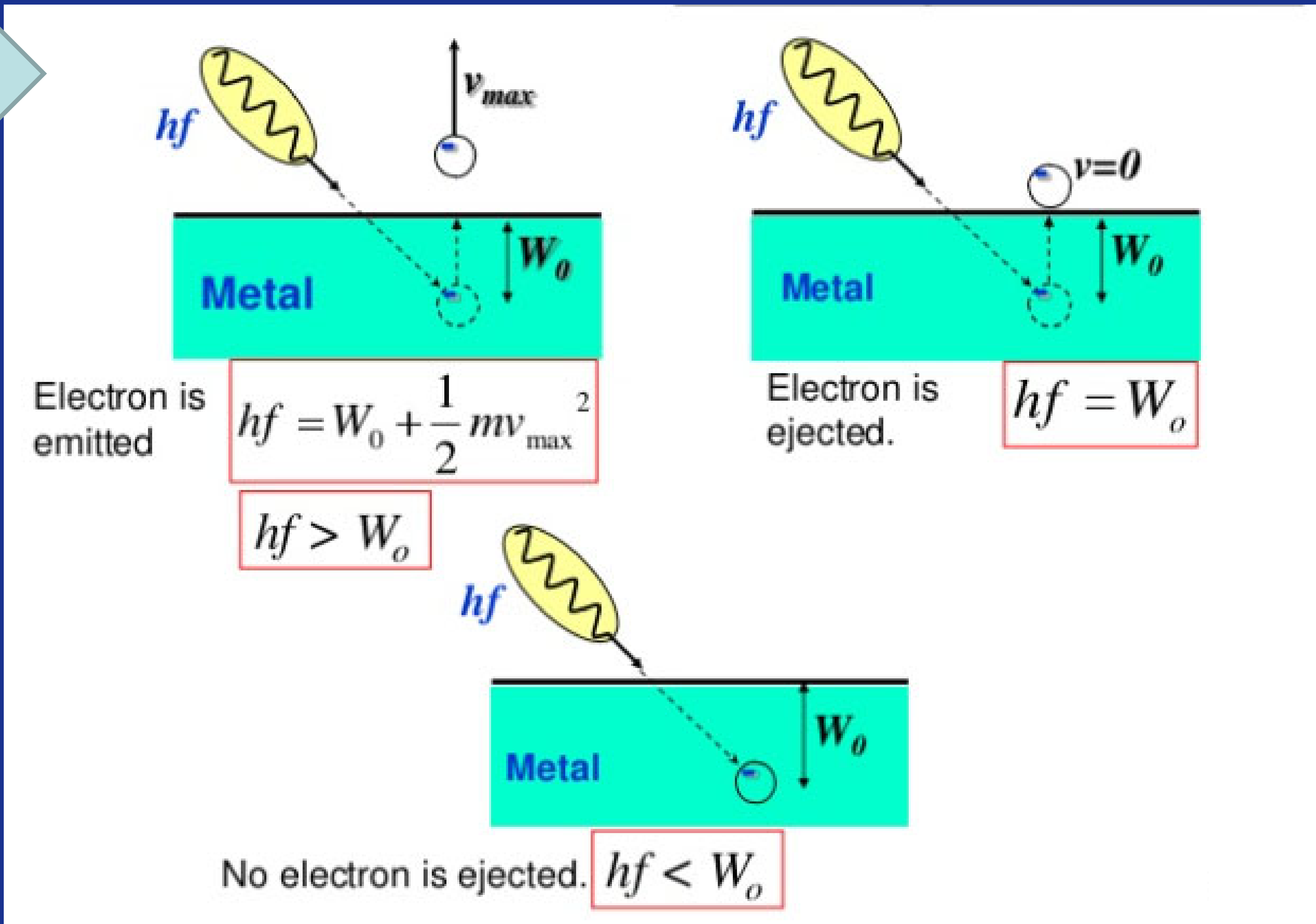
- If $hf > W_o$, then electron will be ejected and energy will be conserved (the excess energy appears as kinetic energy of the ejected electron).

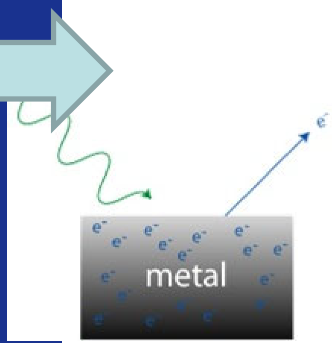
- This is summed up by **Einstein's photoelectric equation ,** $\boxed{E = W_o + K.E_{max}}$

Where $W_o = h\nu_o$

$$\boxed{hf = W_0 + \frac{1}{2} m v_{max}^2}$$




hf
v_{max}
W_0
Metal
Electron is emitted
$hf = W_0 + \frac{1}{2} m v_{max}^2$
$hf > W_o$
hf
$v=0$
W_0
Metal
Electron is ejected.
$hf = W_o$
hf
W_0
Metal
No electron is ejected.
$hf < W_o$



$$\boxed{E = W_o + K.E_{max}}$$

$$\boxed{hf = W_0 + \frac{1}{2} m v_{max}^2}$$

Einstein's photoelectric equation

$E = hf = h\dfrac{c}{\lambda}$ = photon energy

f = frequency of em radiation /incoming light

$K.E_{max} = \dfrac{1}{2} m v_{max}^2$ = maximum kinetic energy of ejected electron.

v_{max} = maximum speed of the photoelectron

Eventually will find Max is speed of light

$\boxed{E = W_o + K.E_{max}}$ $\boxed{hf = W_0 + \frac{1}{2} m v_{max}^2}$ $\boxed{W_o = hf_o = \frac{hc}{\lambda_o}}$

W_o = the **work function** of a metal.

= the minimum energy required (needed) to eject an electron from the surface of target metal.

f_o = **threshold frequency**.

= minimum frequency of e.m. radiation required to eject an electron from the surface of the metal.

$\boxed{f_o = \frac{c}{\lambda_o}}$

λ_o = **threshold wavelength**.

= maximum wavelength of e.m. radiation required to eject an electron from the surface of the target metal.

• Generally, Einstein's photoelectric equation;

There is a critical frequency for each metal, f_o , below which no electrons are emitted. This tells us that the kinetic energy is equal to the frequency of the light times a constant (i.e., the slope of the line). That constant is called Planck's Constant and is given the symbol $h = 6.626 \times 10^{-34}$ *J s.*

.

$$\boxed{E = W_o + K.E_{max}}$$

$$K.E_{max} = E - W_o$$

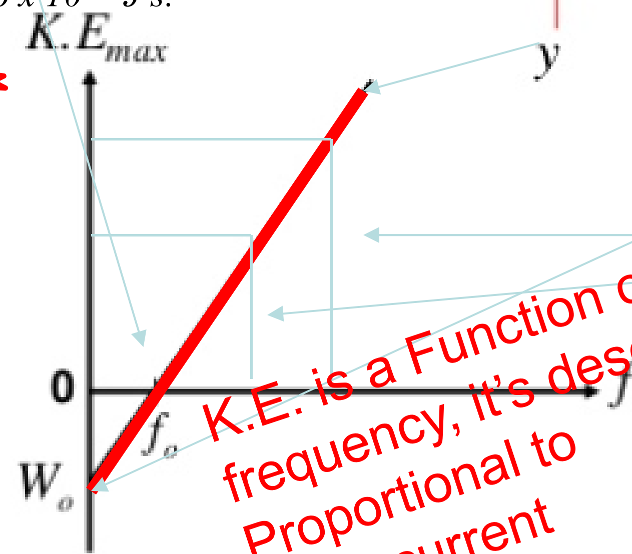
$$K.E_{max} = hf - W_o$$

$$y \qquad = mx + c$$

Linear equation

$$W_o = hf_o$$

$$\boxed{f \uparrow K.E_{max} \uparrow}$$



K.E. is a Function of frequency, it's descrete

Proportional to photocurrent

From the experiment, the kinetic energy of emitted electron shows that it varied with *the **frequency of the light (f)and this changed the kinetic energy of the emitted electron (K.E$_{max}$)***

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In terms of stopping voltage

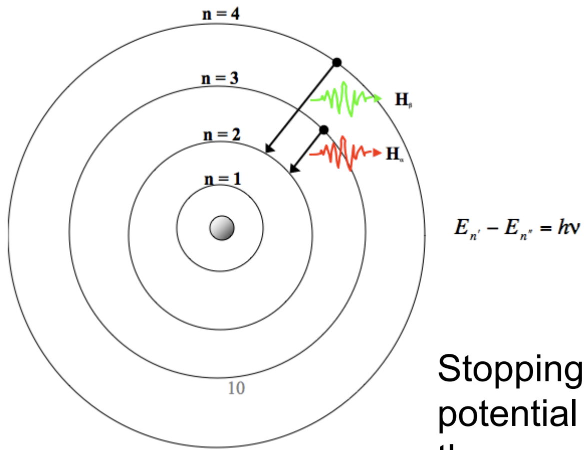
$$K.E_{max} = hf - W_o$$

$$eV_s = hf - W_o$$

$$V_s = \frac{h}{e} f - \frac{W_o}{e}$$

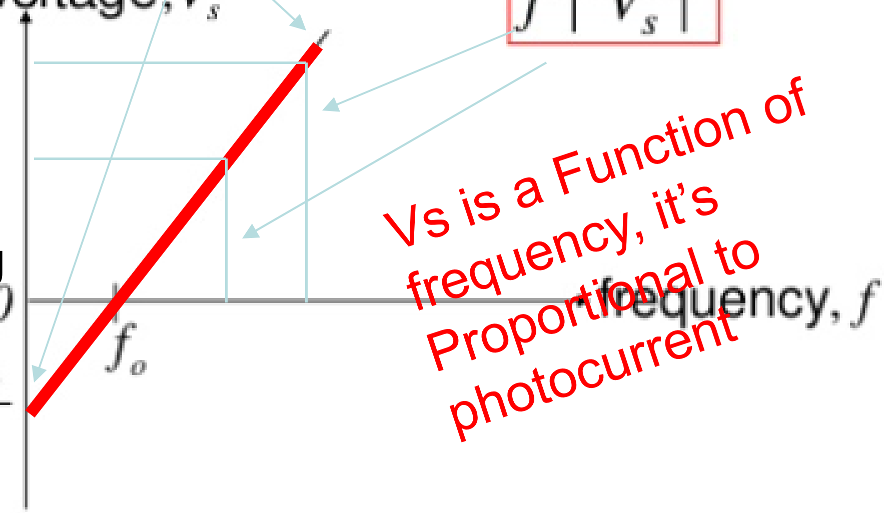
$$y = mx + c$$

Graphs in Photoelectric Effect



Stopping potential is like the energy level

$f \uparrow V_s \uparrow$



Stopping potential is minimum voltage needed from stopping electron from reach anode.

Vs is a Function of frequency, it's Proportional to photocurrent

Example

Ultraviolet light of wavelength 207 nm causes photoemission from a surface. The **stopping potential** is 2 V. Find the work function in eV; (b) the maximum speed of the photon electrons.

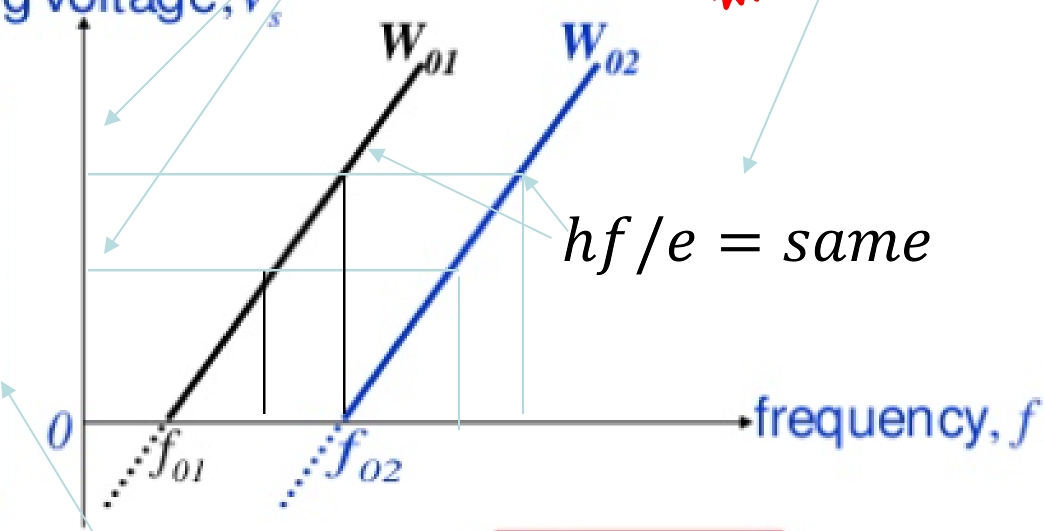
Solution:

$$(a)\ \phi = \frac{hc}{\lambda} - eV_0 = \frac{6.626\times10^{-34}\times3.0\times10^{-8}}{2.07\times10^{-9}\times1.6\times10^{-19}} - 2$$

$$= 6 - 2 = 4\ \text{eV}$$

$$(b)\ \frac{1}{2}mv_{\max}^2 = eV_0 \Rightarrow v_{\max} = \sqrt{\frac{2eV_0}{m}} = \boxed{8\times10^5\ \text{m/s}}$$

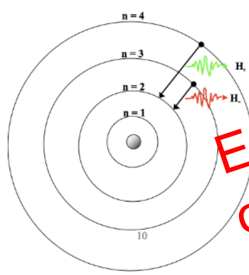
Variation of stopping voltage V_s with frequency f of the radiation for **different metals** but the **intensity** is **fixed**.



$$K.E_{\max} = hf - W_o$$
$$eV_s = hf - W_o$$
$$V_s = \frac{h}{e} f - \frac{W_o}{e}$$
$$y = mx + c$$

$\boxed{W_0 \propto f_0}$ $\boxed{W_{02} > W_{01}}$ $\boxed{f_{02} > f_{01}}$

$W_o = hf_o$



Energy is a Function of materials

Variation of photoelectric current i with voltage V for the radiation of **different intensities** but its **frequency and metal are fixed**.

Experimentally show

intensity is a Function of frequency and increase with number of e^- and independent of bias Voltage when velocity reach max

Photoelectric current, i

$\boxed{i \propto I}$

$$K.Emax = hf - Wo$$

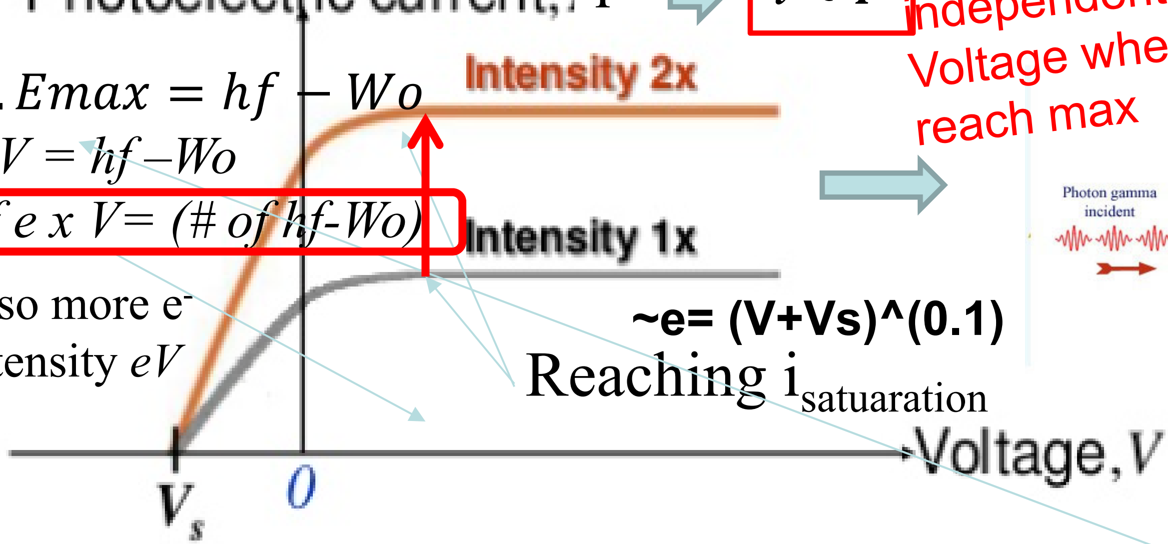
$$eV = hf - Wo$$

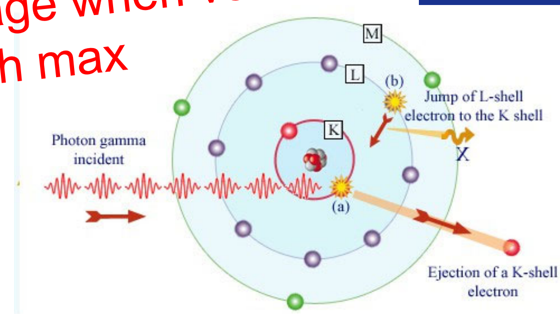
$$\boxed{\#\ of\ e \times V = (\#\ of\ hf - Wo)}$$

is fixed so more e^- more intensity eV

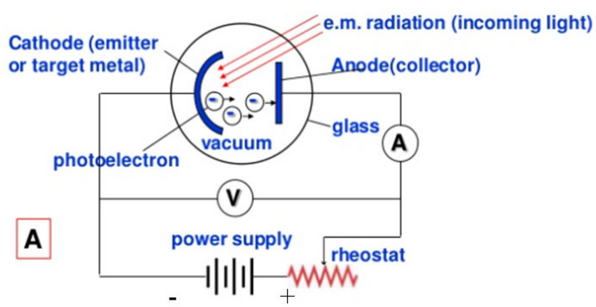
~e= (V+Vs)^(0.1)

Reaching $i_{satuaration}$





Just show e- with very predefined KE



An important feature of this experiment is that <u>the electron is emitted from the metal with a specific kinetic energy</u> (i.e. a specific speed). (if velocity doesn't change, kinetic energy doesn't change)

When light intensity increased (brighter light), <u>the kinetic energy of the emitted electron did not change! The numbers of electrons instead increased! So energy conserved!</u>

Notes:

Classical physics

Light intensity , $I = \dfrac{\text{energy}}{\text{time} \times \text{area}}$

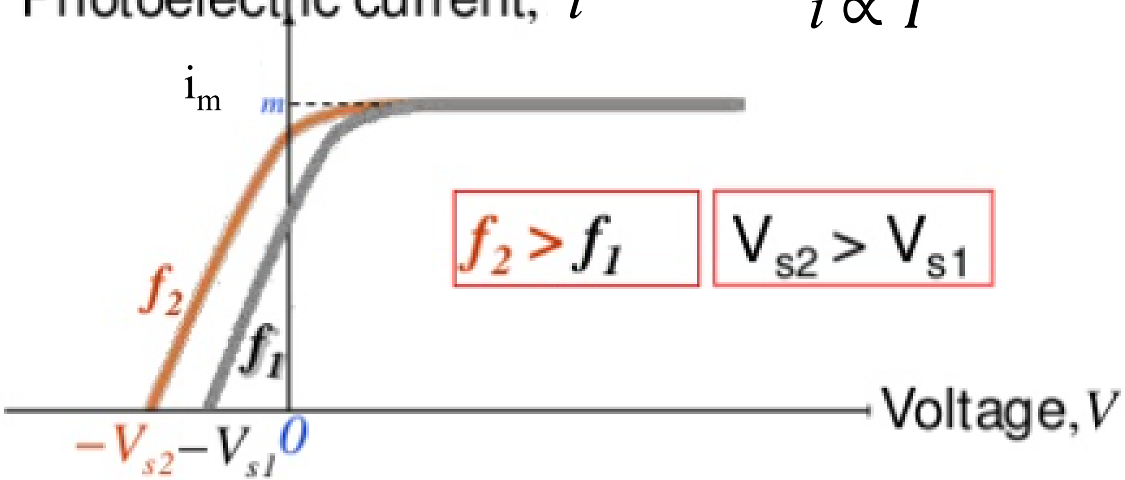
Quantum physics

Light intensity , $I = \dfrac{\text{number of photons}}{\text{time} \times \text{area}}$

Light intensity $\propto$ number of photons

Light intensity $\uparrow$,
number of photons $\uparrow$,
number of electrons $\uparrow$,
current $\uparrow$.
(If light intensity $\uparrow$, photoelectric current $\uparrow$).

Variation of photoelectric current i with voltage *V* for the radiation of **different frequencies** but its **intensity and metal are fixed**.



$i \propto I$

$f_2 > f_1$

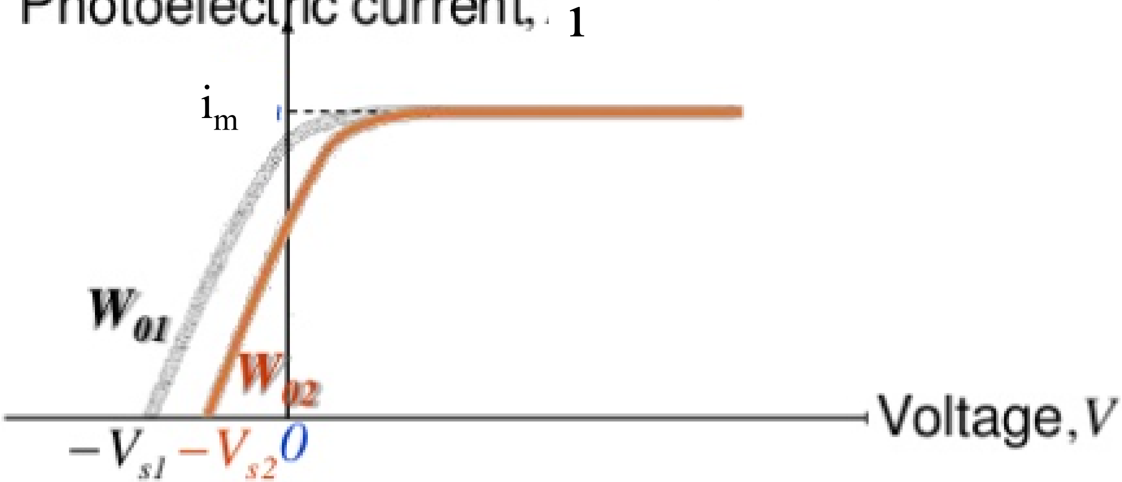
$V_{s2} > V_{s1}$

$$K.E_{\max} = hf - W_o$$
$$eV_s = hf - W_o$$
$$V_s = \frac{h}{e} f - \frac{W_o}{e}$$

$f \uparrow \; V_s \uparrow$

Variation of photoelectric current i with voltage V for the **different metals** but the **intensity** and **frequency** of the radiation are fixed.

$i \propto I$



$$K.E_{\max} = hf - W_o$$
$$eV_s = hf - W_o$$
$$V_s = \frac{h}{e} f - \frac{W_o}{e}$$

$W_o \uparrow, V_s \downarrow$

$W_{02} > W_{01}$

$V_{s1} > V_{s2}$

EXPLAIN the failure of classical theory to justify the photoelectric effect.

1. MAXIMUM KINETIC ENERGY OF PHOTOELECTRON

Clasiccal prediction	Experimental Result	Modern Theory
The higher the intensity, the greater the energy imparted to the metal surface for emission of photoelectrons. •The higher the intensity of light the greater the kinetic energy maximum of photoelectrons.	Very low intensity but high frequency radiation could emit photoelectrons. The maximum kinetic energy of photoelectrons is independent of light intensity.	Based on Einstein's photoelectric equation: $K_{max} = hf - W_0$ The maximum kinetic energy of photoelectron depends only on the light frequency . The maximum kinetic energy of photoelectrons DOES NOT depend on light intensity.

KE of electron not a function of intensity but frequency
a brighter object is more intense but not necessarily more energetic. Remember that a photon's energy depends on the wavelength (or frequency) only, not the intensity. The photons in a dim beam of X-ray light are much more energetic than the photons in an intense beam of infrared light.

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2. EMISSION OF PHOTOELECTRON (energy)

Clasiccal prediction	Experimental Result	Modern Theory
Emission of photoelectrons occur for all frequencies of light. Energy of light is **independent of frequency.**	Emission of photoelectrons occur only when frequency of the light exceeds the certain frequency which value is characteristic of the material being illuminated.	When the light frequency is greater than threshold frequency, a higher rate of photons striking the metal surface results in a higher rate of photoelectrons emitted. If it is less than threshold frequency no photoelectrons are emitted. Hence the **emission of photoelectrons depend** on the **light frequency**.

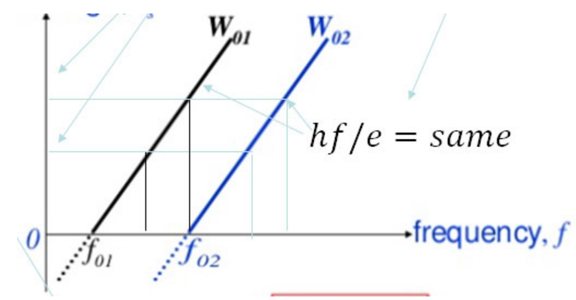
3. EMISSION OF PHOTOELECTRON (time)

Clasiccal prediction	Experimental Result	Modern Theory
Light energy is spread over the wavefront, the amount of energy incident on any one electron is small. An electron must gather sufficient energy before emission, hence **there is time interval** between absorption of light energy and emission. Time interval increases if the light intensity is low.	Photoelectrons are emitted from the surface of the metal almost **instantaneously** after the surface is illuminated, even at very low light intensities.	The transfer of photon's energy to an electron is instantaneous as its energy is absorbed in its entirely, much like a particle to particle collision. The emission of photoelectron is immediate and **no time interval** between absorption of light energy and emission.

4. ENERGY OF LIGHT

Clasiccal prediction	Experimental Result	Modern Theory
Energy of light **depends only on amplitude** (or **intensity**) and not on frequency.	Energy of light depends on frequency	According to Planck's quantum theory which is $E=hf$ Energy of light **depends on its frequency.**

KE



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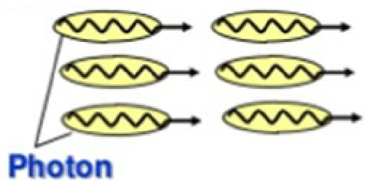
- Experimental observations deviate from classical predictions based on **Maxwell's e.m. theory**. Hence the **classical physics cannot explain** the phenomenon of **photoelectric effect.**
- The **modern theory** based on Einstein's photon theory of light **can explain** the phenomenon of **photoelectric effect.**
- It is because Einstein postulated that **light is quantized** and light is emitted, transmitted and reabsorbed as **photons**.



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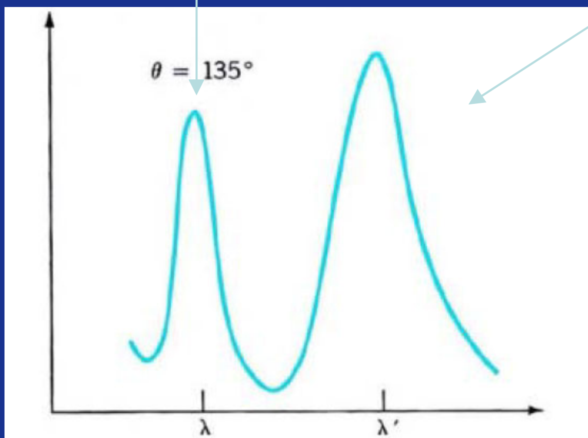
SUMMARY : Comparison between classical physics and quantum physics about photoelectric effect experiment

Feature	Classical physics	Quantum physics
Threshold frequency	An incident light of any frequency can eject electrons (**does not has threshold frequency**), as long as the beam has sufficient intensity.	To eject an electron, the incident light must have a frequency greater than a certain minimum value, (***threshold frequency***) , no matter how intense the light.
Maximum kinetic energy of photoelectrons	Depends on the ***light intensity.***	Depends only on the light ***frequency*** .
Emission of photoelectrons	There should be some ***delays*** to emit electrons from a metal surface.	Electrons are emitted ***spontaneously***.
Energy of light	Depends on the **light intensity.**	Depends only on the light ***frequency*** . 49

Compton Effect

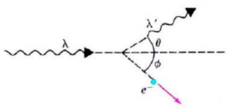
The striking successes in blackbody radiation and the photoelectric effect were still not sufficient to convince some scientists of the validity of the quantum concept. Compton found further evidence for the photon concept while he was studying the scattering of X rays by graphite. He found that the scattered radiation had two components: one at the original wavelength and a second at a longer wavelength.



The derivation of Compotn's scattering: Classically an electromagnetic wave carries moment given by p=E/c

Conservation of linear momentum:

$$p = \frac{hf}{c} = \frac{h}{\lambda}$$



$$\begin{cases} p_x:\ p_\lambda = p_{\lambda'}\cos\theta + p\cos\phi \\ p_y:\ 0 = p_{\lambda'}\sin\theta - p\sin\phi \end{cases}$$

$$\Rightarrow (p_\lambda - p_{\lambda'}\cos\theta)^2 + (p_{\lambda'}\sin\theta)^2 = p^2$$

Conservation of energy:

$$hf = hf' + K;\ K = (\gamma - 1)m_0c^2$$

$$\begin{cases} (cp_\lambda - cp_{\lambda'}) = K \\ K^2 + 2Km_0c^2 = c^2p^2 \end{cases}$$

$$(p_\lambda - p_{\lambda'})^2 + 2(p_\lambda - p_{\lambda'})m_0c = p^2$$

For known X ray frequency and final particle momentums

We can further solve these two equations.

$$(p_\lambda - p_{\lambda'}\cos\theta)^2 + (p_{\lambda'}\sin\theta)^2 = p^2$$

$$(p_\lambda - p_{\lambda'})^2 + 2(p_\lambda - p_{\lambda'})m_0c = p^2$$

Further solving these two equations, we obtain

$$(p_\lambda - p_{\lambda'})m_0c = p_\lambda p_{\lambda'}(1 - \cos\theta)$$

$$\frac{1}{p_{\lambda'}} - \frac{1}{p_\lambda} = \frac{1}{m_0c}(1 - \cos\theta)$$

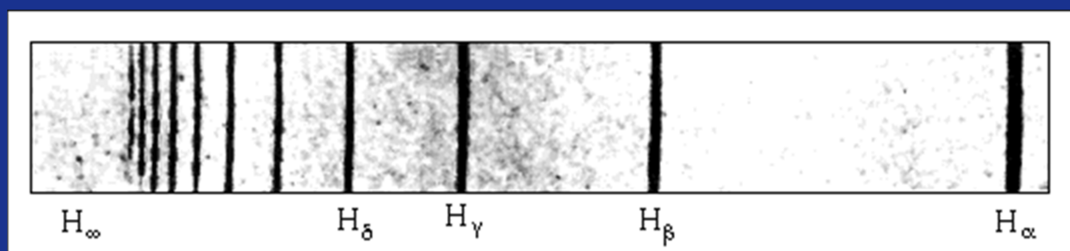
$$\Rightarrow \Delta\lambda = \frac{h}{m_0c}(1 - \cos\theta)$$

$\frac{h}{m_0c} = 0.00243$ nm is called Compton wavelength.

Hydrogen Spectrum

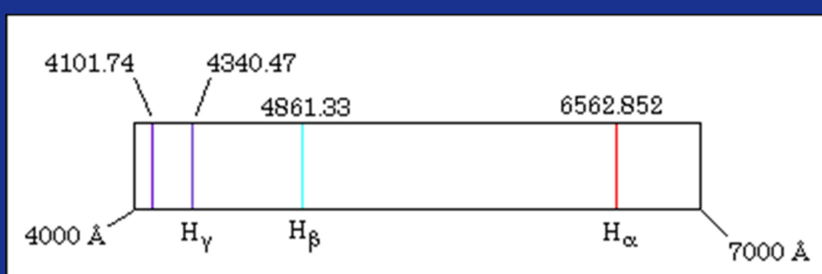
In 1862, Anders Ångström discovered three lines and later on found the 4th line (the 4101.74 violet line). By 1871, he measured all four wavelengths to a high degree of accuracy.

This next diagram is a photograph of a more complete hydrogen spectrum:



H_α, H_β, H_γ, and H_δ are the official designations for the 4 lines of the visible portion of the spectrum. All the other lines to the left of these four are in the ultraviolet portion of the spectrum and so would not be visible to the eye. However, they can be photographed, as they were here.

Here is a drawing of the visible spectrum of hydrogen:

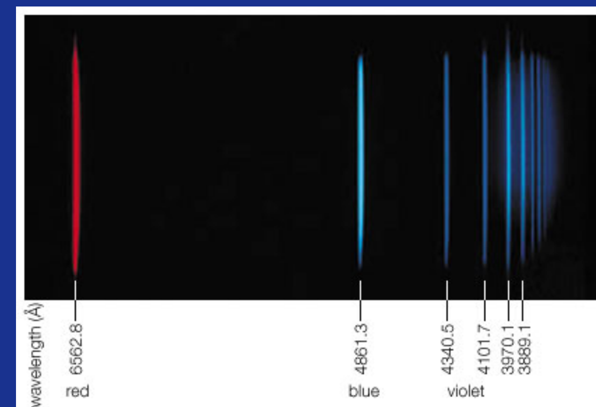


The left edge ends at a wavelength of 4000 Ångströms and the right edge ends at 7000 Ångströms.

Balmer's Principle

The **visible spectrum of hydrogen consists of four lines**. In 1884, Balmer, a Swiss mathematics teacher, found that these wavelengths (in nm) could be represented by a single formula

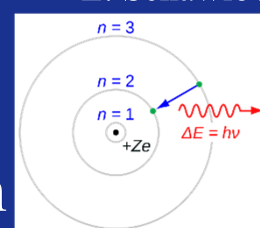
$$\lambda_n = 364.56\frac{n^2}{n^2-4}$$



The Balmer series of hydrogen as seen by a low-resolution spectrometer. n2 to n1
L. Schawlow, Stanford University

An equation that relates wavelength to a photon's electronic transitions

$$1/\lambda \quad = R\left(\frac{1}{n_{final}^2}-\frac{1}{n_{initial}^2}\right)$$

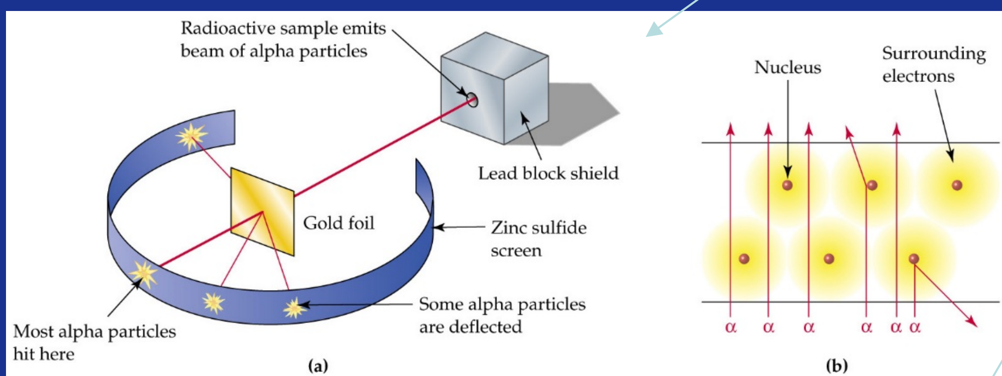


In the simplified Rutherford Bohr model of the hydrogen atom, the Balmer lines result from an electron jump between the second energy level closest to the nucleus, and those levels more distant. Shown here is a photon emission. The 3→2 transition depicted here produces H-alpha, the first line of the Balmer series. For hydrogen (Z = 1) this transition results in a photon of wavelength 656 nm (red).

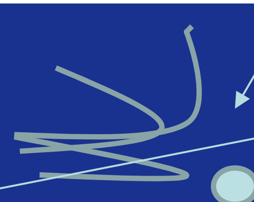
n = infinity to n =2 Balmer emission
$R = 1.0974\text{x } 10^7 \text{ m}^{-1}$

Atomic Model

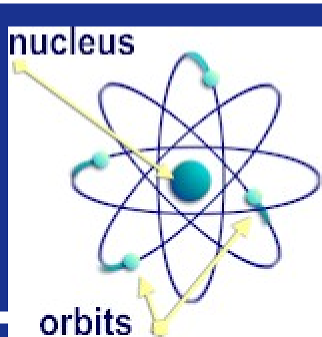
In the 19th century, there was considerable chemical and physical evidence for the existence of atoms, but nothing was known about their structure. In 1909, Rutherford studied the scattering of alpha particles by a very thin gold foil as shown below.



Ruthrtford found that about one in eight thousand of the α particles was scattered through angles larger than 90°. The “head-on’ collision allows us to obtain an estimate the size of the nucleus. Ruthrtford’s work on α particle scattering established the existence of the nucleus.



nucleus



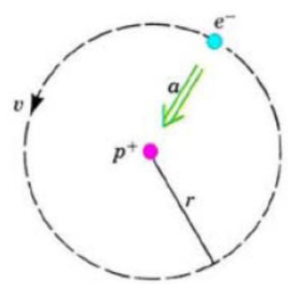
Bohr's Model

In 1913, Bohr presented a model of the hydrogen atom, which has one electron. Bohr stated two postulates:

1. The electron moves only in certain circular orbits, called stationary states. This motion can be described classically.

Force $$\frac{mv^2}{r} = \frac{ke^2}{r^2}$$

Energy $$E = K + U = \frac{1}{2}mv^2 - \frac{ke^2}{r} = -\frac{ke^2}{2r}$$



2. Radiation occurs only when an electron goes from one allowed orbit to another of lower energy. The radiated frequency is $hf = E_m - E_n$, where E_m and E_n are energies of the two states.

Bohr's Model

What is the stationary states? We need a quantum condition that restricts the allowed values of the orbital radius. It was realized only later that this is a fundamental aspect of quantum theory, and so it serves as our "third" postulate:

3. The angular momentum of the electron is restricted to integer multiples of h/2π.

Angular momentum $mvr = n\hbar \Rightarrow v = \dfrac{n\hbar}{mr}$

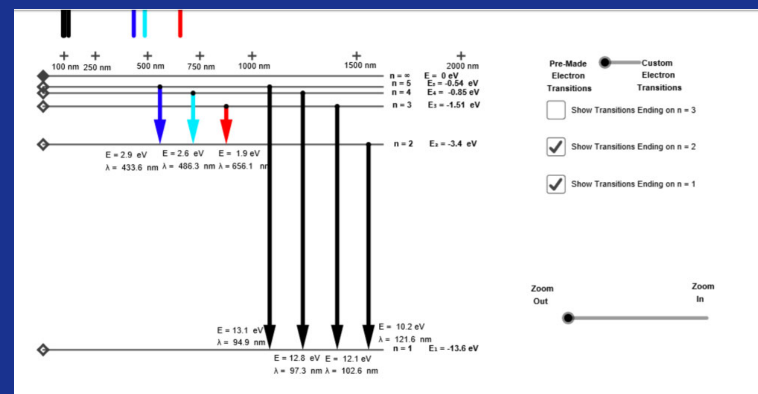
Energy $\dfrac{1}{2}mv^2 = \dfrac{ke^2}{2r} \Rightarrow r_n = \dfrac{n^2\hbar^2}{mke}$

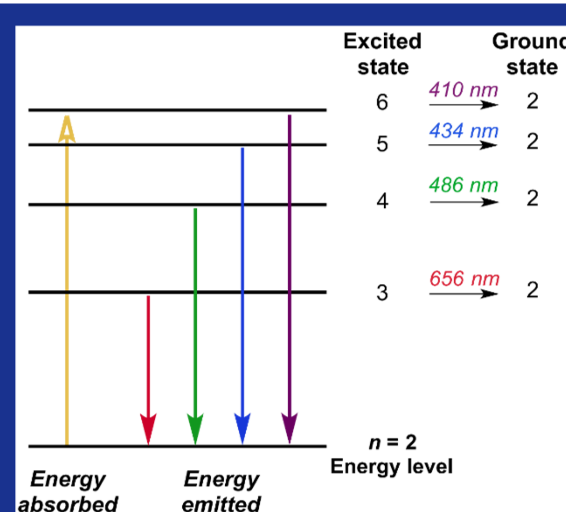
The total energy of the nth orbit is

$$E_n = -\frac{mk^2e^4}{2\hbar^2}\left(\frac{1}{n^2}\right) = -\frac{13.6}{n^2}\ \text{eV}$$

Bohr's model Predict Energy Level Diagram for Hydrogen Correctly

Bohr's theory correctly predicted the frequencies of the H_2, it provided no information regarding the relative intensities of the lines or the spectra of multi-electron atoms. <u>Later it was show n that Bohr's theory has been replaced by quantum mechanics.</u> The second and third postulates remain valid, <u>***but the picture of an electron in well-defined orbit is not correct (heidenbger theory).***</u>

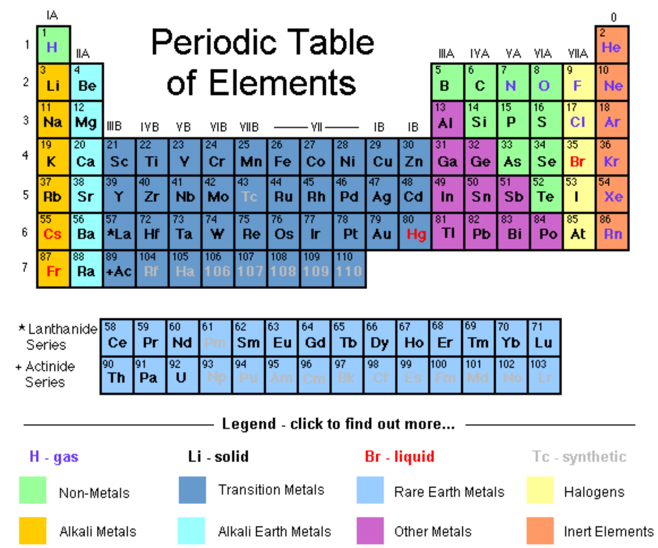


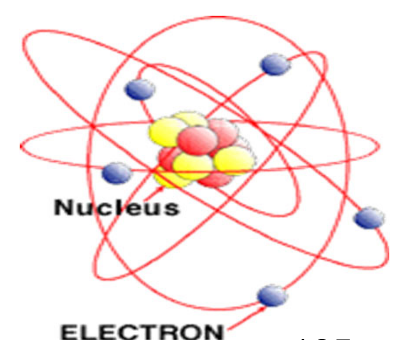


Bohrs' mode of hydrogen

Atoms and Electrons

- There are many different kinds of atoms, one for each type of element, There are 118 different known elements that make up every thing!
- Each atom has a **specific number of electrons, protons and neutrons**. But no matter how many particles an atom has, the number of electrons usually needs to be the same as the number of protons. If the numbers are the same, the atom is called balanced, and it is very stable.
- Sometimes **atoms have loosely attached electrons.** An atom that loses electrons has more protons than electrons and is positively charged. An atom that gains electrons has more negative particles and is negatively charge. **A "charged" atom is called an "ion."**





Wave-Particle Duality of Light

- Wave nature: Young's double-slit interference and single slit diffraction.
- Particle nature: Photoelectric effect (shows phonons and electrons are particle function of wavelength) and Compton scattering (show direction of impingement affects the wavelength emission).

Light exhibits a wave-particle duality. Depending on the experiment performed, it will behave either as a particle, or as a wave. Generally speaking, at lower frequency it behaves like a wave, while at higher frequency it acts like a particles.

Wave Mechanics

Partly success: **Bohr's theory was successful in explaining the spectrum of hydrogen. However, it could not predict the relative intensities of spectral lines or explain why, with increased resolution, some lines were found to consist of two or more finer lines.**

More findings: **Sommerfeld** refined Bohr's theory by incorporating **special relativity and the possibility of elliptical orbits**. With the addition of two new quantum numbers, **the Bohr-Sommerfeld theory accounted for many features of spectra and showed how the periodic table is built up in a systematic way.**

Proper foundation: The rules that were used had no proper foundation and had limited explanatory power. Radical reform was needed in quantum theory.

De Broglie Waves

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De Broglie put forward an astounding proposition that "nature is symmetrical". Einstein had shown that a complete description of cavity radiation requires both the particle and wave aspects of cavity radiation.

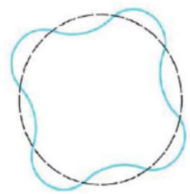
De Broglie guessed that ***a similar wave-particle duality might apply to material particles***. That is, **matter may also display wave nature.** He used a combination of quantum theory and special relativity to propose that ***the wavelength, λ, associated with a particle is related to its linear momentum, $p=mv$, by***

$$\lambda = \frac{h}{p}$$

De Broglie Waves

The physical significance of the “matter wave” was not clear, but he gained courage from the following demonstration. In the Bohr’s model, **the angular momentum of the electron is quantized**:

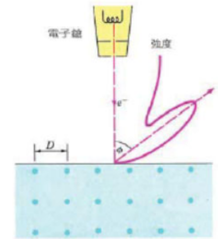
$$mvr = \frac{nh}{2\pi} \Rightarrow 2\pi r = n\lambda$$



This looks like the condition for a standing wave!

Stationary orbits: Only those orbits that can fit an integral number of wavelengths around the circumference are allowed.

Davisson studied the scattering of electrons off nickel surface, reported the curious result that the reflected intensity depends on the orientation of the sample. Why? Matter wave.



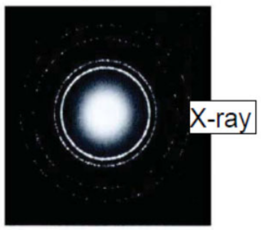
Electrons were produced by a heated filament, accelerated by a potential difference, V, and then directed at the Ni target.

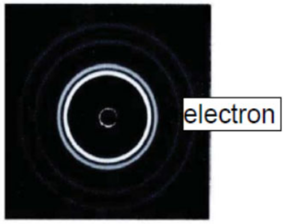
X-ray diffraction like: If the electrons had interacted with the atoms on a one-to-one basis, this would have produced random scattering. The pronounced reflections implied that the electrons were interacting with an array of atoms.

When a particle of mass m and charge q is accelerated from rest by a potential difference V, its kinetic energy is given by $K=p^2/2m=qV$. The de Broglie wavelength takes the form

$$p = \sqrt{2mgV}$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mgV}}$$



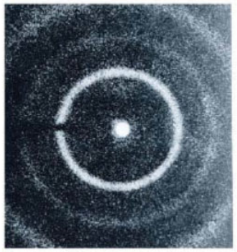


By an analysis similar to that for X-ray, it is found that the angular positions of the diffraction maxima are given by

$$D\sin\phi = n\lambda$$

Where D is the spacing between atoms, which in the case of nickel is 0.215 nm.

Wavelike behavior is exhibited by all elementary particles.



A diffraction pattern produced by 0.07-eV neutrons passing through a polycrystal-line sample of iron.

Example

What is the de Broglie wavelength of (a) an electron accelerated from rest by a potential difference of 54 V, and (b) a 10 g bullet moving at 400 m/s?

Solution:

$$\text{(a) } \lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}} = \frac{6.626\times10^{-34}}{\sqrt{(2\times9.11\times10^{-31}\times1.6\times10^{-19}\times54}} = 0.167\ \text{nm}$$

$$\text{(b) } \lambda = \frac{h}{p} = \frac{6.626\times10^{-34}}{0.01\times400} = 1.66\times10^{-34}\ \text{m}$$

There is no chance of observing wave phenomena, such as diffraction, with macroscopic objects.

Schrodinger's Equation (Wave function)

When Schrodinger realized that Einstein took the matter wave seriously, he decided to look for an equation to describe these matter waves.

The derivation of Schrodinger's equation is not straight forward. A simplified discussion can be based on the wave equation.

wave equation $$\frac{\partial^2 y}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2} = 0$$

general solution $$y(x,t) = \psi(x)\sin\omega t$$

time - independent $$\frac{\partial^2 \psi}{\partial x^2} + \frac{\omega^2}{v^2}\psi = 0$$

matter wave $$\frac{\omega^2}{v^2} = \frac{p^2}{\hbar^2} = \frac{2m(E-U)}{\hbar^2}$$

Schrodinger's Eq. $$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m(E-U)}{\hbar^2}\psi = 0$$

Schrodinger's Equation

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m(E-U)}{\hbar^2}\psi = 0$$

This is the one-dimension **time-independent Schrodinger wave equation**. The wave function $\psi(x)$ represents stationary states of an atomic system for which E is constant in time.

How can a *continuous* description lead to *discrete* quantities, such as the energy level of the hydrogen atom?

The boundary condition.

Wave Function

Schroding's success in tackling several problems confirmed that the wave mechanics was an important advance**. But how was the "wave associated with the particle" to be interpreted?**

De Broglie suggested that the **wave might represent the particle itself**.

Schrodinger believed that a particle **is really a group of waves, a wave packet.**

Einstein though the intensity of **a light wave at a given point is a measure of the number of photons that arrive at the point. In other word, the wave function for the electromagnetic field determines the probability of finding a photon.**

By analogy, Born suggested that the square of the wave function tells us the probability per unit volume of finding the particle. Born's interpretation of the wave function has now been generally accepted.

Wave Function

The square of the wave function tells us the probability per unit volume of finding the particle.

$$\psi^2 dV = \text{probability of finding a particle with a volume dV}$$

The quantity $\psi 2$ is called the probability density.

Normalization: Since the particle has to be found somewhere, the sum of all the probabilities along the x axis has to be one:

$$\int_{-\infty}^{\infty} \psi^2(x)dx = 1$$

A wave function that satisfies this condition is said to be normalized.

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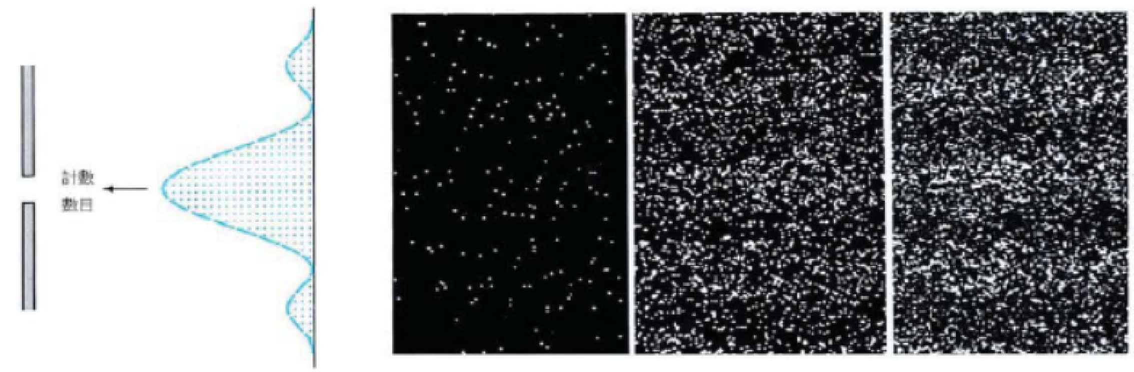
Wave Function

The classical physics and special relativity are based on the principle of determinism.

Quantum mechanics correctly predicts average value of physical quantities, not the result of individual measurements.



diffraction interference

Application of Wave Mechanics: Particle is a box

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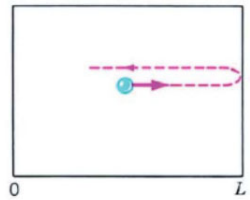
Consider a particle of mass m that bounces back and forth in a one-dimensional box of side L. The potential U is zero within the box and infinite at the wall.



wave equation $\quad \dfrac{\partial^2\psi}{\partial x^2} + k^2\psi = 0, \;$ where $k = \sqrt{2mE}/\hbar$

solutions $\quad \psi(x) = A\sin(kx + \phi)$

boundary condition $\quad \psi(x) = A\sin(\dfrac{n\pi x}{L}), \; n = 1, \; 2, \; 3, \ldots$

The wave length has to satisfy standing wave condition

$$k = 2\pi/\lambda = n\pi/L, \quad \lambda = 2L/n$$

From de Broglie's equation, we can derive the momentum and find out the velocity.

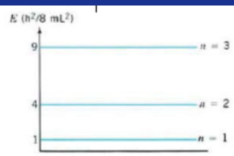
$$p = mv = \frac{h}{\lambda} = \frac{nh}{2L} \Rightarrow v = \frac{nh}{2mL}$$

The particle's energy, which is purely kinetic, K=1/2mv^2, is thus also quantized.

$$E_n = \frac{n^2h^2}{8mL^2} \qquad n = 1, 2, 3, ..$$

The quantized energy level:

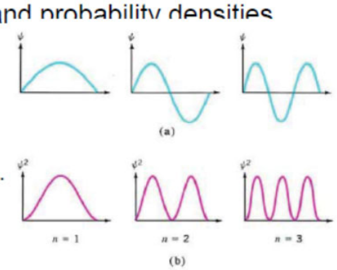
$$E_n = \frac{n^2h^2}{8mL^2} \qquad n = 1, 2, 3, ..$$



The first three wave functions and probability densities

$$\psi(x) = A\sin(\frac{n\pi x}{L}), \; n = 1, \; 2, \; 3, \ldots$$

$$\psi^2(x) = A^2\sin^2(\frac{n\pi x}{L}), \; n = 1, \; 2, \; 3, \ldots$$



An electron is trapped within an infinite potential well of length 0.1 nm. What are the first three energy levels?

Solution:

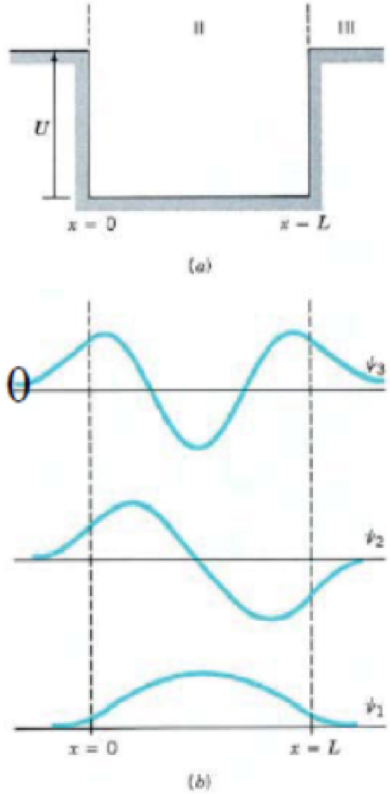
$$E_n = \frac{n^2h^2}{8mL^2} \qquad n = 1, 2, 3, .. \qquad L = 0.1\,\text{nm} = 1\times10^{-10}\,\text{m}$$

$$E_n = \frac{n^2(6.626\times10^{-34})^2}{8(9.1\times10^{-31})\times10^{-20}} = n^2(6.03\times10^{-18})\text{J} = 37.7n^2\ \text{eV}$$

$$E_1 = 37.7\,\text{eV}, \; E_2 = 151\,\text{eV}, \; E_3 = 339\,\text{eV}$$

Application of Wave Mechanics: Finite potential well

Consider a particle of mass *m* that bounces back and forth in a one dimensional box of side *L*. The potential *U* is zero within the box and infinite at the wall.



$$\text{wave equation } \frac{\partial^2\psi}{\partial x^2} + k^2\psi = 0, \text{ or } \frac{\partial^2\psi}{\partial x^2} + K^2\psi = 0$$

$$\text{where } k = \sqrt{2mE}/\hbar \text{ or } K = \sqrt{2m(U-E)}/\hbar$$

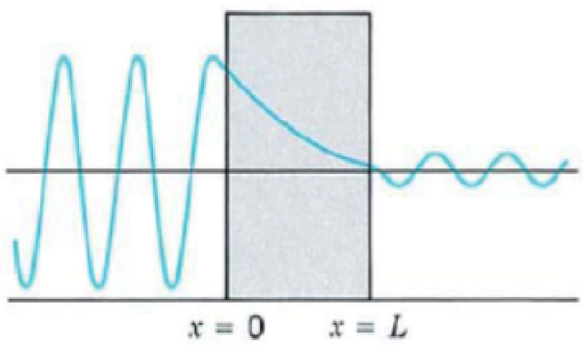
$$\psi(x)\begin{cases} = A\exp(Kx) & x \le 0 \\ = B\exp(ikx) + C\exp(-ikx) & 0 \le x \le L \\ = D\exp(-Kx) & L \le x \end{cases}$$

What is the boundary conditions?

Application of Wave Mechanics: Barrier Penetration: Tunneling

When a particle with energy *E* encounters a potential energy barrier of height *U* (>*E*), what would happens?

Partly reflected and partly transmitted (tunneling).

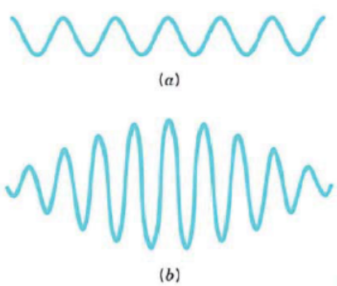


Examples: Tunnel diode, Josepheson junction, and scanning tunneling electron microscope.

Can we reproduce such effect in classical electromagnetism?

Heisenberg Uncertainty Principle

The wavelength of a wave can be specified precisely only if the wave extends over many cycles. But if a matter wave is spread out in space, the position of the particle is poorly defined. Thus to reduce the uncertainty in position of the particle, Δx, one can propose many wave length to form a reasonably well-localized ***wave-packet***.



From the de Broglie matter wave relation, we see that a spread in wavelengths, $\Delta\lambda$, means that the wave packet involves a spread in momentum, Δp. According to the Heisenberg uncertainty principle, the uncertainties in position and in momentum are related by

$$\Delta x \Delta p \geq h$$

It is not possible to measure both the position of a particle and its linear momentum simultaneously to arbitrary precision.

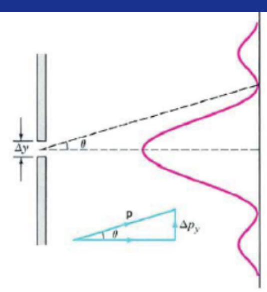
For a wave packet, the uncertainty relation is an intrinsic property, independent of the measuring apparatus.

Consider the electron diffraction by a single slit.

We know that the position of the first minimum is given by

$$\begin{cases} \sin\theta = \dfrac{\lambda}{a} = \dfrac{\lambda}{\Delta y} \Rightarrow \Delta y = \dfrac{\lambda}{\sin\theta} \\ \Delta p_y = p\sin\theta \end{cases}$$

$$\Rightarrow \Delta y \Delta p_y \approx h$$



A finer slit would locate the particle more precisely but lead to a wider diffraction pattern---that is, to a greater uncertainty in the transverse momentum.

The Heisenberg uncertainty principle also applies to other pairs of variables.

$$\Delta x \Delta p_x \geq h, \qquad \Delta y \Delta p_y \geq h, \qquad \Delta z \Delta p_z \geq h$$

Among the most important are energy and time:

$$\Delta E \Delta t \geq h$$

To minimize the uncertainty in measuring the energy of a system, one must observe it for as long as possible.

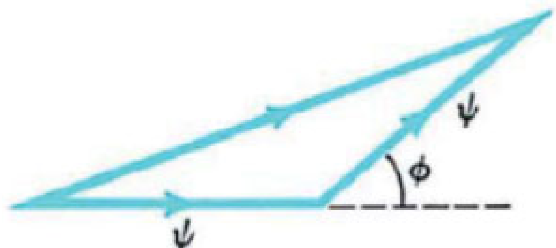
The energy of a system can fluctuate from the value set by the conservation of energy---provided the fluctuation occurs within the time interval specified by above Eq.

Wave-Particle Duality

Let's reconsider Young's double-slit experiment, but now conducted with electrons.

Let us say that $\psi 1$ is the wave function that applies to the passage of an electron through slit S1 whereas $\psi 2$ applies to slit S2. When both slits are open, the distribution will display the familiar interference fringes.

$$\psi^2 = \left|\boldsymbol{\Psi}_1 + \boldsymbol{\Psi}_2\right|^2 = \psi_1^2 + \psi_2^2 + 2\psi_1\psi_2 \cos\phi$$



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Atoms and Solids

The theory of quantum mechanics has been applied with great success to a wide variety of phenomena. To specify the state of the electron, we need the principal quantum number, n, the orbital quantum number, l, and the orbital magnetic quantum number, m_l. In addition, particles have an intrinsic spin angular momentum that is specified by a spin magnetic quantum number, m_s.

Pauli Exclusion Principle: No two electrons in an atom can have the same four quantum numbers.

Quantum numbers for the hydrogen atom

$$E_n = -\frac{mk^2e^4}{2\hbar^2 n^2} = -\frac{13.6}{n^2} \text{ eV}$$ principal quantum number, n

$$L = \sqrt{\ell(\ell+1)}\hbar$$
$$\ell = 0, 1, 2, \ldots, (n-1)$$ orbital quantum number, l

$$L_z = m_\ell \hbar$$
$$m_\ell = 0, \pm 1, \pm 2, \ldots, \pm \ell$$ orbital magnetic quantum number, m_l

W. Wang

W. Wang

W. Wang

Summarize What is Light

- Behave like mechanical particle (reflection, refraction)
- Behave like wave (interfere, diffract, partly electric, partly magnetic)
- Energy transfer between light and matter (behave like particle with wavelike behavior)

Light as particle

- A photon is like a particle, but it has small mass
- Think of a photon as a grain of sand.
- We see so many photons at the same time it's like seeing all the sand on a beach; we don't notice the single grains
- When it hit a reflective surface, it reflects like particle and when it travel through different optical medium, it refract to different angle.

Light as a wave

- But sometimes light acts like a wave
- A wave has a wavelength, a speed and a frequency.
- We'll learn more about wave behavior when we talk about polarization, interference and diffraction
- All light travels same speed (in vacuum, index independent of wavelength)
- Wavelength gets shorter as frequency goes up ($C=f\lambda$)

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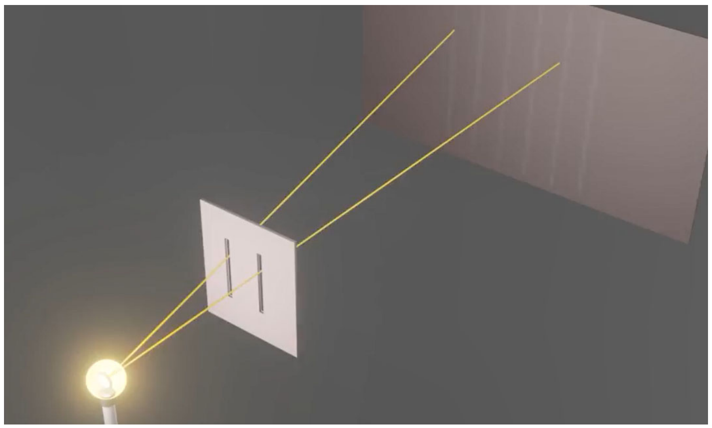
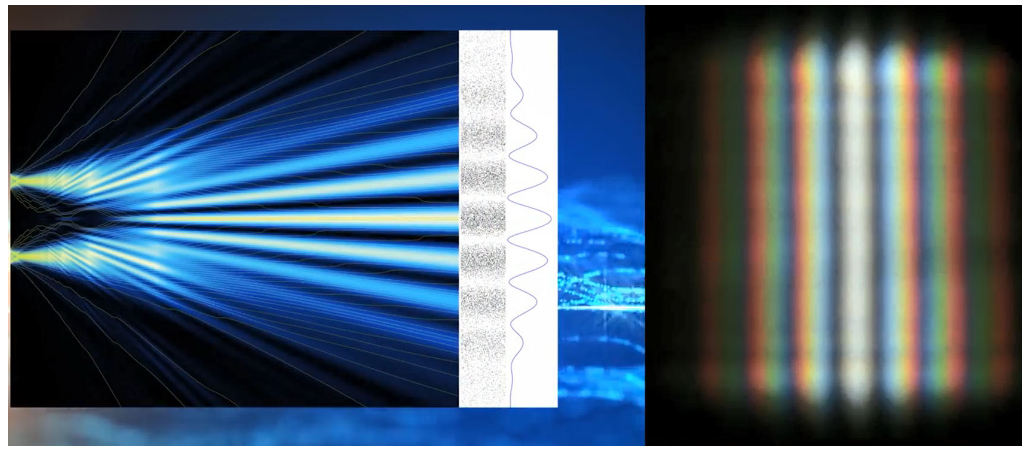
Light as a photon

- Particle with zero mass
- Particles which have wavelike properties
- Consisting of a quantum of electromagnetic radiation where energy is concentrated
- The energy goes up as frequency goes up (E = h x f = h x C_o/ λ)
- Color depends on frequency in nonvacuum medium (E = h x f = h x C_o/ λ)

Double Slits Mystery

Behave like Wave

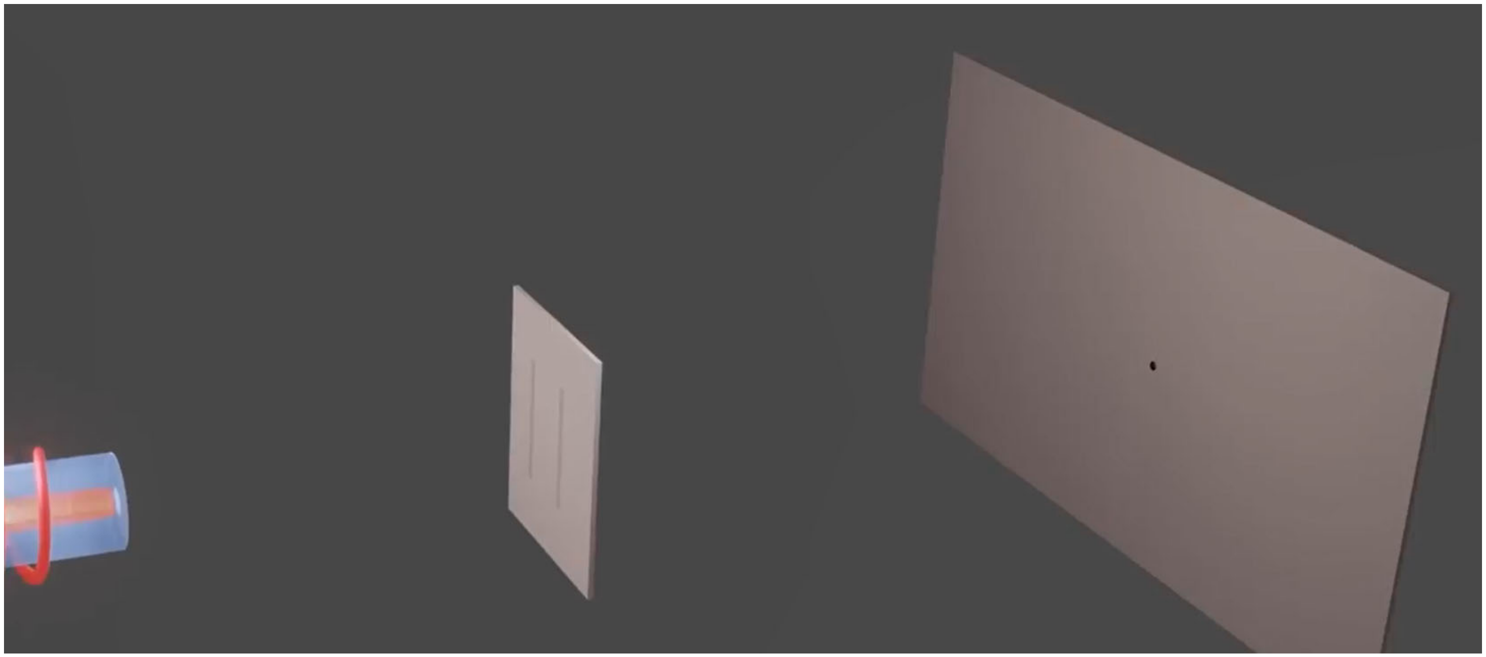




Wave say light interfere going through double slits

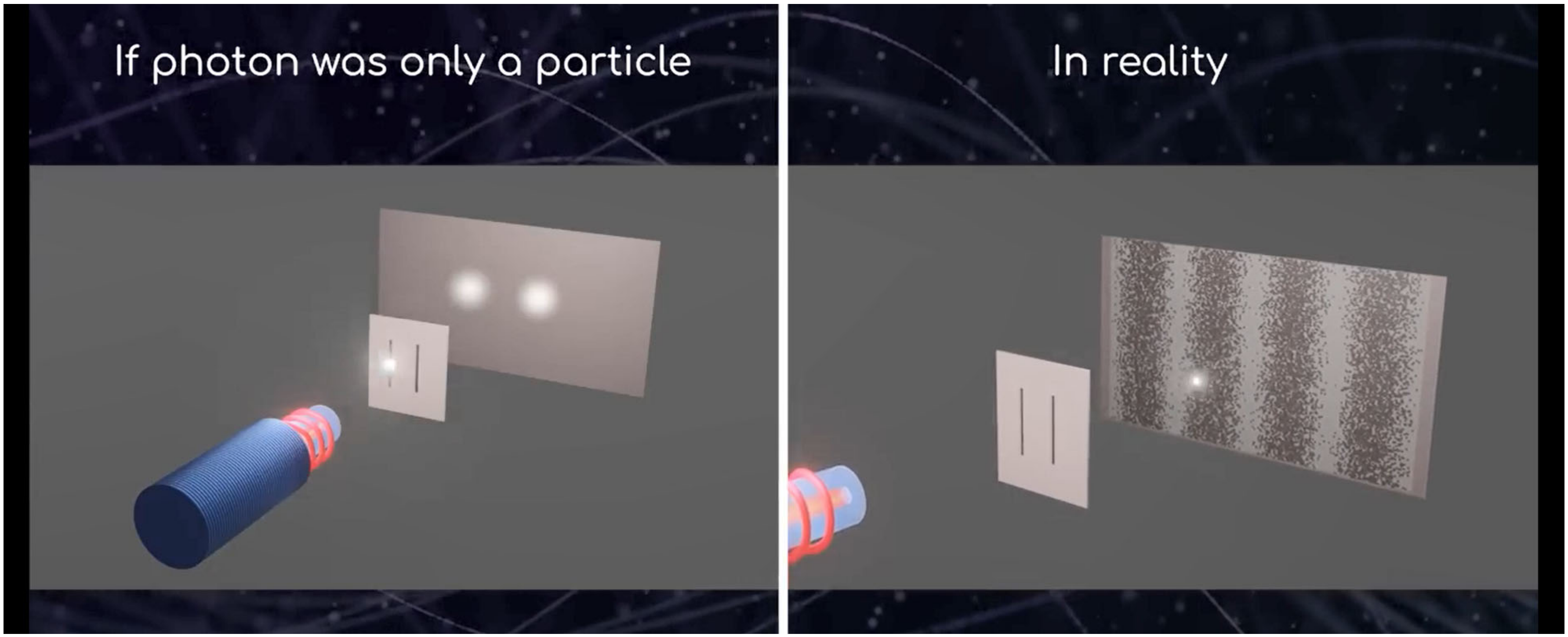
Double Slits Mystery

Behave like particle



Single photon experiment shows photon do collect through gap!!!

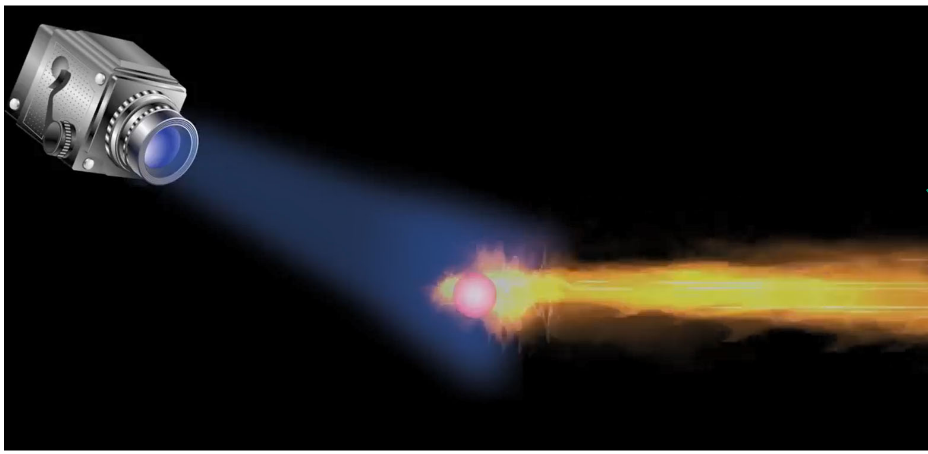
Double Slits Mystery

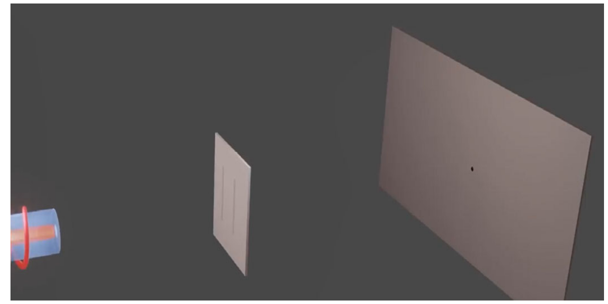


However, if photons are continuously firing, we see particle do behave like wave meaning interference starts to show up.

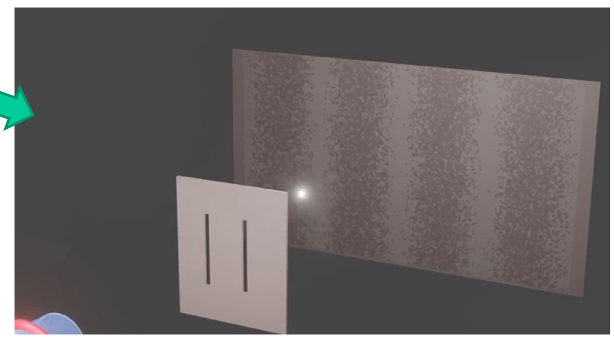
Double Slits Mystery

When we observe it then





Single electron firing

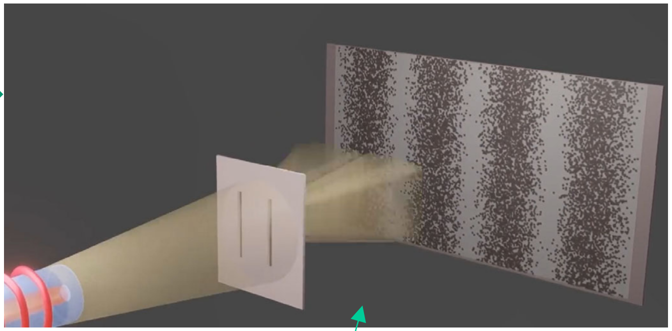


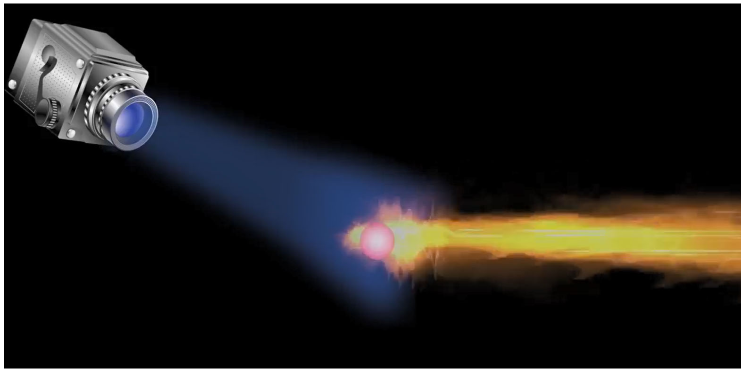
Continuous electrons firing

The moment light interact with someway by any particles, which is the only way we can detect light

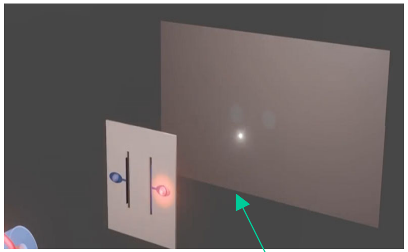
Double Slits Mystery

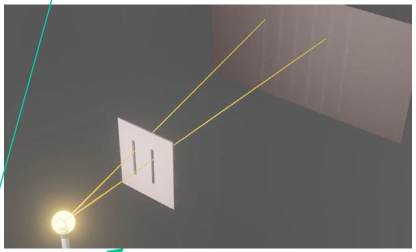
In reality when we are not observing





The moment light interact with someway by any particles, which is the only way we can detect light, no other way to observe it, it starts to behave differently.





In reality, More like wave of probability rather than discrete of particle or wave (Every time it asked, it gives definitive answer it was detected at specific location).

w.wang

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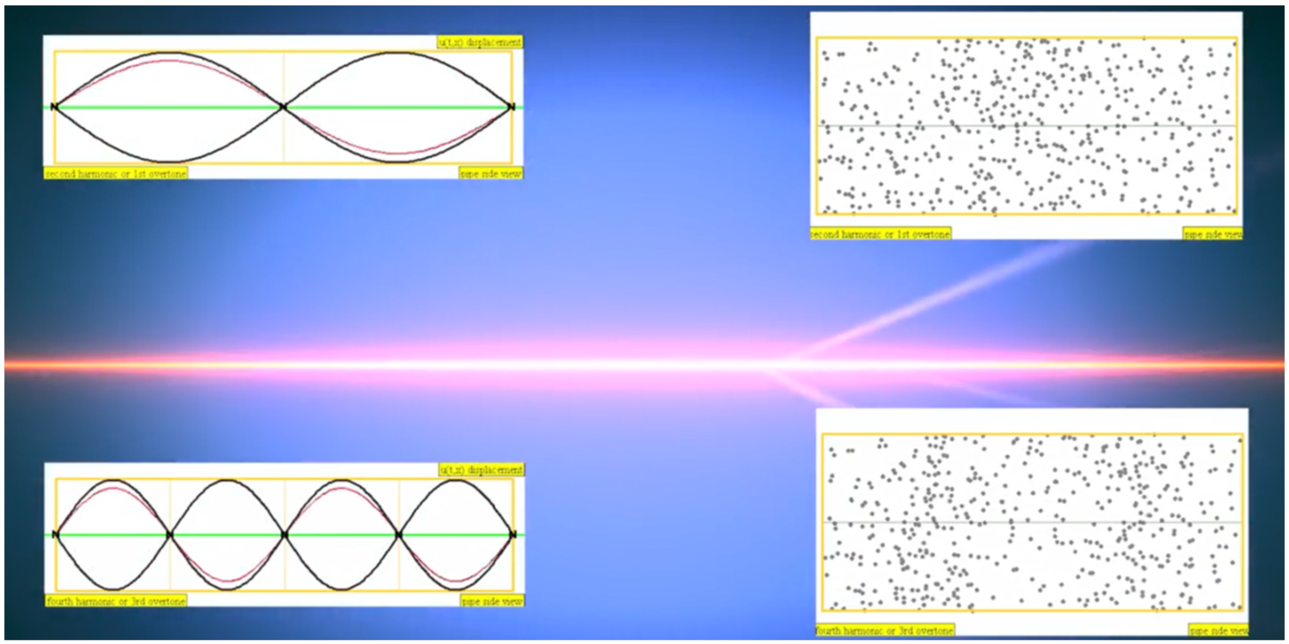
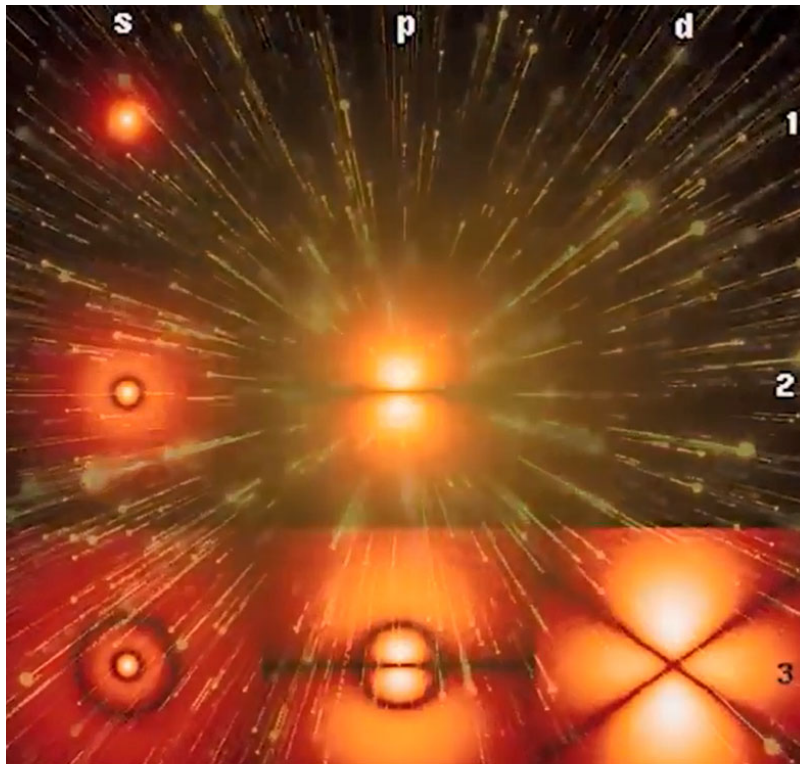
Harmonic theory



Harmonics occurs on a bounded string, what mechanism drive it is unknown
This applies to all form of energy and particles (e.g. electrons)

Particles have wavelengths



Atoms and molecules have been Shown to have wavelengths

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Quote of the week

"Our <u>relationship with China</u> will be competitive when it should be, collaborative when it can be and adversarial when it must be. The <u>common denominator</u> is the need to engage China <u>from a position of **strength**</u>,"

Antony Blinken, US Secretary of State

Week 2

- New Course Website: http://courses.washington.edu/me557/optics
- Reading Materials:

  - Week 1 and 2 reading materials can be downloaded from :

  http://courses.washington.edu/me557/readings/
- Lab sign up (arrangement between you and TAs, general building II 802, contact Gary or Daniel for more information)

  - Lab can be done in a small group (a group of 3 or 4 people), but lab report must be turn in individually.
- Assignments are due two weeks after they are assigned.
- Website updated weekly, make sure you upload the latest lecture notes and homework
- Week 17 Mon (6/10, Final Presentation) schedule to change
- Design Project 1 assigned and due Week 7 (4/1)
- No class on 3/25 (conference)

Week 3

- Course Website: http://courses.washington.edu/me557/optics
- Reading Materials:
  - Week 3 and 4 reading materials can be found:
    http://courses.washington.edu/me557/readings/
- **Design project 1 is due 4/1. Prepare a 5 minutes PowerPoint presentation for the third hour and also send me an electronic copy of the PowerPoint slides and memo for the project (send me the PPT and memos electronically to abong@uw.edu).**
- First part of the Lab 1 starts this week (after the lecture)
  - Do problem 1 in assignment before doing lab 1.
- **Monday sometimes might be three hours if we need to finish up the lecture that week, but most of the time is 2 hours.**
- **Special Career presentation 3/25 (the week I will be away for conference)**
- **HW1 assigned due on 4/1**
- Useful website:
  http://refractiveindex.info/?shelf=organic&book=polycarbonate&page=Sultanova

This week

- Ray optics
  - law of reflection (introduction and derivation)
  - mirrors (derivation of different reflective surface and structure)
  - paraxial approximation, mirror equation

Recap

Three Models

- Geometric Optics: Explain light and matter interaction using mechanical model and assume light as a small non interfering particles.
- EM wave: Explain light and matter interaction using wave theory and assume light as a wave and matter consists of array of electric and magnetic dipoles.
- Quantum: Explain light matter interaction in atomic level and assume light as a particle with wavelike behavior.

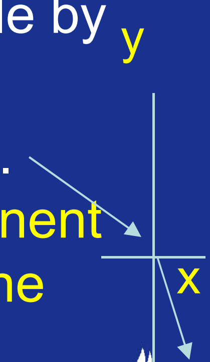
Corpuscular Model

The corpuscular model is the simplest model of light. According to the theory, a **luminous body emits stream of particles in all direction**. Issac Newton, in this book Opticks also wrote,” Are not the ray of light every small bodies emitted from shinning substance?”

Based on this, light is assumed to be **consisted of very small particles so that when two light beams overlap**, a **collision between the two particles rarely occurs**. Using this model, one can explain the laws of reflection and refraction (Snell’s law):

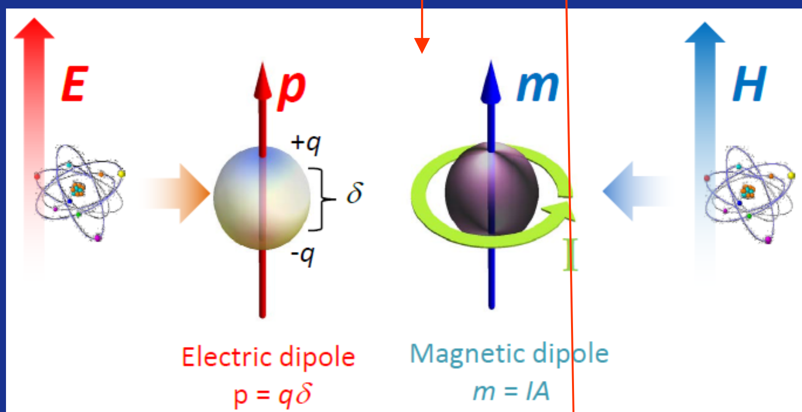
-Reflection law follows considering the elastic reflection of a particle by plane surface.

- Refraction law assume that the motion is confined to the xy plane. The trajectory of particle is determined by conservation of x component momentum (component parallel to the interface, perpendicular to the direction of propagation)

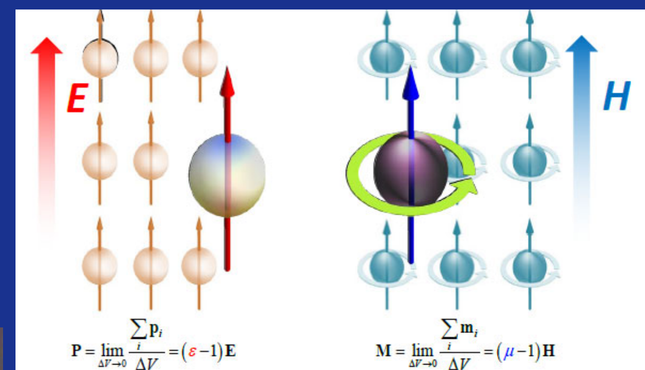


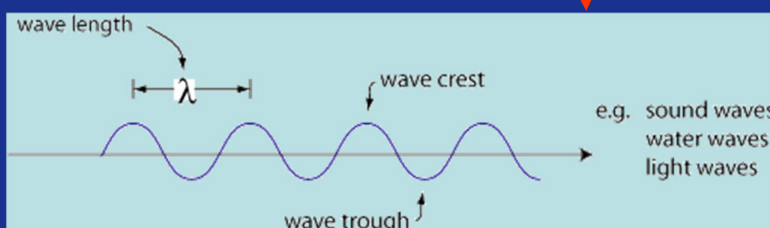
Electromagnetic Wave

From the electromagnetic point-of-view, an atom is just an electric or magnetic, *polarizable* dipole. Light is just an electromagnetic wave



A material is a collection of electric and magnetic dipoles. Homogenization allows this collection to be *continuous*.





<u>Abstract Mathmatical model that helps explain when light behave like wave or how it propagate in space or how it interact with each other</u>

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Quantum Theory of Light

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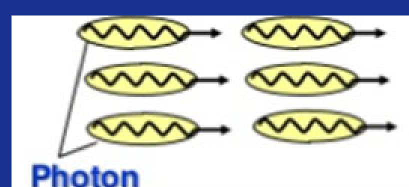
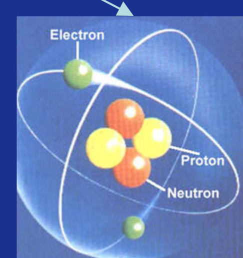
Wave-Particle Duality of Light

Quantum theory tells us that both **light and matter consists of tiny *particles* *which have* *wavelike properties*** associated with them. **Light** is composed of particles called photons, and **matter** is composed of particles called electrons, protons, neutrons. It's only when the mass of a particle gets small enough that its wavelike properties show up.

Generally speaking, at lower frequency it behaves like a wave, while at higher frequency it acts like a particles.





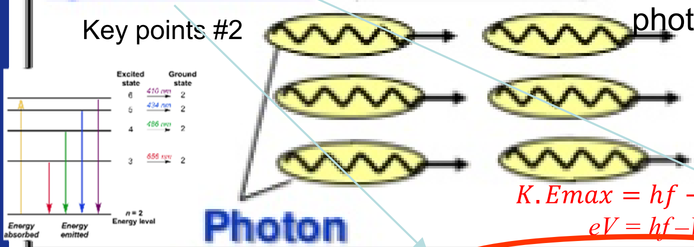
For example, at large quantum number the physics becomes to classical.

In summary

Planck's Quantum Theory	Classical theory EM Theory
Energy of the e.m radiation is **quantised**. (**discrete**)	Energy of the e.m radiation is **continously**.
Energy of e.m radiation ***depends*** on its **frequency or wavelength**	Energy of e.m radiation ***does not*** depend on its **frequency or wavelength (depends on Intensity)** $I \propto A^2$
$\boxed{E = hf}$	$\boxed{E_{classical} = k_B T}$ $k_B = \text{Boltzman's constant}$ $T = \text{temperature}$

Energy level of matter and photon

Key points #2

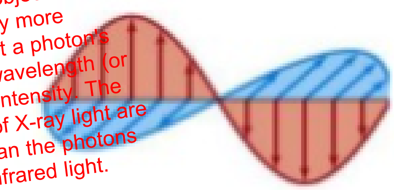


$K.Emax = hf - Wo$

$eV = hf - Wo$

# of e x V_s = (# of hf-Wo)

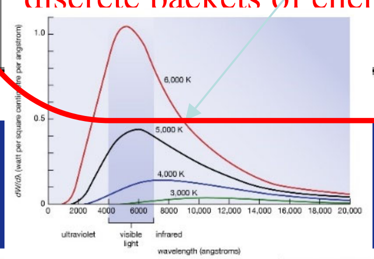
KE of electron not a function of intensity but frequency. a brighter object is more intense but not necessarily more energetic. Remember that a photon's energy depends on the wavelength (or frequency) only, not the intensity. The photons in a dim beam of X-ray light are much more energetic than the photons in an intense beam of infrared light.

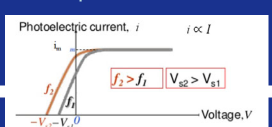


Key points #1

The EM radiation emitted by the black body is discrete packets of energy known as quanta..

Applies to all mechanical and classical equations

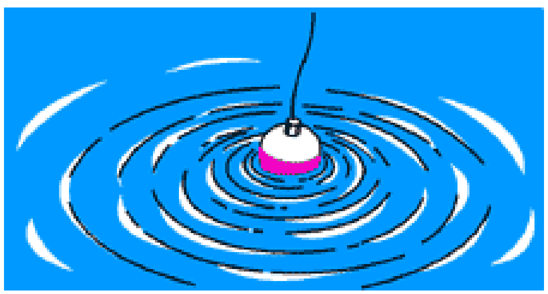




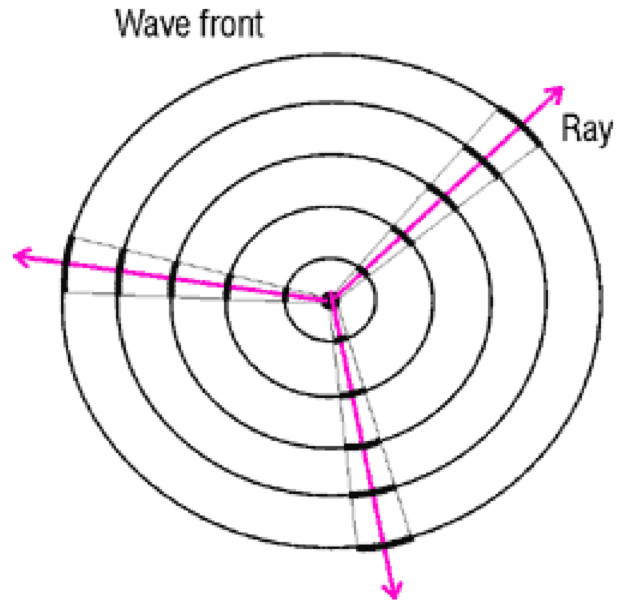
Geometric Optics

Light rays and light waves

We can classify optical phenomena into one of three categories: **ray optics, wave optics**, and **quantum optics**.

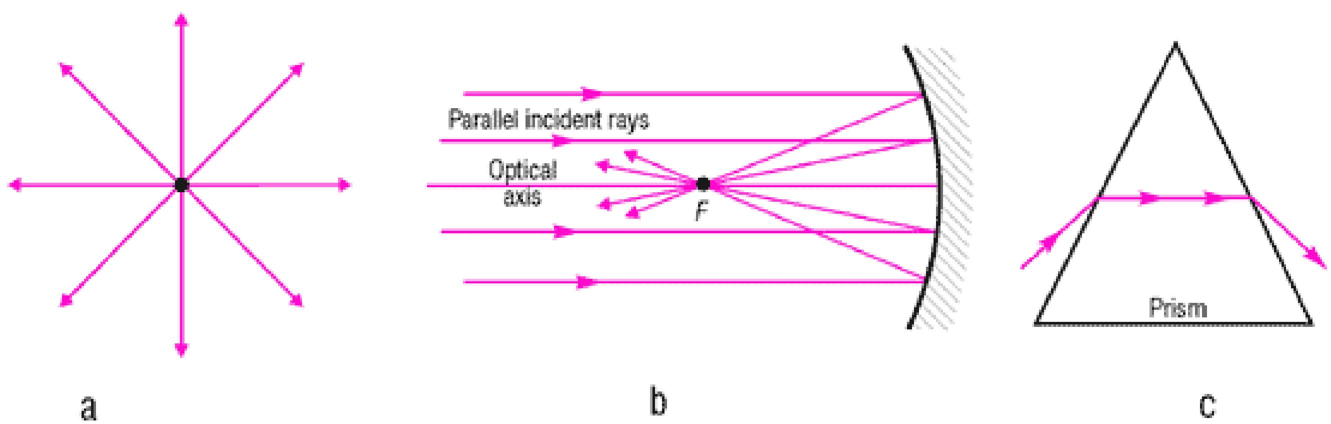


Wave from the bubble



Light rays and wavefronts

Geometric construct of a light ray we can illustrate propagation, reflection, and refraction of light



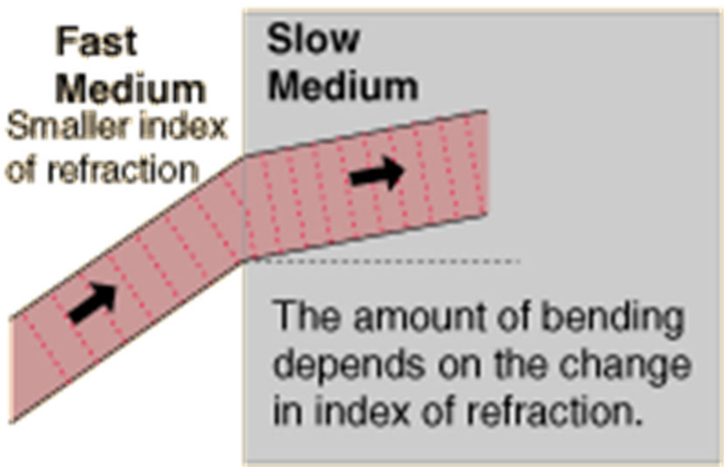
Light as a particle

Typical light rays in (a) *propagation,* (b) *reflection, and* (c) *refraction*

Geometric optics is also called ray optics. Light travels in the form of rays. Ray optics only concern with the location and direction of light rays.
Geometric optics completely ignore the finiteness of the wavelength (independent of λ)

Index of Refraction

In a material medium the effective speed of light is slower and is usually stated in terms of the index of refraction of the medium. The index of refraction is defined as the speed of light in vacuum divided by the speed of light in the medium.



HyperPhysics

$$n = \frac{C_o}{C}$$

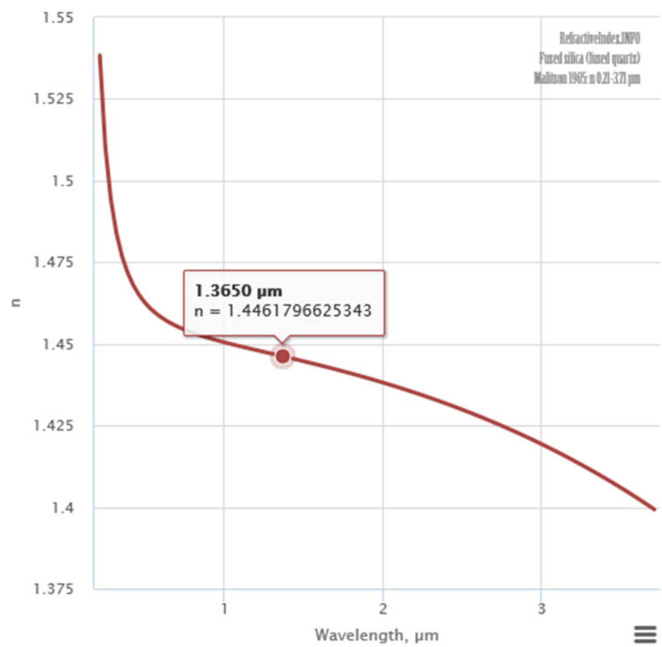
Co/n = d x t

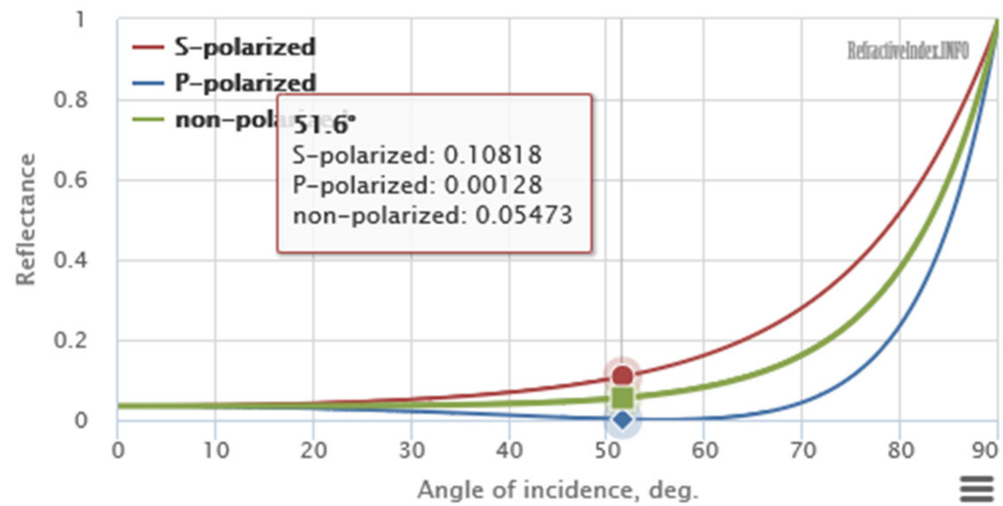
The indices of refraction of some common substances

Vacuum	1.000	Ethyl alcohol	1.362
Air	1.000277	Glycerine	1.473
Water	4/3	Ice	1.31
Carbon disulfide	1.63	Polystyrene	1.59
Methylene iodide	1.74	Crown glass	1.50-1.62
Diamond	2.417	Flint glass	1.57-1.75

The values given are approximate and do not account for the small variation of index with light wavelength which is called dispersion (n = function of wavelength).
(useful link for indices of different materials:
http://refractiveindex.info/?shelf=organic&book=polycarbonate&page=Sultanova

Optical constants of Fused silica (fused quartz)





http://refractiveindex.info/?shelf=organic&book=polycarbonate&page=Sultanova

Index in wave theory, a function of wavelength, polarization and also angle of incident as well as B.C. and EM properties of the materials

S polarization (TE):

$$1 + R_l = T_l$$

$$1 - R_l = \frac{\mu_1 k_{tz}}{\mu_2 k_z} T_l$$

$$\Rightarrow$$

$$R_l = \frac{\mu_2 k_z - \mu_1 k_{tz}}{\mu_2 k_z + \mu_1 k_{tz}}$$

$$T_l = \frac{2\mu_2 k_z}{\mu_2 k_z + \mu_1 k_{tz}}$$

P polarization (TM):

We get,

$$1 + R_{ll} = T_{ll}$$

$$1 - R_{ll} = \frac{\varepsilon_1 k_{tz}}{\varepsilon_2 k_z} T_{ll}$$

$$\Rightarrow$$

$$R_{ll} = \frac{\varepsilon_2 k_z - \varepsilon_1 k_{tz}}{\varepsilon_2 k_z + \varepsilon_1 k_{tz}}$$

$$T_{ll} = \frac{2\varepsilon_2 k_z}{\varepsilon_2 k_z + \varepsilon_1 k_{tz}}$$

Method for achieving polarizing light

TE and TM mode reflection and transmission coefficient:

$$r_s = \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t},$$

$$t_s = \frac{2n_1 \cos\theta_i}{n_1 \cos\theta_i + n_2 \cos\theta_t},$$

$$r_p = \frac{n_2 \cos\theta_i - n_1 \cos\theta_t}{n_2 \cos\theta_i + n_1 \cos\theta_t},$$

$$t_p = \frac{2n_1 \cos\theta_i}{n_2 \cos\theta_i + n_1 \cos\theta_t}.$$

If we put $n_2=n_1\sin\theta_i/\sin\theta_t$ (Snell's law and multiply the numerator and denominator by $(1/n_1)\sin\theta_t$

$$r_s = -\frac{\sin(\theta_i - \theta_t)}{\sin(\theta_i + \theta_t)}.$$

$$t_\perp = \frac{2\sin\theta_t \cos\theta_i}{\sin(\theta_i + \theta_t)}$$

If we do likewise with the formula for r_p, the result is easily shown to be equivalent to

$$r_p = \frac{\tan(\theta_i - \theta_t)}{\tan(\theta_i + \theta_t)}.$$

$$t_\parallel = \frac{2\sin\theta_t \cos\theta_i}{\sin(\theta_i + \theta_t)\cos(\theta_i - \theta_t)}$$

Geometric Optics

We define a **ray** as the path along which light energy is transmitted from one point to another in an optical system. It represents location and direction of energy transfer and direction of light propagate. The basic laws of geometrical optics are the law of reflection and the law of refraction.

Law of reflection: $|\theta_r| = |\theta_i|$
Snell's law, or **the law of refraction**: $n_i \sin\theta_i = n_t \sin\theta_t$.

If not being reflected or refracted, a light ray travels in a straight line.

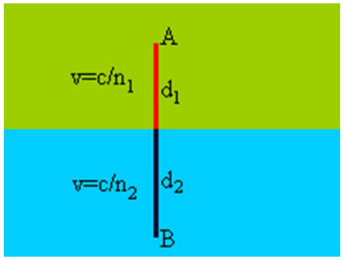
$$C = C_o/n = d / t$$

The **optical path length** of a ray traveling from point A to point B is defined as c_o times the time t it takes the ray to travel from A to B **= nd** (assume n=1 in air or vacuum).

Assume a ray travels a distance d_1 in a medium with index of refraction n_1 and a distance d_2 in a medium with index of refraction n_2.

The speed of light in a medium with index of refraction n is c/n. The travel time from A to B therefore is

$$t = n_1 d_1 / c_o + n_2 d_2 / c_o.$$



The optical path length is OPL = $n_1 d_1 + n_2 d_2$.

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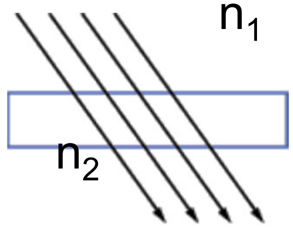
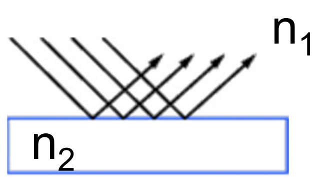
Light and matter

- When light hits something (air, glass, a green wall, a black dress), it may be:
- Transmitted and refracted (if the thing is transparent)
- Reflected or scattered (off mirror or raindrops)
- Absorbed (off a black velvet dress, not in ray theory)
- Often it's some combination. Take a simple piece of paper: you can see some light through, white reflects, black print absorbs.



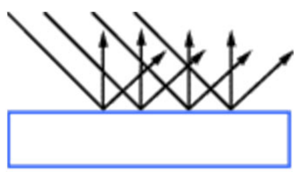
$n_1 = n_2$

The waves can pass **through** the object



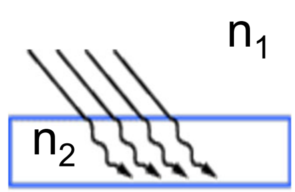
$|R| = 1$

The waves can be **reflected** off the object.



Surface roughness

The waves can be **scattered** off the object.

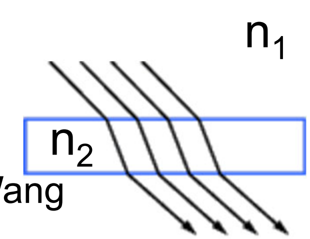


real

$n_2 = n_2' + j\, n_2''$

Loss due imaginary part of n

The waves can be **absorbed** by the object.



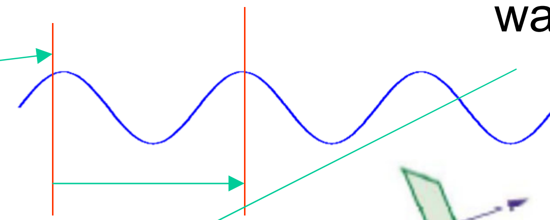
$n_1 \neq n_2$

The waves can be **refracted** through the object.

Geometric Optics

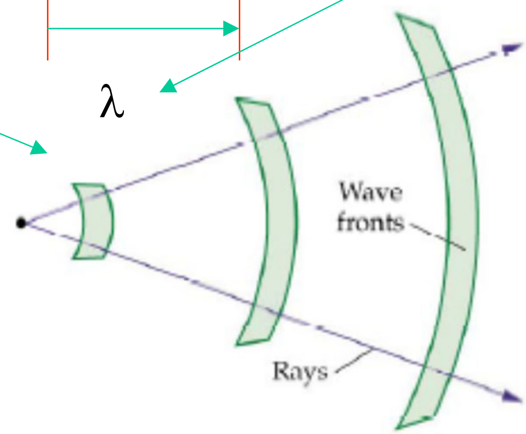
wavelength

Wave front = wave crest



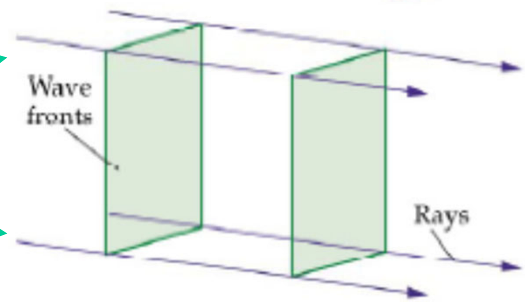
λ

To describe the light as ray, we need to borrow some of the properties from wave equation:

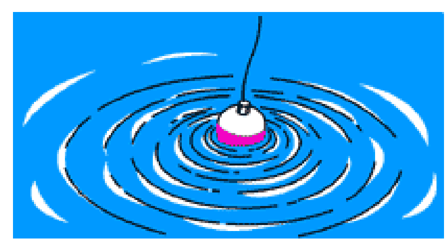


Wavefronts: a surface passing through points of a wave that have the same phase and amplitude

Rays (wave vector): a ray describe the direction of the wave propagation. A ray is a vector perpendicular to the wavefront



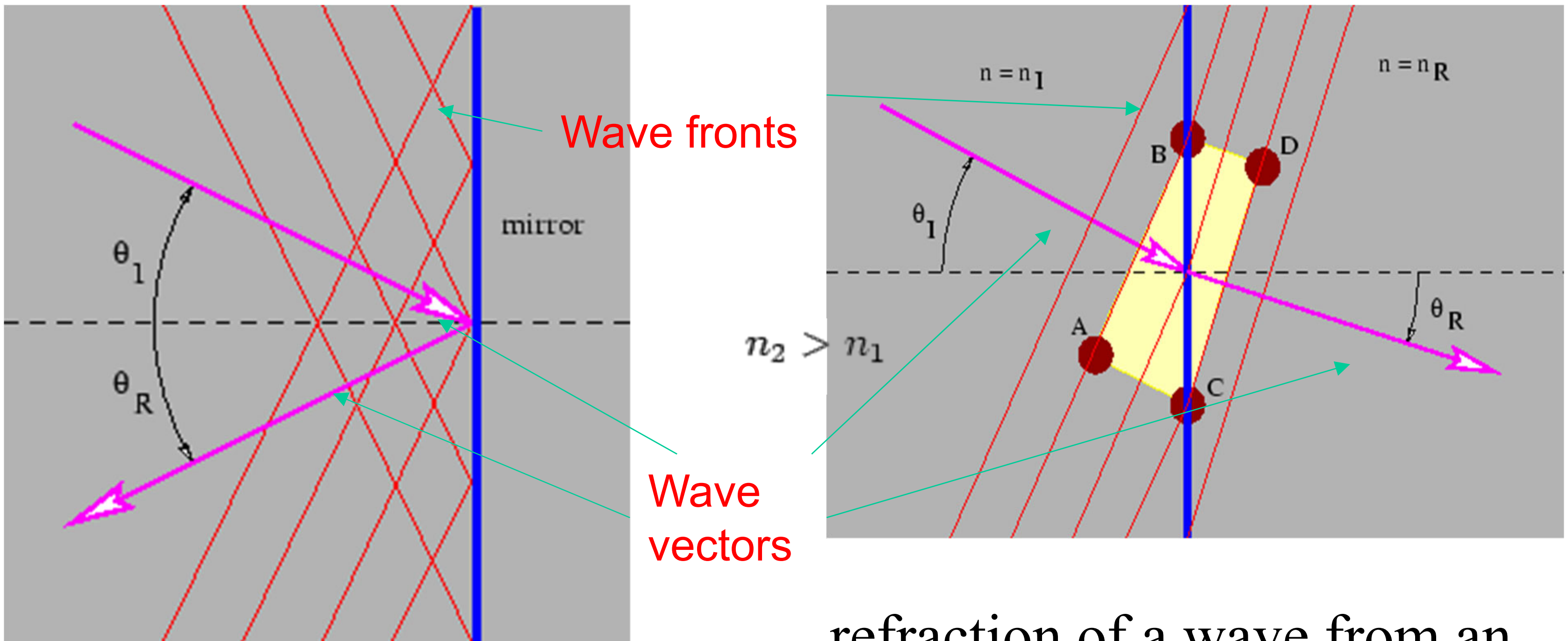
Relating ray with wave



Most of what we need to know about geometrical optics can be summarized in two rules:

1) **the laws of reflection :** $|\theta_r| = |\theta_i|$

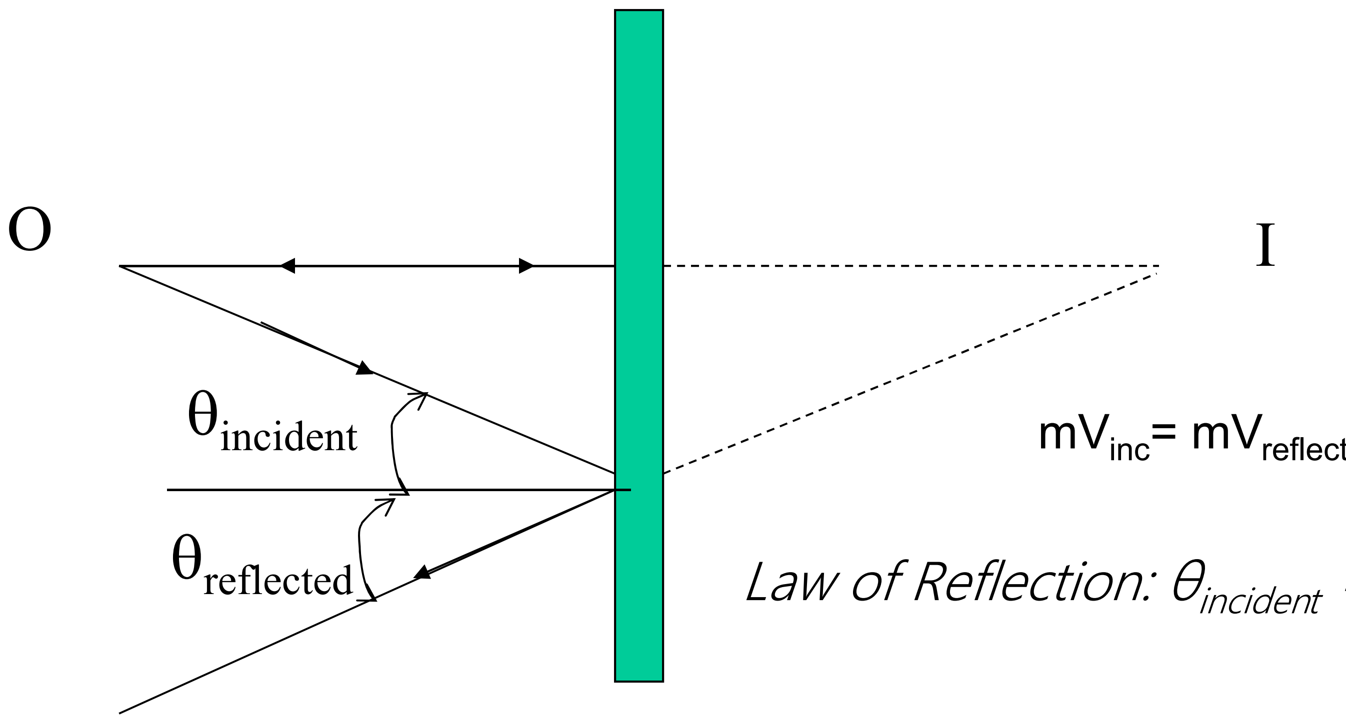
2) **The law of refraction: Snell's law, or the law of refraction:** $n_i \sin\theta_i = n_t \sin\theta_t$.



wave vector and wave front of a wave being reflected from a plane mirror

refraction of a wave from an interface between two dielectric media

Law of Reflection



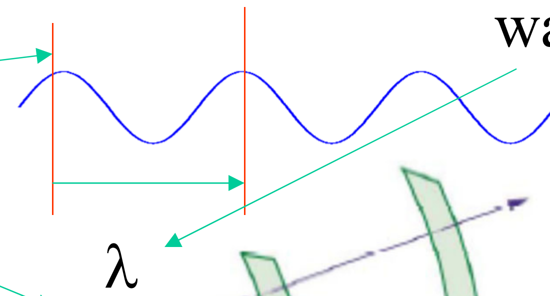
Law of Reflection: $\theta_{incident} = \theta_{reflected}$

The angles of incidence, $\theta_{incident}$, and reflection, $\theta_{reflected}$, are defined to be the angles between the incoming and outgoing wave vectors respectively and the line normal to the mirror. The law of reflection states that $\theta_{incident} = \theta_{reflected}$. This is a consequence of the need for the incoming and outgoing wave fronts to be in phase with each other all along the mirror surface. This plus the equality of the incoming and outgoing wavelengths is sufficient to insure the above result.

Borrow Geometric Optics

wavelength

Wave front = wave crest



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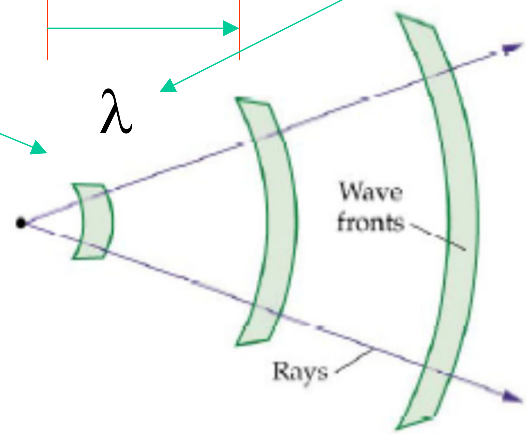

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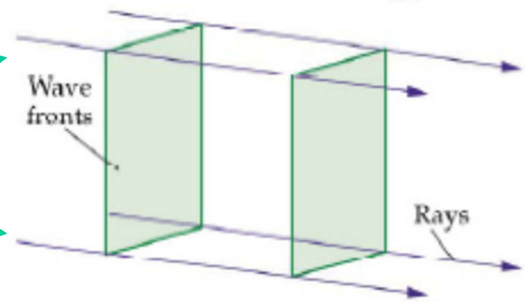

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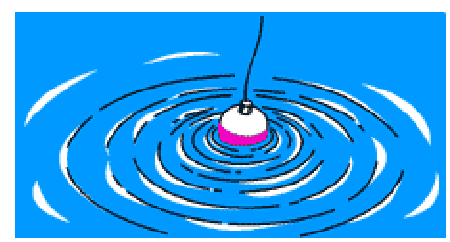
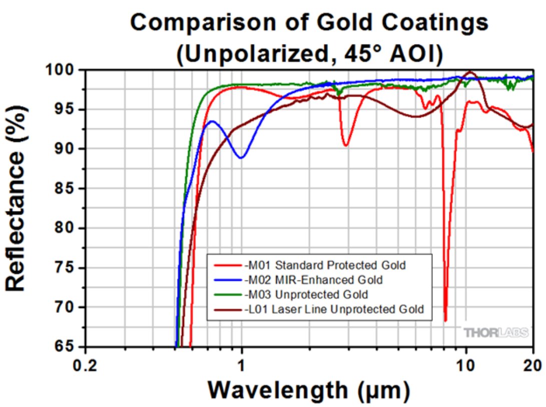
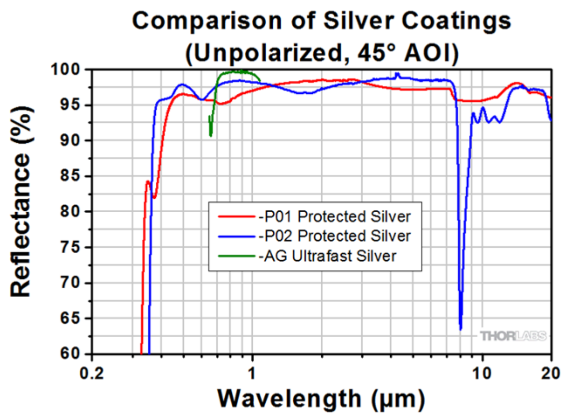
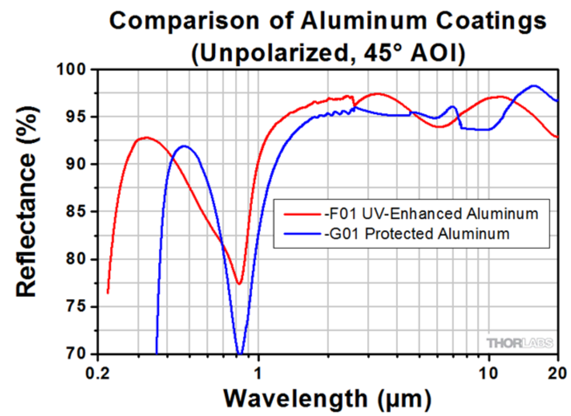


Different Mirror Reflectivity

$R \neq 1$!!!







Thorlab mirrors

Relate particle to wave

Law of Reflection

Assume 100% Reflection!!!
Not true

$\lambda_I = AB$
$AA' = AB/\sin(\theta_I)$

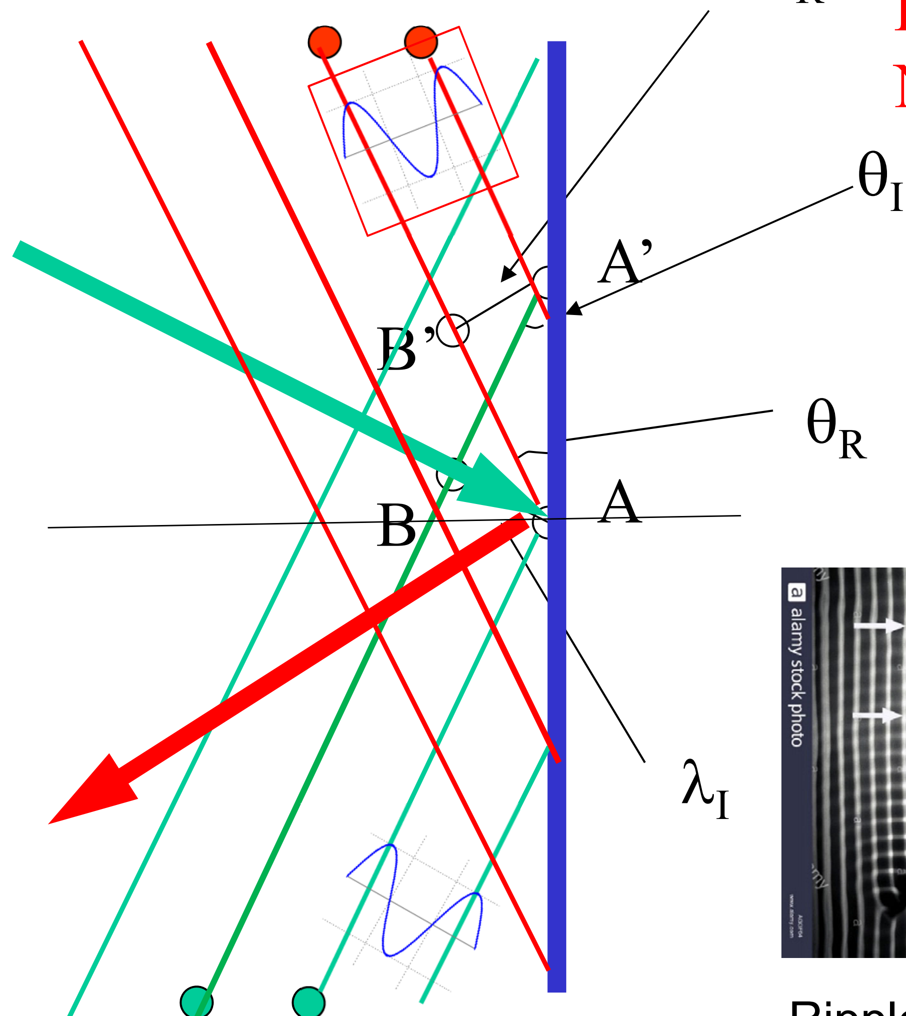
$\lambda_R = A'B'$
$AA' = A'B'/\sin(\theta_R)$

Since $\lambda_I = \lambda_R$
(elastic collision)
Then
$\lambda_I \sin(\theta_I) = \lambda_R \sin(\theta_R)$

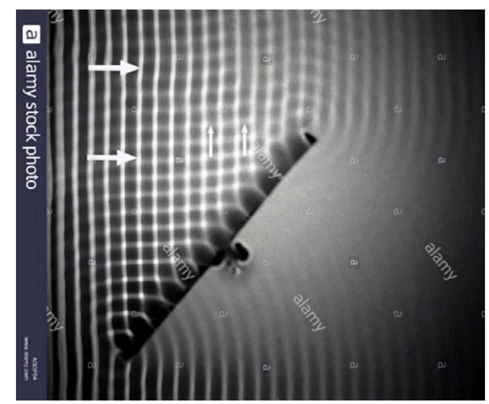
So

$\boxed{\theta_I = \theta_R}$

Law of Reflection



Green – incident wave fronts
Red – reflection wave fronts

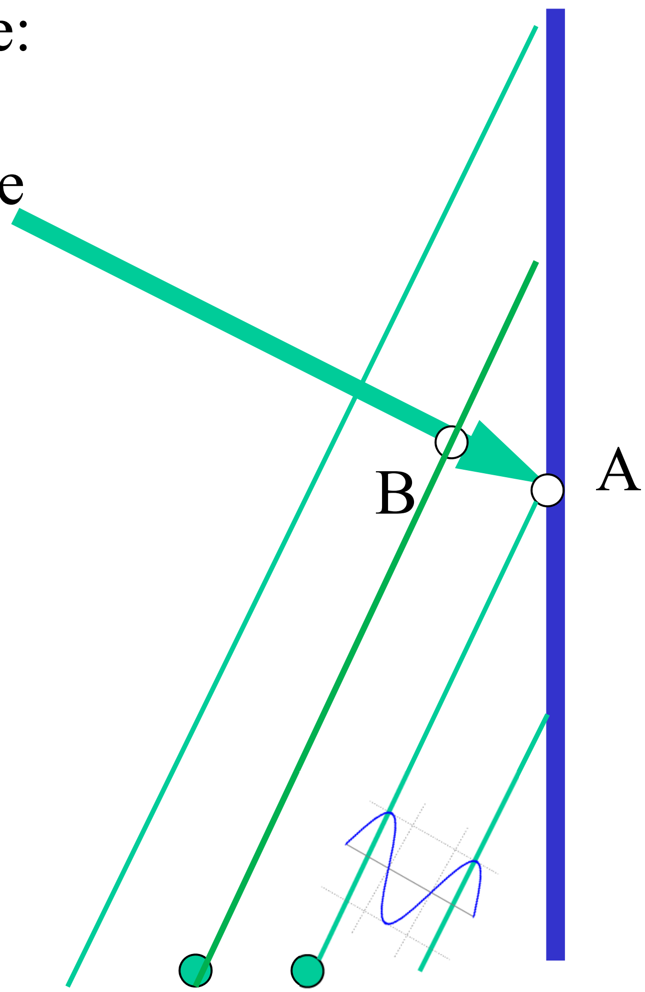


Ripple tank exp: showing wave fronts going and reflect off the wall

Assume 100% Reflection!!! Not true

Looking at two consecutive wave fronts at the interface:

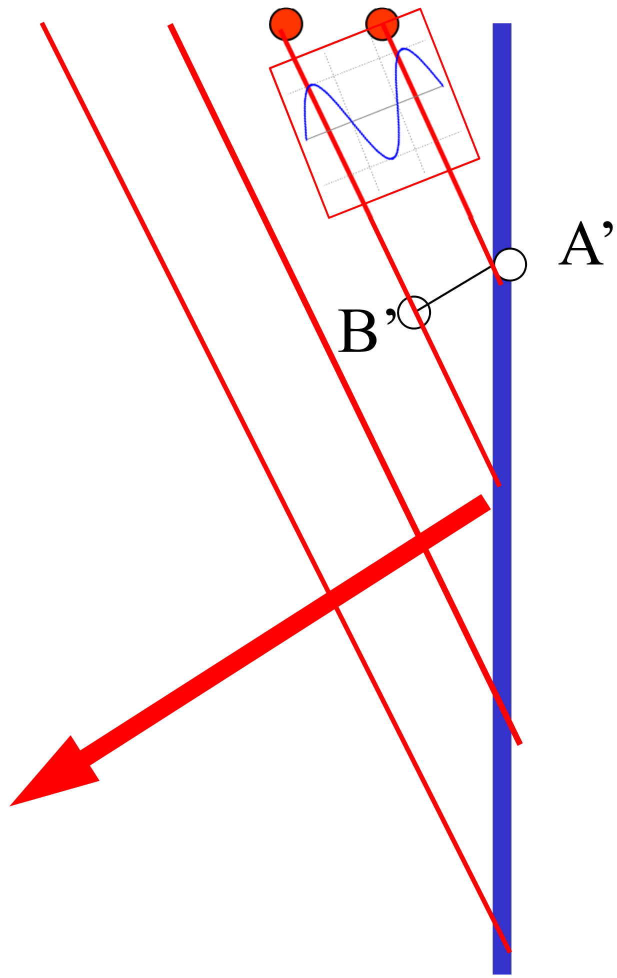
For the incident wave



Green – incident wave fronts

Looking at two consecutive wave fronts at the interface:

For the reflection wave

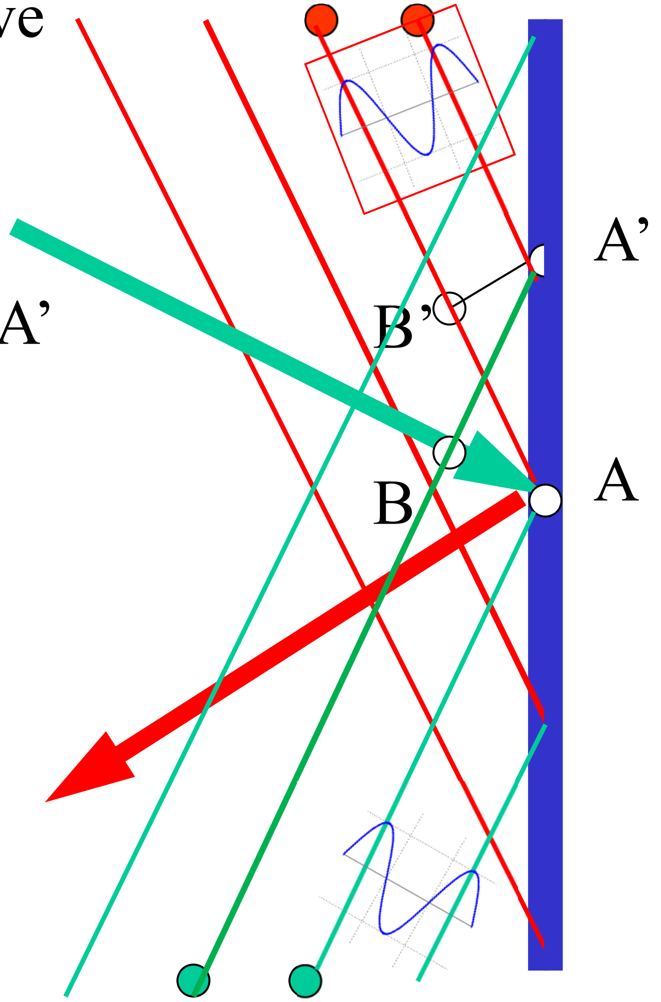


Assume 100% Reflection!!! Not true

Red – reflection wave fronts

Looking at two consecutive wave fronts at the interface:

When combine together, we see they share the same interface A A’



Green – incident wave fronts
Red – reflection wave fronts

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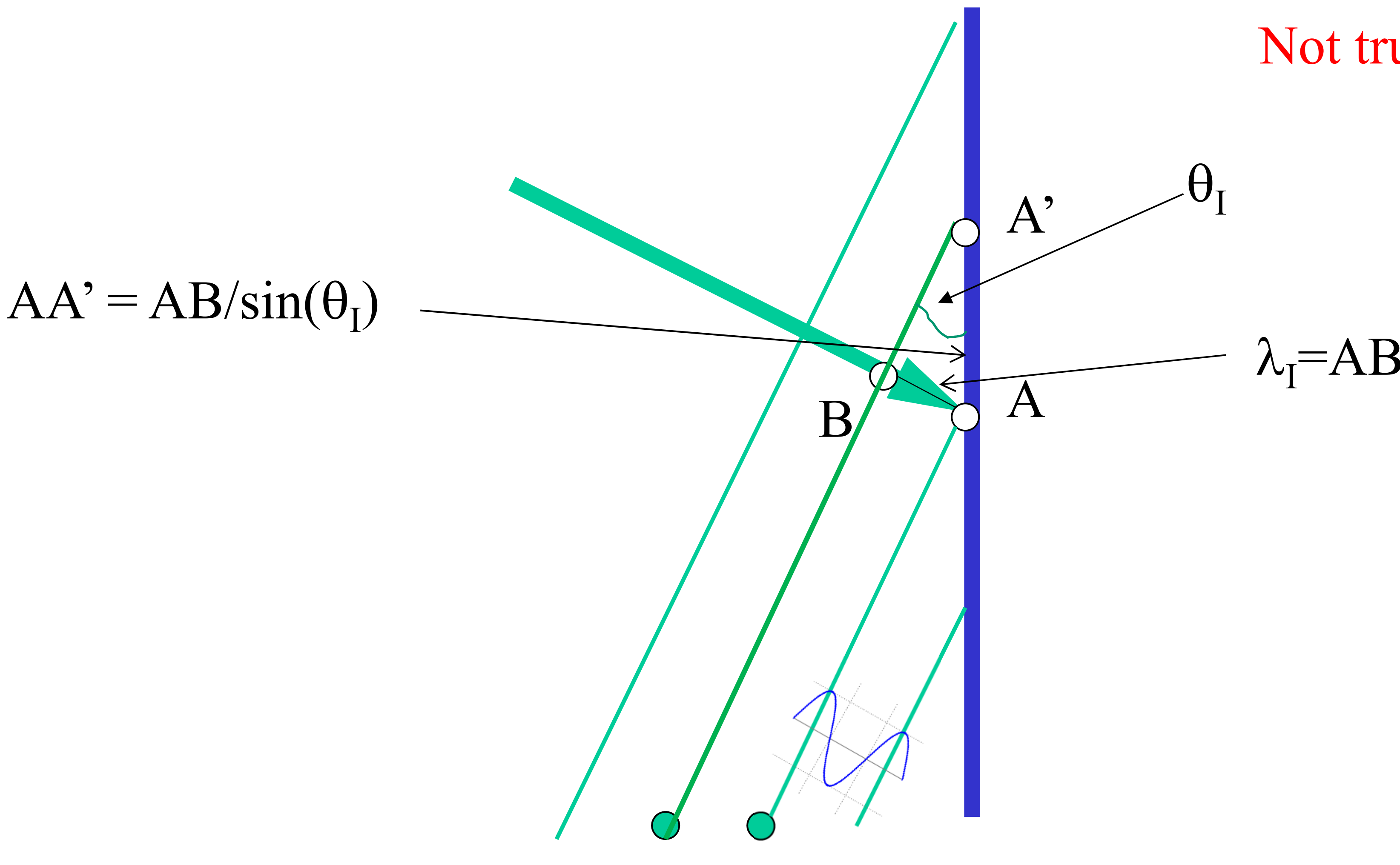
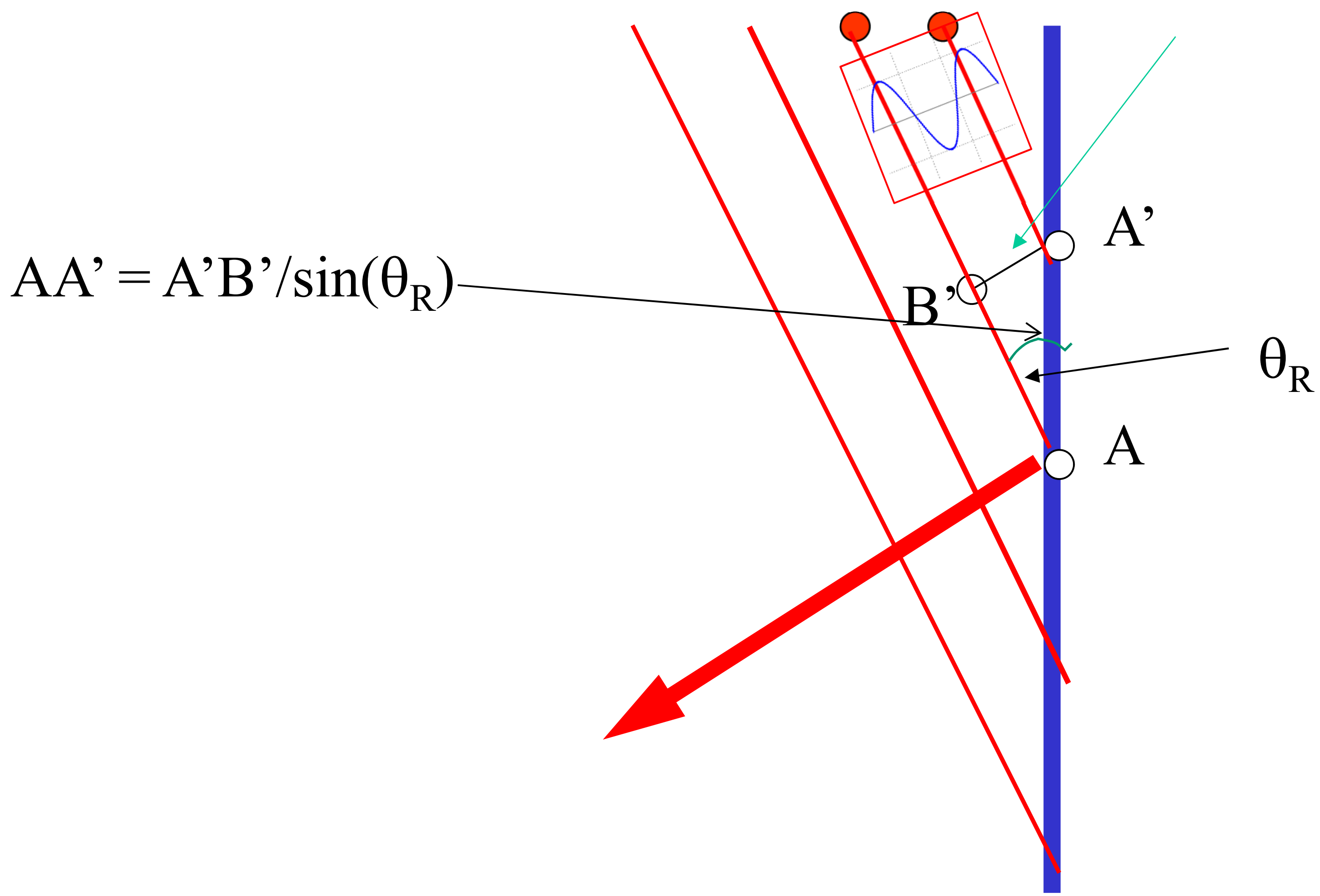
For the incident wave

Assume 100% Reflection!!! Not true



Green – incident wave fronts

For the reflection wave



Assume 100%
Reflection!!!
Not true

Red – reflection wave fronts

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λ_I=AB

AA' = AB/sin(θ_I)

λ_R=A'B'

AA' = A'B'/sin(θ_R)

Since we are assuming an elastic collision, all wave will bounce back the same way,

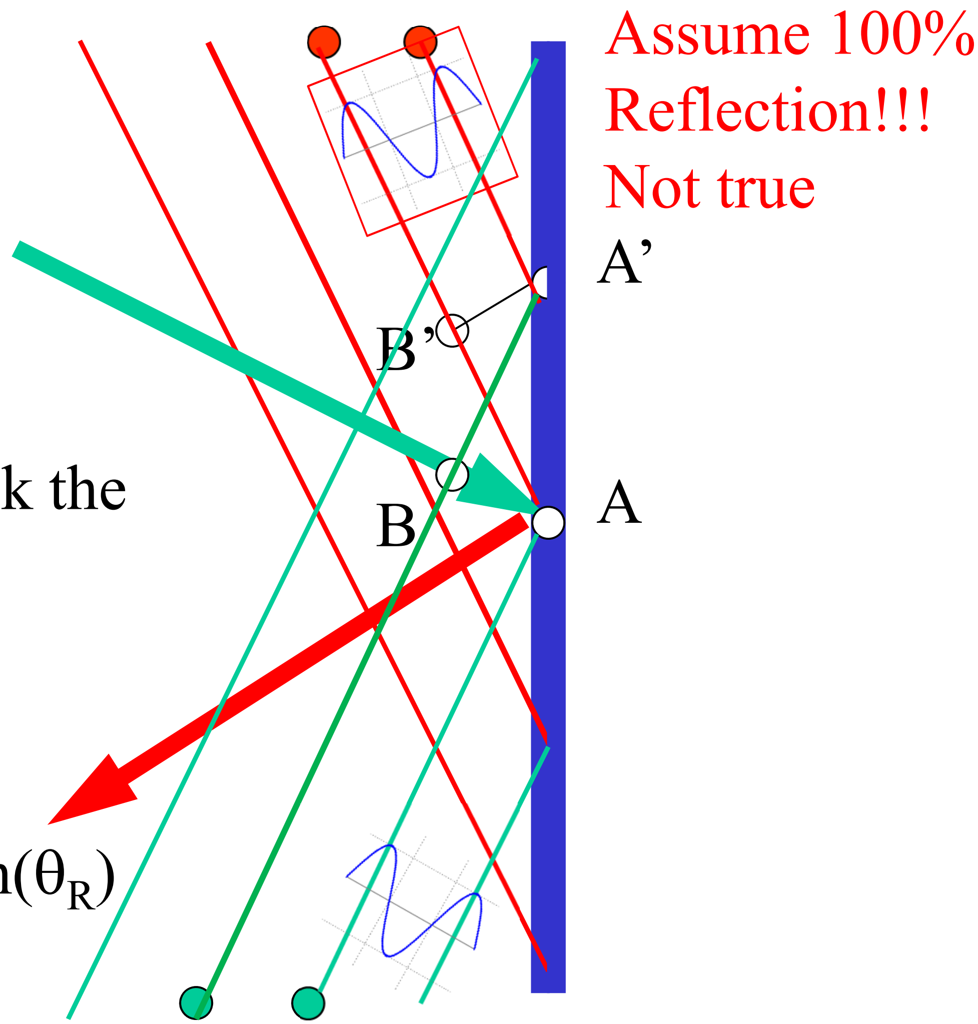
$\Rightarrow$ AB= λ_I = A'B'= λ_R

Since AA'= AB/sin(θ_I) =A'B'/sin(θ_R)

and $\lambda_I \sin(\theta_I) = \lambda_R \sin(\theta_R)$

Then $\boxed{\theta_I = \theta_R}$

Law of Reflection



Green – incident wave fronts
Red – reflection wave fronts

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Relate particle to wave

Conservation of Energy and Momentum

$KE_o + Pe_o = KE_f + Pe_f$ Conservation of energy)

$P_o = P_f$ (Conservation of momentum)

$KE = \frac{1}{2} mV^2$
$P = mV$

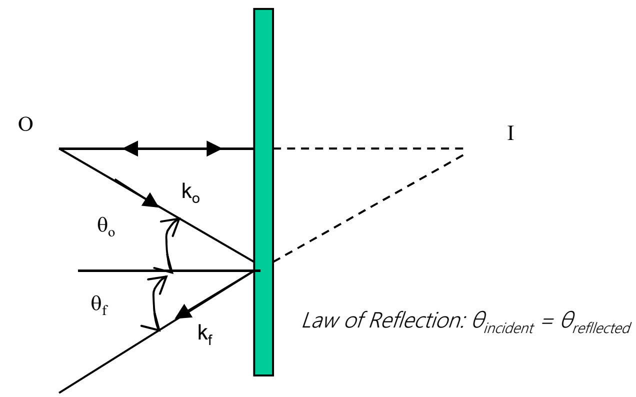
$V = f\lambda$ (speed of light)

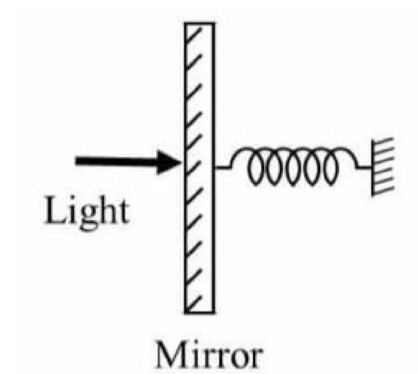
$\lambda_o = 2\pi/k_o$ or $\lambda_f = 2\pi/k_f$ (wavelength in term of wave vector)

$k = k_o \, n \sin\theta$ (wave vector or propagation constant)

Use $P = mV \Rightarrow 2\pi f/k_o = 2\pi f/k_f \Rightarrow k_o = k_f$
Or use both energy and momentum to get both x and y components and you will get the $\theta_o = \theta_f$

$mV_{inc} = mV_{reflect}$





$\frac{1}{2} mV_o^2 = \frac{1}{2} mV_f^2$
$mV_o \sin\theta_o = mV_f \sin\theta_f$

Light reflecting off of a polished or mirrored surface obeys the law of reflection: the angle between the incident ray and the normal to the surface is equal to the angle between the reflected ray and the normal.

Precision optical systems use first surface mirrors that are aluminized on the outer surface to avoid refraction, absorption, and scatter from light passing through the transparent substrate found in second surface mirrors.
When light obeys the law of reflection, it is termed a *specular* reflection. Most hard polished (shiny) surfaces are primarily specular in nature. Even transparent glass specularly reflects a portion of incoming light.

Diffuse reflection is typical of particulate substances like powders. If you shine a light on baking flour, for example, you will not see a directionally shiny component. The powder will appear uniformly bright from every direction.
Many reflections are a combination of both diffuse and specular components.
One manifestation of this is a *spread* reflection, which has a dominant directional component that is partially diffused by surface irregularities (surface roughness)

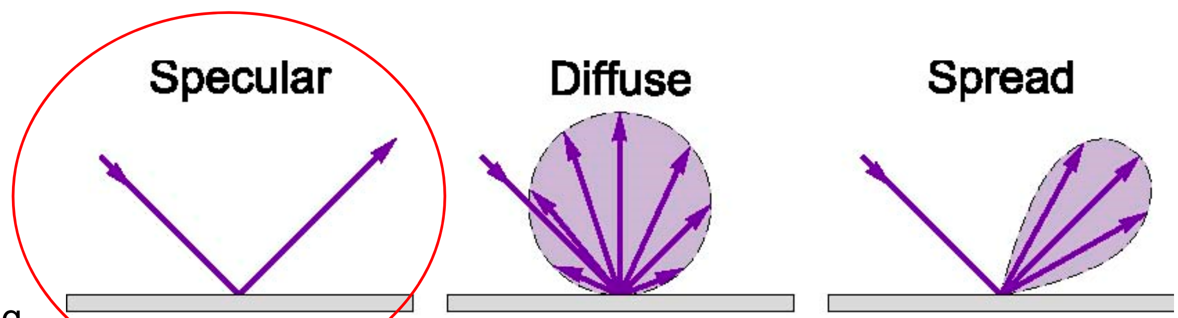


Fig. 3.2 Specular, diffuse, and spread reflection from a surface.

Plane Mirror

ray tracing

Reflection

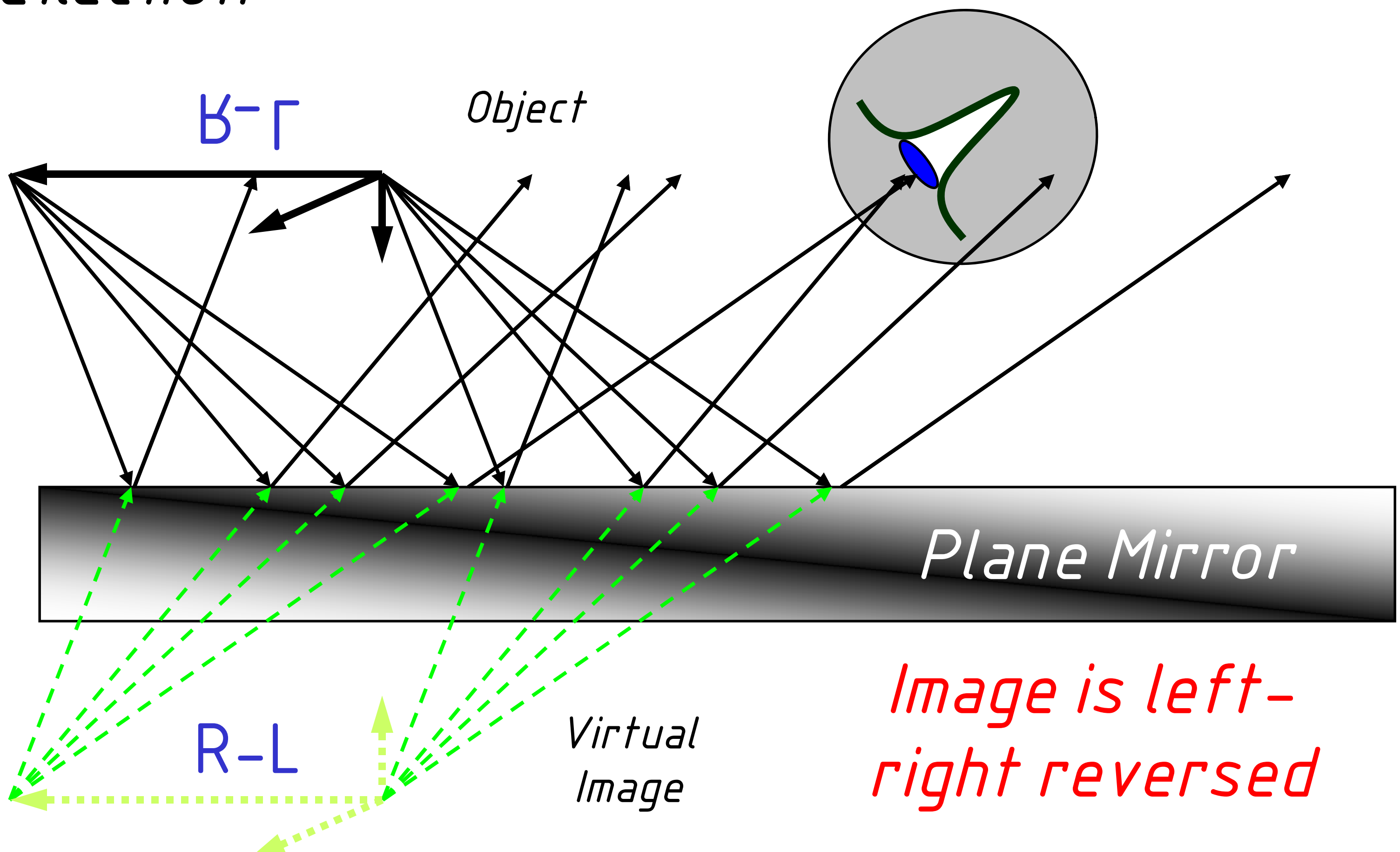


Image is left-right reversed

W.Wang

Recap

Geometric Optics

- Geometric optics is also called ray optics. Light travels in the form of rays. Ray optics only concern with the location and direction of light rays.
- Geometric optics completely ignore the finiteness of the wavelength (independent of λ)

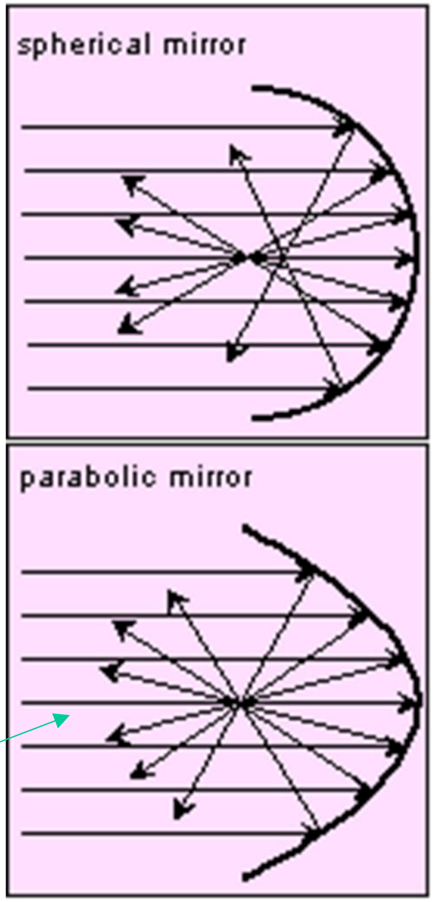
Most of what we need to know about geometrical optics can be summarized in two rules:

1) the laws of reflection : $|\theta_r| = |\theta_i|$

2) The law of refraction: Snell's law, or the law of refraction: $n_i \sin\theta_i = n_t \sin\theta_t$.

Curved Mirrors

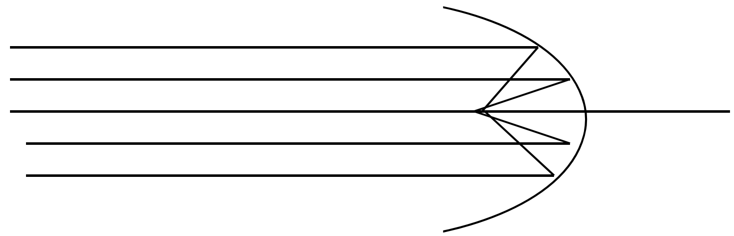
The ideal shape for a "**converging mirror**" is that of a parabola. Parabolic mirrors do not produce spherical aberrations; all incident rays parallel to the principal axis (optial axis) converge at the focus.



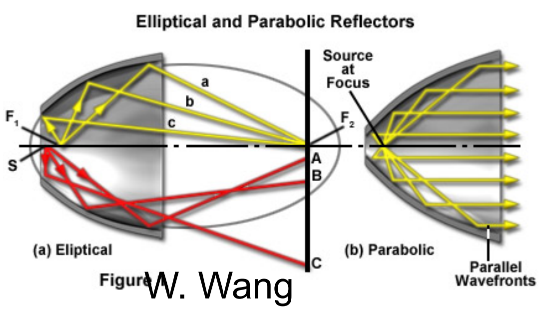
Curve Mirrors

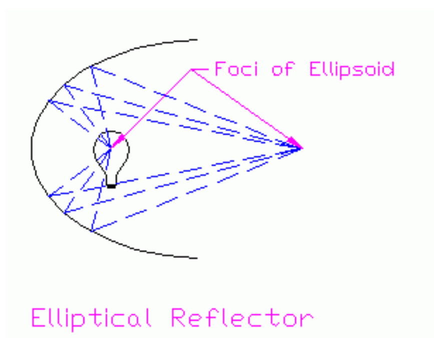
Parabolical mirrors: main purpose focusing all incident rays parallel to its axis to a single point. Using in telescope for light collecting element or reflector in flashlights.

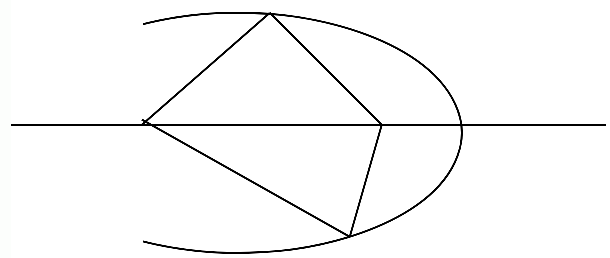




Elliptical mirrors: reflects all rays emitted from one of its two foci. The distances traveled by the light from two foci along any of the paths are all equal.







Geometric Optics

Assume parallel rays

Close to the axis so all

Rays approximately focus

to a single point f at

distance $R/2$ from

mirror center.

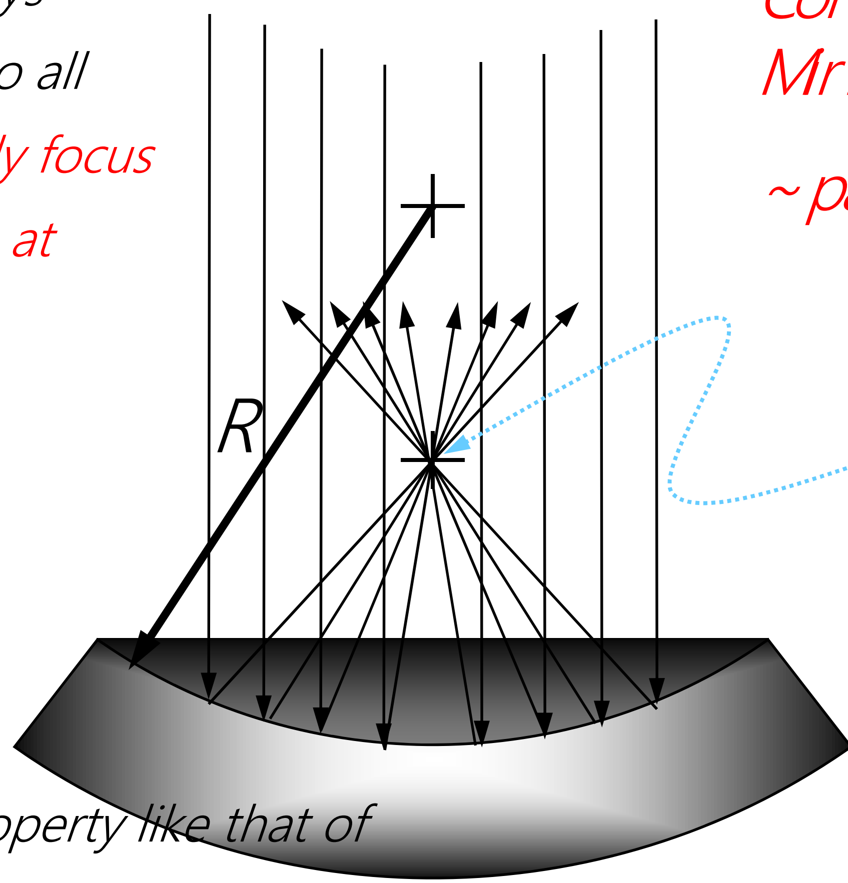
Using paraxial

Approximation,

Spherical mirror

Has a focusing property like that of

Paraboloidial mirror and imaging property like that of elliptical mirror.



How to make

Concave Spherical

Mirror

~ parabolic mirror

$f = R/2?$

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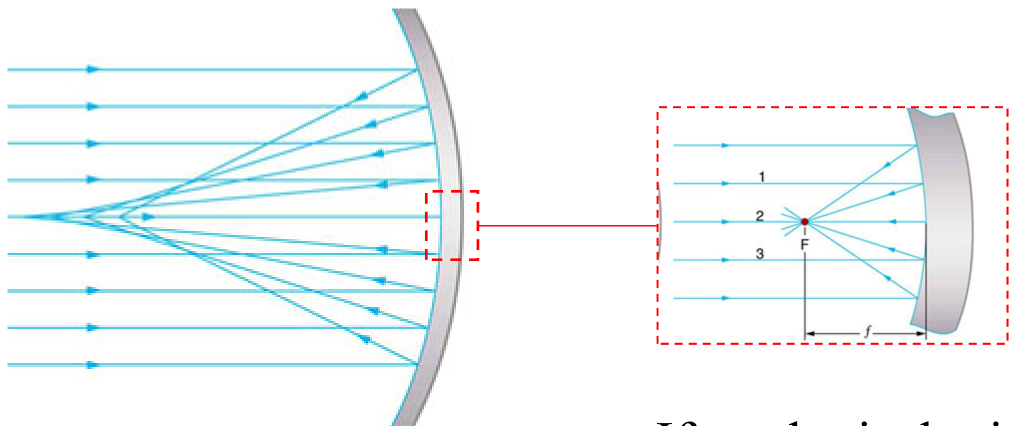
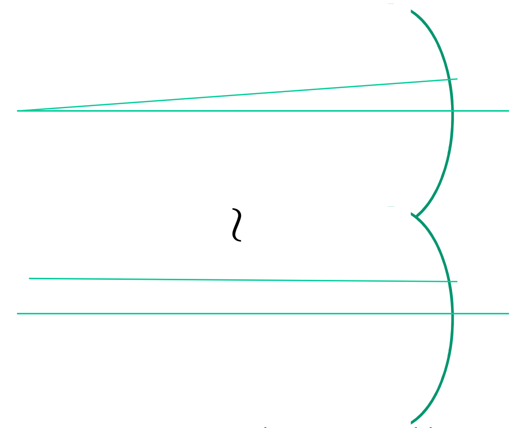
Paraxial Approximation

- Mirror is much larger than incoming rays
  (very large mirror curvature)
- Ray makes small angles with mirror axis
  (ray propagating really close to optical axis)





If spherical mirror is relatively large compared with its curvature of radius, parallel rays don't focus on a common point.

If a spherical mirror is small compared with its radius of curvature, parallel rays are focused to a common point.

Rays make small angle with mirror axis

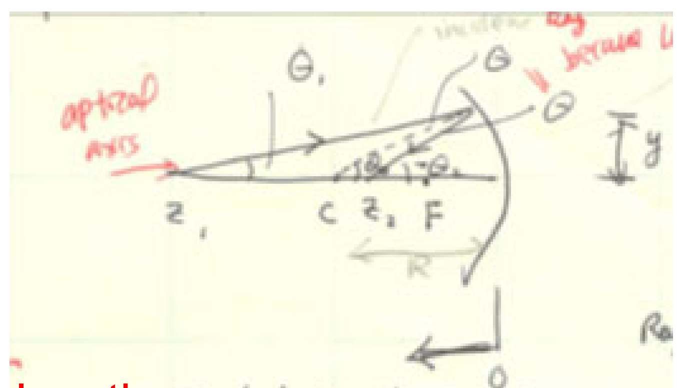
Derivation of Spherical Mirror Equation

- Please read the hand written lecture notes on reflection and refraction in week1 for mirror equation derivation:

depts.Washington.edu/me557/reading/reflection+refraction.pdf

$1/z_1+1/z_2 = 1/f$
Or
$1/p+1/q = 1/f$

Spherical mirror equation



Relates image length with focal and object length

How do we get f = R/2?

Spherical mirro equation?

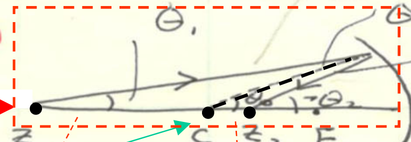
$1/p+1/q=1/f$

spherical concave mirror

incident Ray

because law of reflection

optical axis



Dotted line normal to the curve surface

reflected Ray

25

y

assume rays are travel close to axis

* Assume paraxial approximation (small θ_1)

Ray makes small angles with mirror axis

Relates Incident & reflected Ray in term of position & angle

how ?

Try to find a relationship between object distance, image distance & focal length

$$\theta_1 = \theta_0 - \theta$$

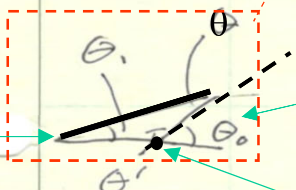
$$-\theta_2 = \theta_0 + \theta$$

in terms of incident and reflected angles

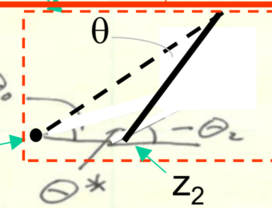
$-\theta_2$

reflection angle neg – away from mirror (sign convention to show incoming & outgoing)

Assume 100% Reflection!!! Not true



Same angle



$$\pi - \theta_1 = \theta + \theta'$$

$$\pi - \theta' = \theta_0$$

(Supplementary Angles)

$$\theta' = \pi - \theta_0$$

$$\Downarrow$$

$$\pi - \theta_1 = \theta + (\pi - \theta_0)$$

$$\boxed{-\theta + \theta_0 = \theta_1}$$

$$\pi - \theta_0 = \theta + \theta^*$$

$$\pi - \theta^* = -\theta_2$$

(Supplementary Angles)

$$\theta^* = \pi + \theta_2$$

$$\Downarrow$$

$$\pi - \theta_0 = \theta + \pi + \theta_2$$

$$\boxed{-\theta_2 = \theta_0 + \theta}$$

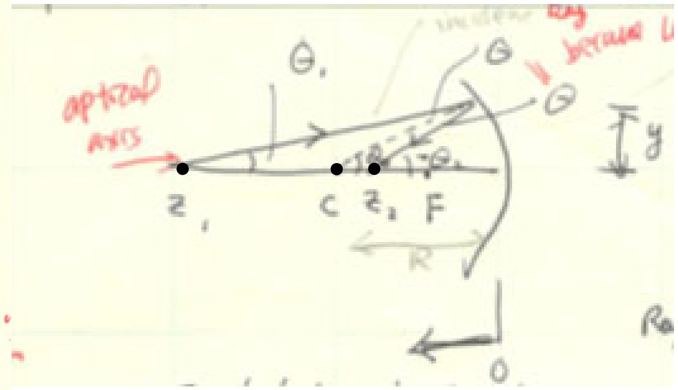
$$\boxed{\theta_1 - \theta_2 = 2\theta_0} \quad \text{— eq(1)}$$

If $\theta_0 \simeq$ small $\quad \tan\theta_0 \simeq \theta_0$

$$\theta_0 \simeq \frac{y}{R}$$

$$\boxed{(-\theta_2)+\theta_1 \simeq \frac{2y}{R}} \quad \text{— eq 2}$$

(R neg because mirror is concave)



If θ_1 & θ_2 are small (since θ small)

$$\begin{cases} \theta_1 \simeq y/z_1 \\ \theta_2 \simeq -\dfrac{y}{z_2} \end{cases}$$

plug back into ~~above~~ eq. 2

$$\frac{y}{z_1} + \frac{y}{z_2} \simeq \frac{2y}{R}$$

$$\boxed{\frac{1}{z_1} + \frac{1}{z_2} \simeq \frac{2}{R}}$$ or

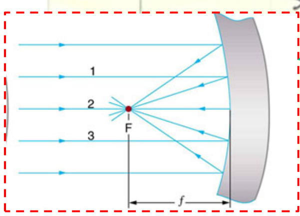
where p = object length, q = image length

$$\frac{1}{p} + \frac{1}{q} = \frac{2}{R}$$

$1/z_1+1/z_2 = 1/f$
Or
$1/p+1/q = 1/f$

Spherical mirror equation

So if $z_1 \simeq \infty$ then all rays will converge to a single pt



$$\frac{1}{f} = \frac{1}{z_2} = \frac{2}{R} \qquad \boxed{f = \frac{R}{2}}$$

(focal length)

Focal length of the curve mirror

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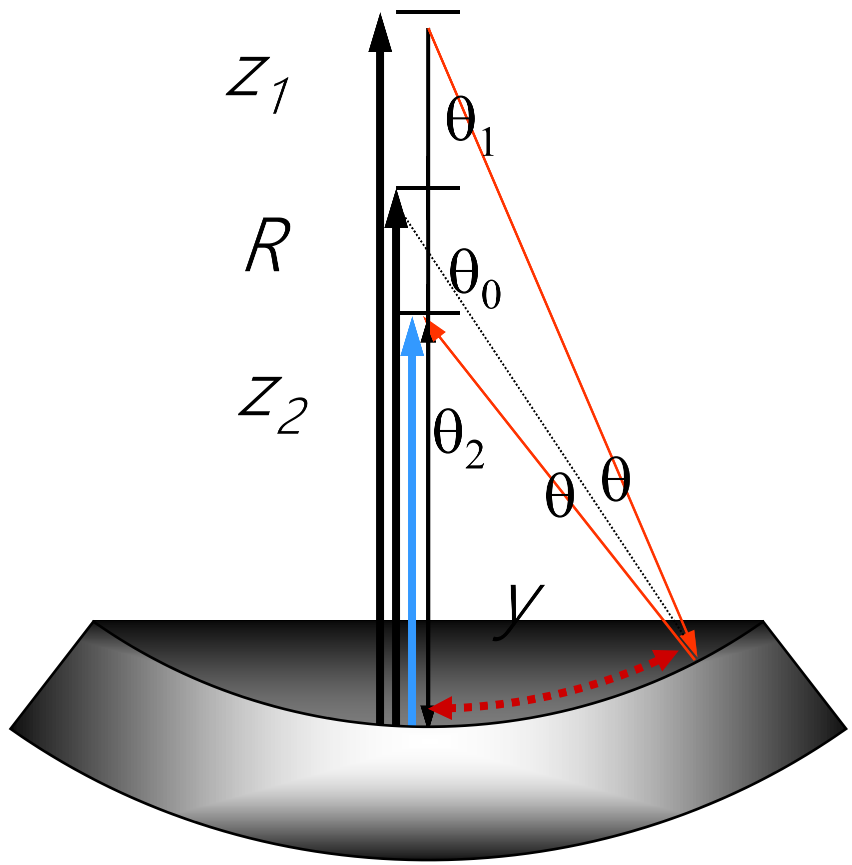
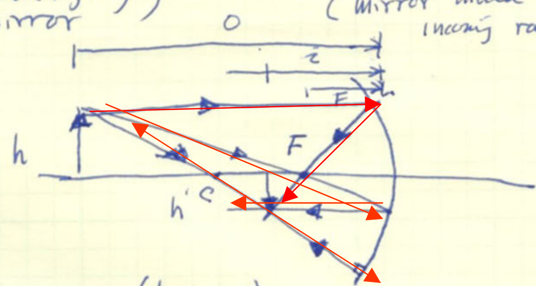
Concave Spherical Mirror

Law of Reflection



concave mirror (converging mirror)

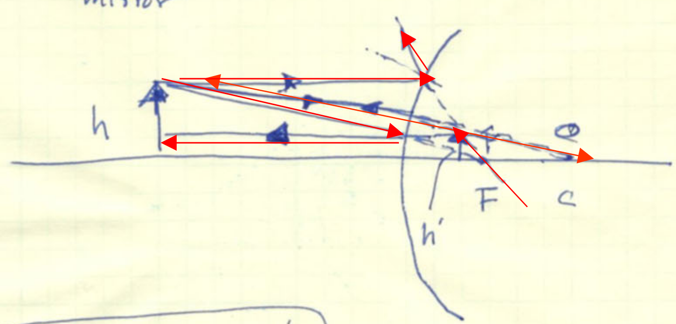
parallel axial approximation (mirror much larger than incoming ray)



How to find image:

all based on Law of reflection
- parallel ray reflects & to ~~thru~~ focal pt.
- ray go thru center reflects directly back to the same direction
- ray go thru focal pt, reflect at mirror & becomes parallel ray

convex mirror (diverging mirror)



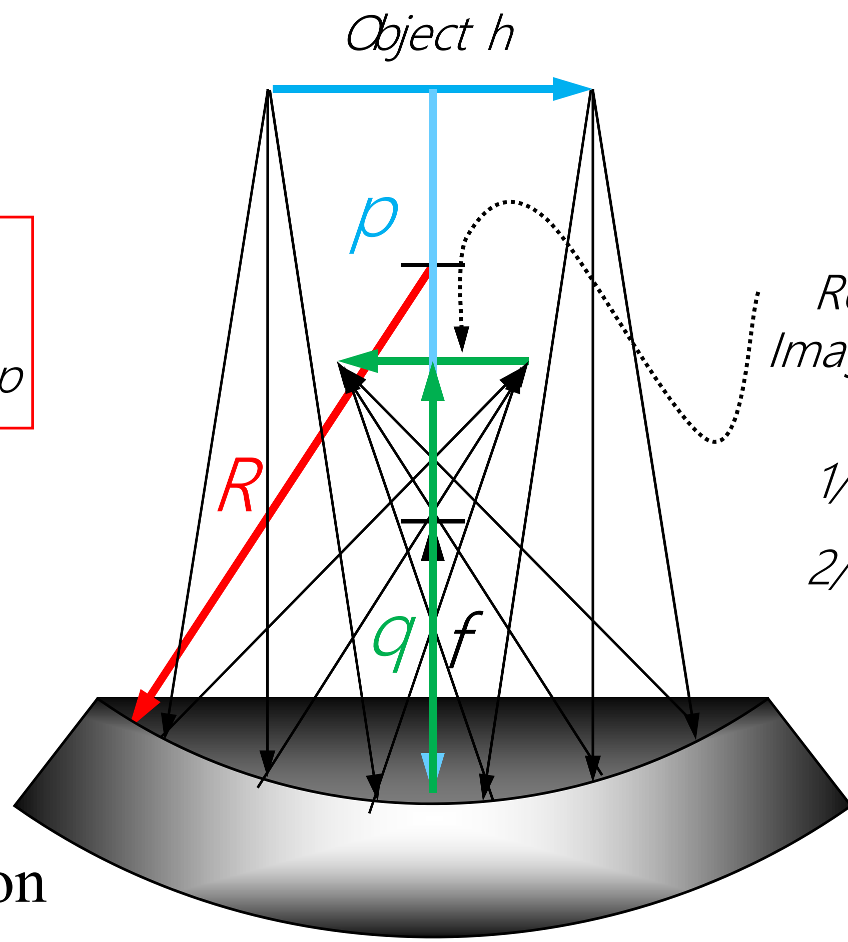
$$\boxed{m = -\frac{i}{o} = \frac{h'}{h}}$$

Concave Spherical Mirror

Reflection

Magnification

$M = h'/h = -q/p$



Real Image h'

$1/f = 1/p + 1/q$

$2/R = 1/p + 1/q$

Spherical mirror equation

p= object length

q = image position

W.Wang

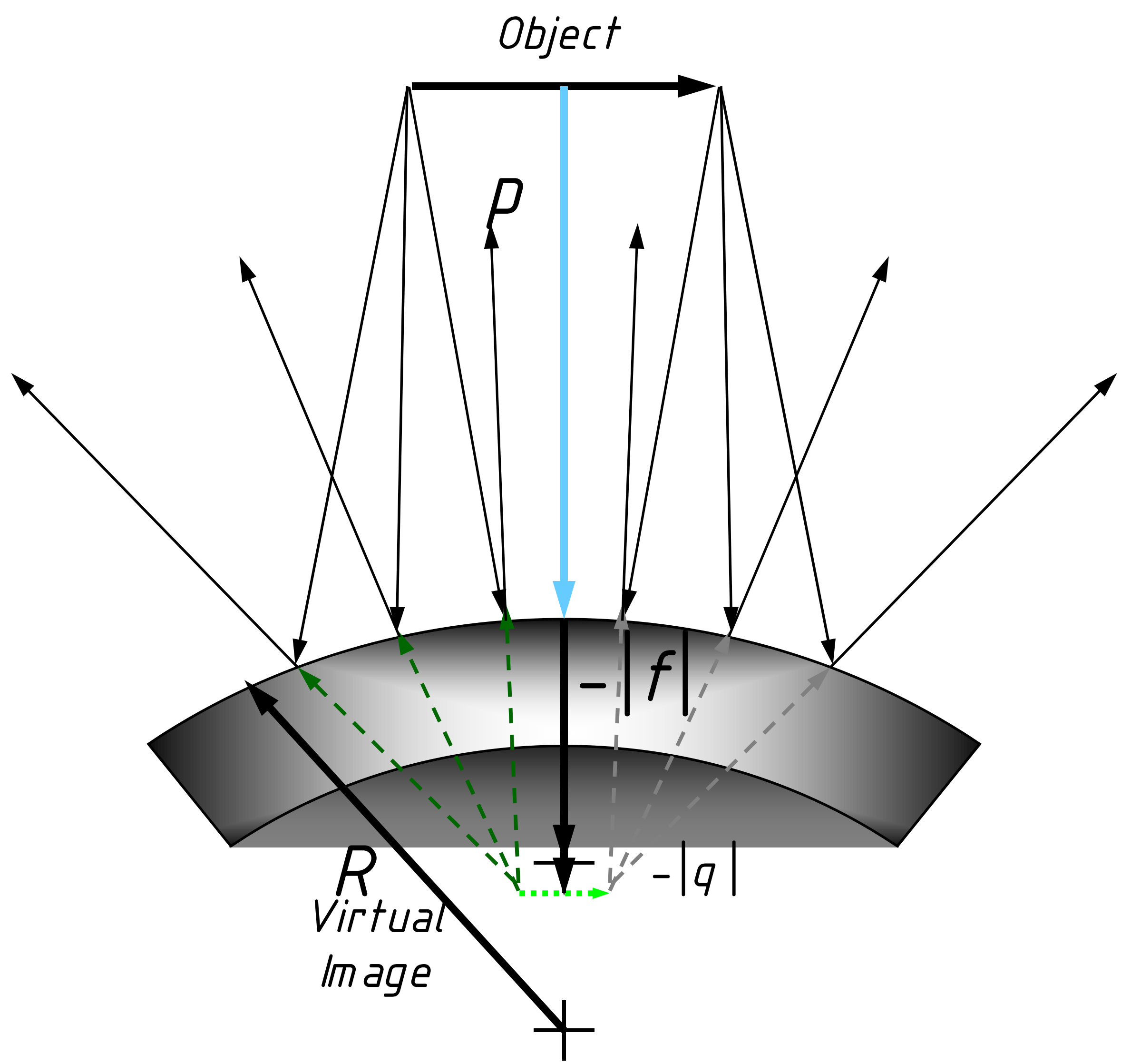
Convex Spherical Mirror

Spherical mirror equation

$$1/f = 1/p + 1/q$$

$$-1/|f| = 1/p - 1/|q|$$

$$M = h'/h = |q|/p$$



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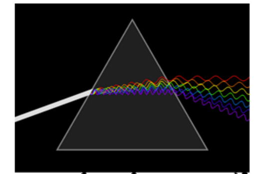
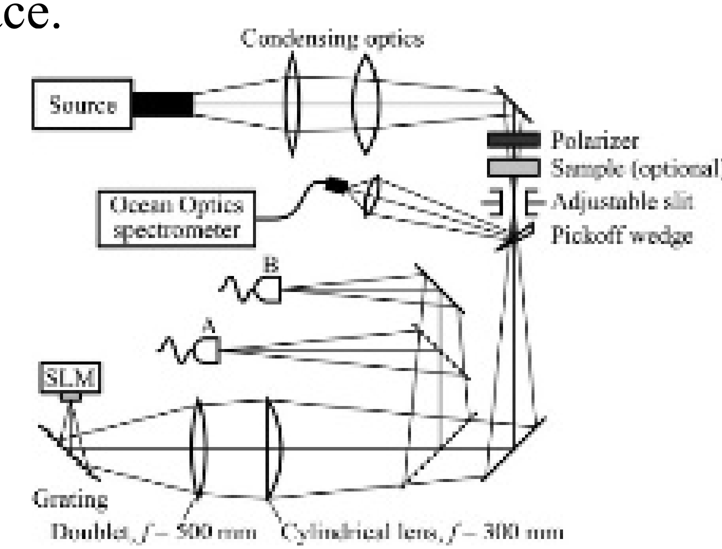
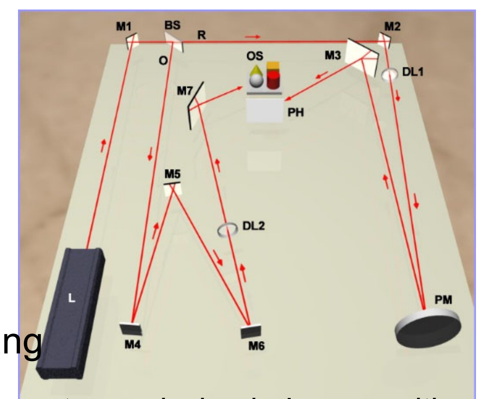
Why Mirrors?



- Reflected light preserves many or most of the detailed physical characteristics of the original light (minimum dispersion effect **<u>normal or paraxial</u>**!!)
- Most mirrors are designed for visible light; however, mirrors designed for other wavelengths of electromagnetic radiation are also used (e.g. metal, Silicon, multilayer dielectric mirror)
- Curved mirrors produce magnified or diminished images or focus light or simply distort the reflected image.(e.g. image reduction or magnification, reduce focal length, funny mirror)
- Allow long working distance in a limited space.
- Allow light to change direction
- Allow Beam splitting (how?)
- Minimized dispersion and heat effect



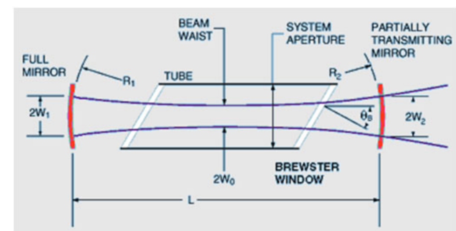
Multiple-output multivariate optical computing for spectrum recognition

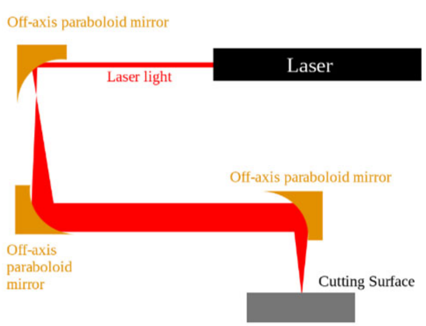


multi-beam transmission hologram with a single object beam

Application

- Laser cavity end mirrors (especially high power laser such as Co_2 laser excimer laser)
- Hot and cold mirrors,
- Thin-film beam splitters
- Coatings on modern mirror shades
- Telescope and Camera lenses





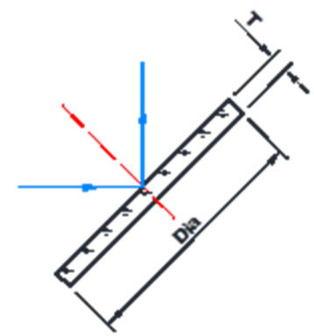
Parabolic mirrors used in beam expansion and convergence

Silicon zinc selenide o copper mirrors

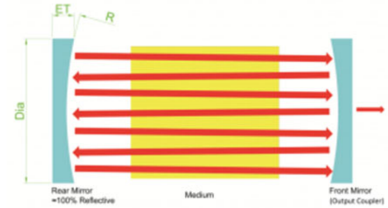
Co2 laser curved mirrors

Co2 laser Flat Mirror

Product Type	Part Number	Wavelength (nm)	Material	Dia (mm)	ET (mm)
Reflective Mirror	RSI-0.75-3	10600	Silicon	19.1	3.0
Reflective Mirror	RSI-1-3	10600	Silicon	25.4	3.0
Reflective Mirror	RSI-1.1-3	10600	Silicon	27.9	3.0
Reflective Mirror	RSI-1.5-4	10600	Silicon	38.1	4.0
Reflective Mirror	RSI-2-5	10600	Silicon	50.8	5.1
Reflective Mirror	RSI-2-9.5	10600	Silicon	50.8	9.5
Reflective Mirror	RMO-0.75-3	Polished Surface	Molybdenum	19.0	3.0
Reflective Mirror	RMO-1-3	Polished Surface	Molybdenum	25.4	3.0



Co2 laser cavity mirrors



Product Type	Part Number	Wavelength (nm)	Material	Dia (mm)	ET (mm)	Radius	Reflectivity (%)
Rear Mirror	RSI-1-4.5-3MCC	10600	Znse	25.4	4.5	3M Concave	>99.7%
Rear Mirror	RSI-1-4.5-5MCC	10600	Znse	25.4	4.5	5M Concave	>99.7%
Output Coupler	OCZ-0.5-2-80%R	10600	ZnSe	12.7	2.0	Plano	80+/-3%
Output Coupler	OCZ-0.5-3-92%R	10600	ZnSe	12.7	3.0	Plano	92+/-3%
Output Coupler	OCZ-0.75-2-70%R	10600	ZnSe	19.1	2.0	Plano	70+/-3%
Output Coupler	OCZ-0.75-3-85%R	10600	ZnSe	19.1	3.0	Plano	85+/-3%
Output Coupler	OCZ-0.75-2-95%R-5MCC	10600	ZnSe	19.1	2.0	5M Concave	95+/-3%
Output Coupler	OCZ-20-85%R-3MCC	10600	ZnSe	20.0	3.5	3M Concave	85+/-3%
Output Coupler	OCZ-25-3-70%R	10600	ZnSe	25.0	3.0	Plano	70+/-3%

Output Coupler

OCZ-25-3-95%R

10600

ZnSe

25.0

3.0

Plano

95+/-3%
Showing 1 to 10 of 12 entriesPreviousNext
CO_2 Laser cavity optics consist of Rear mirror and Front mirror (also called Output coupler or Partial reflector). Rear Mirrors, typically ZnSe, with very high reflectivity (>99.7%) are key optical components in laser resonator. Output Couplers are partially reflective mirrors to extract a portion of the laser beam from the laser resonator. They often require a slight wedge to prevent interference from multiple reflections inside the component.

Diameter Tolerance: +0/-0.13mm
Thickness Tolerance: ±0.25mm
Centration: < 3 arc min
Clear Aperture: >90%
Surface Quality: 40-20 S-D
Angle of Incidence: 0°
LIDT: 5.1 MW/cm² @ 10.6μm

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New PN: Infra-LW1001.0-21 & Infra-LW751.0-21 have been added to LWIR series
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Output Coupler
OCZ-25-3-95%R
10600
ZnSe
25.0
3.0
Plano
95+/-3%
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CO_2 Laser cavity optics consist of Rear mirror and Front mirror (also called Output coupler or Partial reflector). Rear Mirrors, typically ZnSe, with very high reflectivity (>99.7%) are key optical components in laser resonator. Output Couplers are partially reflective mirrors to extract a portion of the laser beam from the laser resonator. They often require a slight wedge to prevent interference from multiple reflections inside the component.
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Clear Aperture: >90%
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New PN: Infra-LW1001.0-21 & Infra-LW751.0-21 have been added to LWIR series
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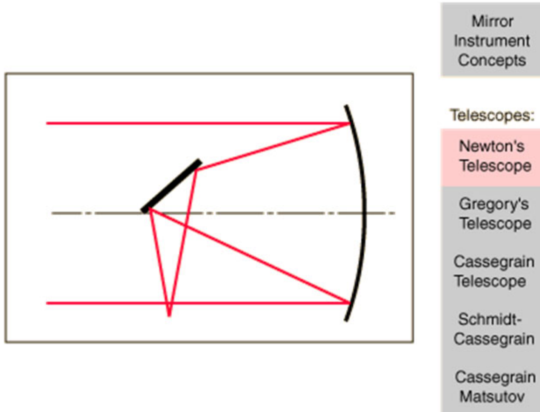
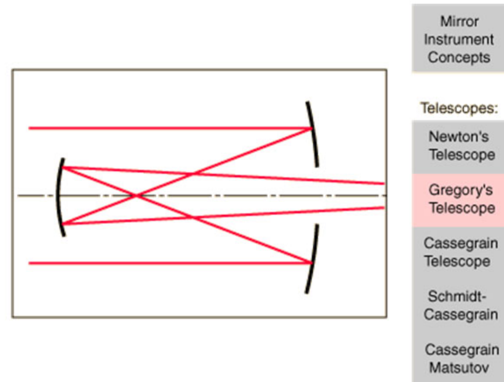
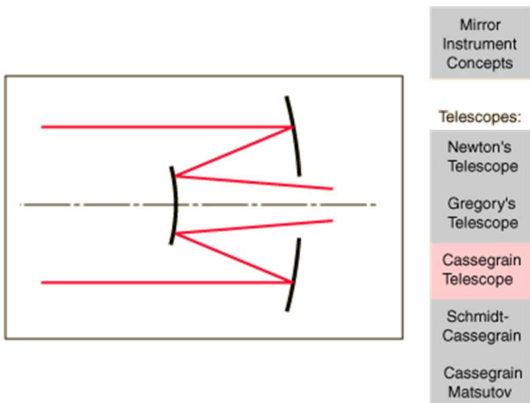
Mirror Systems for Telescope Applications

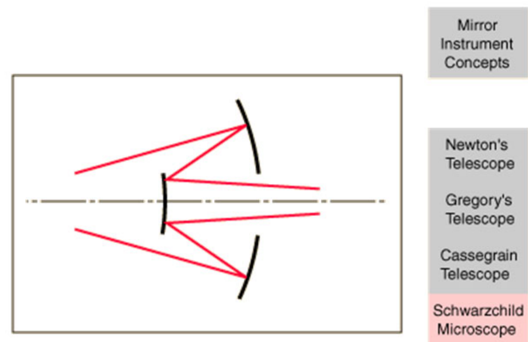
Instruments which use only mirrors to form images are called catoptric systems, while those which use both lenses and mirrors are called catadioptric systems (dioptric systems being those with lenses only).

Catoptric Systems





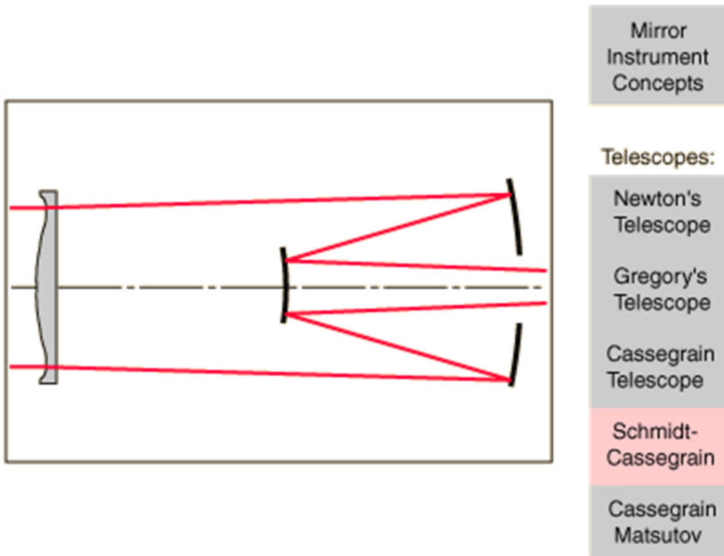


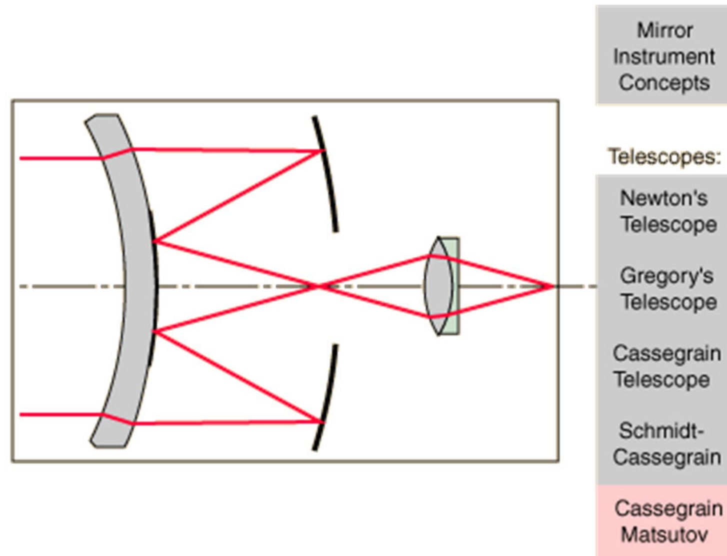


hyperphyics

Catoptric systems are those which use **only mirrors** for image formation. They contrast with catadioptric systems which use both mirrors and lenses and with pure dioptric systems which use only lenses.

Catadioptric Systems



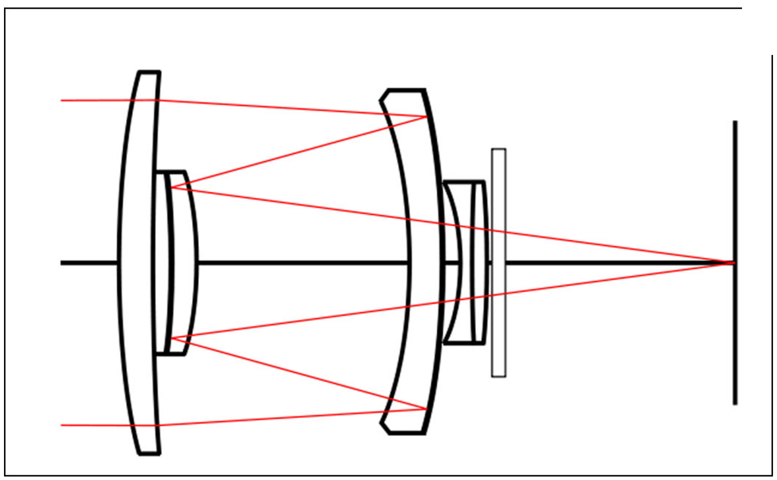


hyperphyics

Catadioptric systems are those which make **use of both lenses and mirrors** for image formation. This constrasts with catoptric systems which use only mirrors and dioptric systems which use only lenses. (i.e. Nikon 500mm mirror lens)

Mirror Lens
(Reflex Lens)

- Reduce Lens length
- Simpler lens design



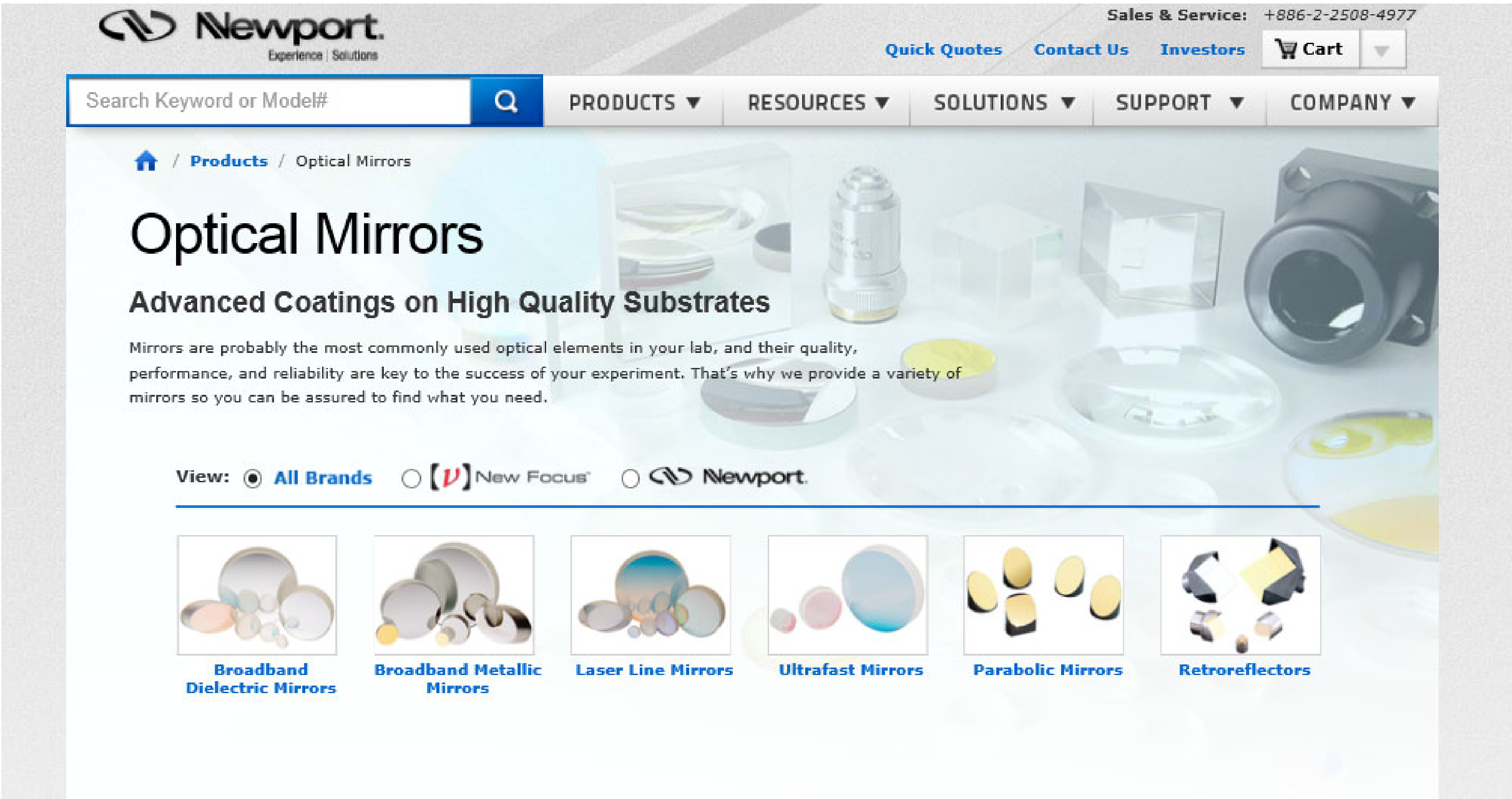
Nikon 500mm

Tokina 300mm

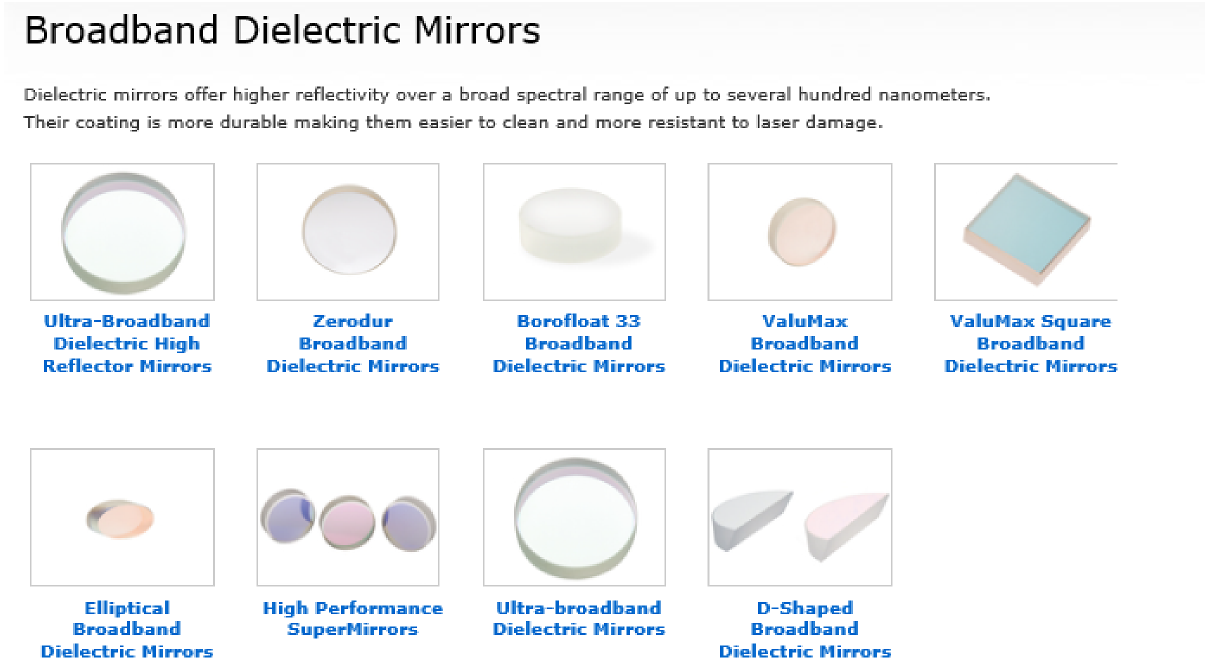
Mirror Lenses

Contax Zeiss 500mm f8 – Sigma 600mm f8 – Canon 100-400 mkll f4-5.6

Different Types of Mirrors



Dielectric Mirrors

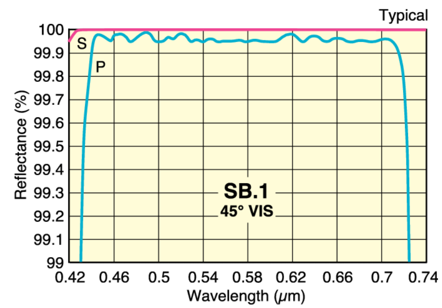


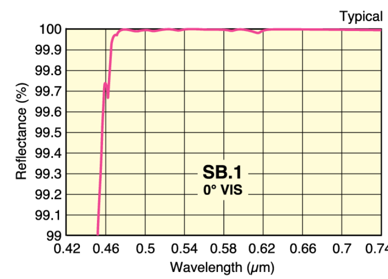
A dielectric mirror, also known as a Bragg mirror, is a type of mirror composed of multiple thin layers of dielectric material, typically deposited on a substrate of glass or some other optical material. By careful choice of the type and thickness of the dielectric layers, one can design an optical coating with specified reflectivity at different wavelengths of light. Dielectric mirrors are also used to produce ultra-high reflectivity mirrors: values of **99.999% or better over a narrow range of wavelengths can be produced using special techniques.** Alternatively, they can be made to reflect a broad spectrum of light, such as the entire visible range or the spectrum of the Ti-sapphire laser. Mirrors of this type are very common in optics experiments, due to improved techniques that allow inexpensive manufacture of high-quality mirrors. Examples of their applications **include laser cavity end mirrors, hot and cold mirrors, thin-film beamsplitters, and the coatings on modern mirrorshades.**

Example of a dielectric mirror

Newport Broadband SuperMirror, 25.4mm, Ref>99.9%, 0-45 Deg AOI, 485-700nm

Model	10CM00SB.1
Wavelength Region	VIS
Wavelength Range	485-700 nm
Mirror Shape	Round
Substrate Size	Ø25.4 mm
Material	UV Grade Fused Silica
Coating Type	Broadband IBS Coating
Surface Quality	20-10 scratch-dig
Surface Flatness	λ/10 at 632.8 nm
Coating Code	SB.1
Angle of Incidence	0-45°
Clear Aperture	≥central 50% of diameter
Reflectivity	R_s, R_p>99.9% @ 485-700 nm
Thickness	6.35 mm
Thickness Tolerance	±0.38 mm
Diameter Tolerance	+0/-0.1 mm
Chamfers	±0.25 mm
Chamfers Angle/Tolerance	45° ± 15°
Diameter	25.4 mm
Cleaning	Cleaning techniques specifically developed for ultra high performance optics may be used by qualified personnel

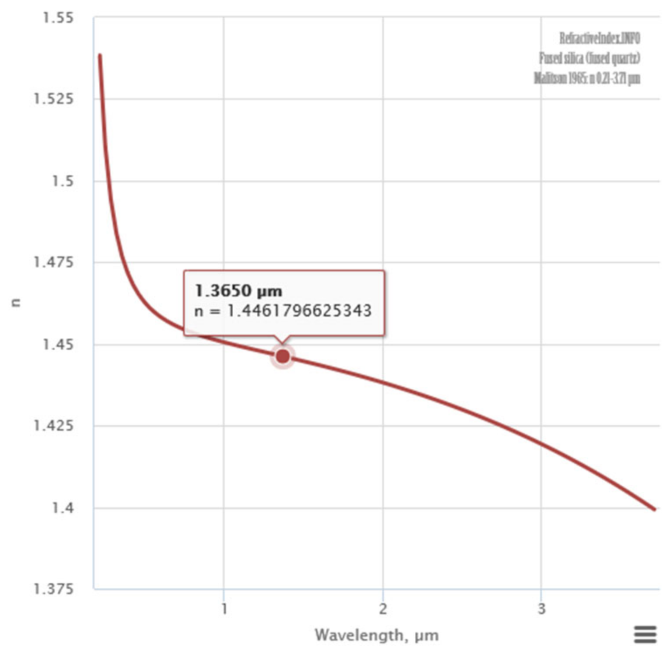


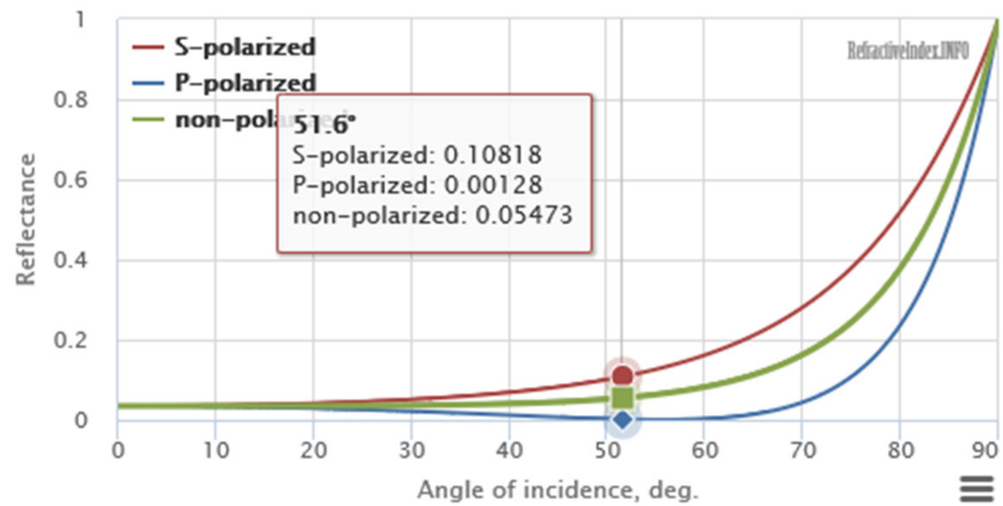


The 10CM00SB.1 is a Broadband Super Mirror, 25.4 mm in diameter, 6.35mm thick, and coated to have a reflectivity of > 99.9%, 0-45 °, from 485 to 700 nm.

Broadband SuperMirrorsTM are extremely high performance dielectric mirrors. They are produced with ion beam sputtered (IBS) coatings on superpolished fused silica substrates to achieve very low scatter and absorption. These mirrors have reflectivity greater than 99.9%, independent of polarization, at any angle of incidence from 0–45°.

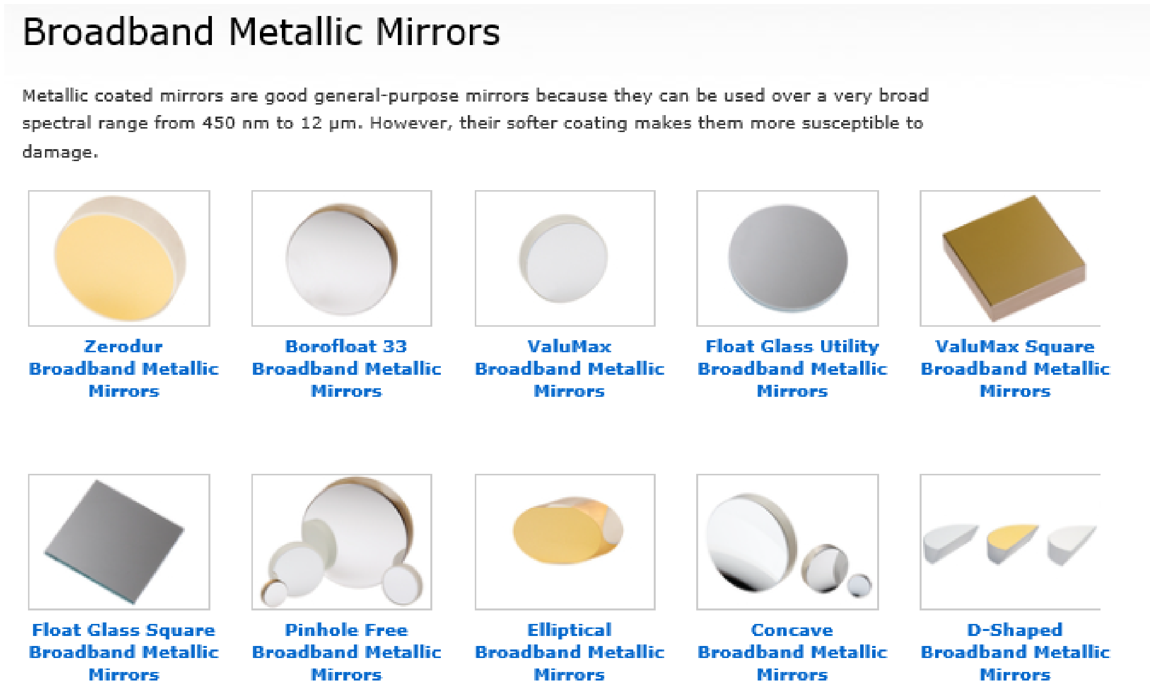
Optical constants of Fused silica (fused quartz)





http://refractiveindex.info/?shelf=organic&book=polycarbonate&page=Sultanova

Metallic Mirror



Mirror made of very thinly coated metal deposition made of aluminum coating (reflectivity of 87% to 93%) or silver mirror coating (reflectivity of 95% to 98%) and gold coating (` 95%) in visible band is used. Most commonly used mirrors.

Example of a metallic mirror

Newport General Purpose Silver Coated Mirror, 1.0 in., 450 - 12,000 nm

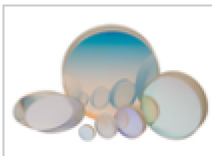
Model	5103
Wavelength Region	Broadband
Diameter	1.00 in. (25.4 mm)
Mirror Shape	Round
Material	Borofloat® 33
Wavelength Range	450-12000 nm
Surface Quality	25-10 scratch-dig
Surface Flatness	$\lambda/10$ at 632.8 nm
Clear Aperture	≥central 80% of diameter
Damage Threshold	30 kW/cm^2 (cw), 0.5 J/cm^2 (10-ns pulse)
Thickness	0.32 in. (8.0 mm)

The 5103 General Purpose Silver Coated Mirror can be used over a broad spectral range with better than **95% reflectivity from 450 nm to beyond 12 µm**. It's proprietary silver-based coating makes this 1.0 inch (25.4 mm) diameter mirror highly reflective from 0° to 45° and virtually **insensitive to polarization**. Protective dielectric coatings make it resistant to tarnish and oxidation. Plus it has minimal phase distortion, so it is useful for ultrafast-pulsed applications with Ti:Sapphire and other lasers.

Laser Mirrors

Laser Line Mirrors

Our laser line mirrors are highly efficient reflectors optimized for a narrow wavelength range.

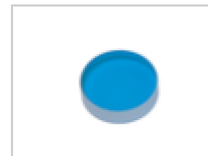


Zerodur Laser Line Dielectric Mirrors

Borofloat 33 Laser Line Dielectric Mirrors

Elliptical ND:YAG Laser Line Dielectric Mirrors

Long Lived Deep UV Excimer Mirrors



High Energy ND:YAG Laser Mirrors

High Energy Excimer Laser Mirrors

Example of a Laser mirror

Newport Femtosec Optimized Silver Mirror, 0-45° AOI, 25.4 mm, 600-1100 nm

Model	5103
Wavelength Region	Broadband
Diameter	1.00 in. (25.4 mm)
Mirror Shape	Round
Material	Borofloat® 33
Wavelength Range	450-12000 nm
Surface Quality	25-10 scratch-dig
Surface Flatness	λ/10 at 632.8 nm
Clear Aperture	≥central 80% of diameter
Damage Threshold	**30 kW/cm^2 (cw), 0.5 J/cm^2 (10-ns pulse)**
Thickness	0.32 in. (8.0 mm)

The 10B20EAG.1 Femtosecond Optimized Silver Mirror is 25.4 mm in diameter, 6.35 mm thick, and broadband optimized for 540 - 1100 nm. **This ultrafast mirror is designed for zero to 45 degrees angle of incidence for S and P polarization**. The absolute value of GDD (Group Delay Dispersion) is an astounding less than 5 fs from 550 to 1050 nm. Ideal for use with ultrashort pulse lasers, this high reflectance beamsteering mirror gives R>99% 600-1000 nm at 0 deg AOI, Rp>98.5% 580-1000 nm at 45 deg AOI and Rs>99% 540-1000 nm. Careful thin film coating design and advanced coating processes result in a mirror with maximum reflectivity and bandwidth with minimum effect on pulse dispersion.

Group Delay Dispersion

The group delay dispersion (also sometimes called *second-order dispersion*) of an optical element is the derivative of the group delay with respect to the angular frequency, or the second derivative of the change in spectral phase:

$$D_2(\omega) = \frac{\partial T_g}{\partial \omega} = \frac{\partial^2 \varphi}{\partial \omega^2}$$

It is usually specified in fs^2 or ps^2. Positive (negative) values correspond to normal (anomalous) chromatic dispersion. For example, the group delay dispersion of a 1-mm thick silica plate is +35 fs^2 at 800 nm (normal dispersion) or −26 fs^2 at 1500 nm (anomalous dispersion). Another example is given in Figure 1.

If an optical element has only second order dispersion, i.e., a frequency-independent D_2 value, its effect on an optical pulse or signal can be described as a change of the spectral phase:

$$\Delta \varphi(\omega) = \frac{D_2}{2}(\omega - \omega_0)^2$$

where ω_0 is the angular frequency at the center of the spectrum.

Note that the group delay dispersion (GDD) always refers to some optical element or to some given length of a medium (e.g. an optical fiber). The GDD per unit length (in units of s^2/m) is the *group velocity dispersion* (GVD).

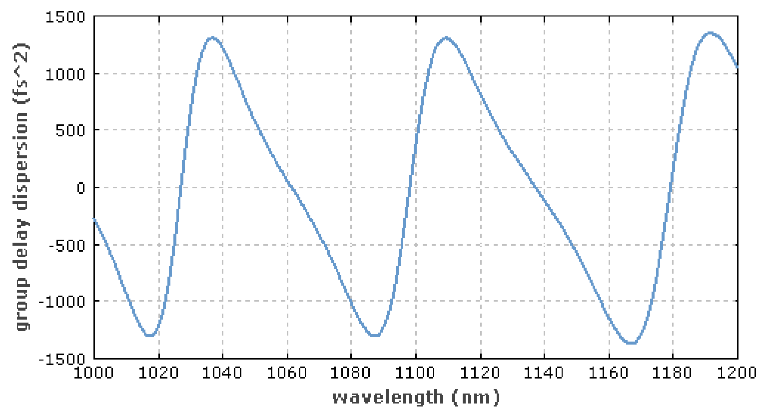
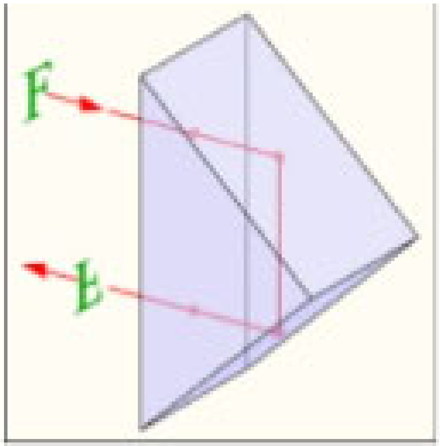


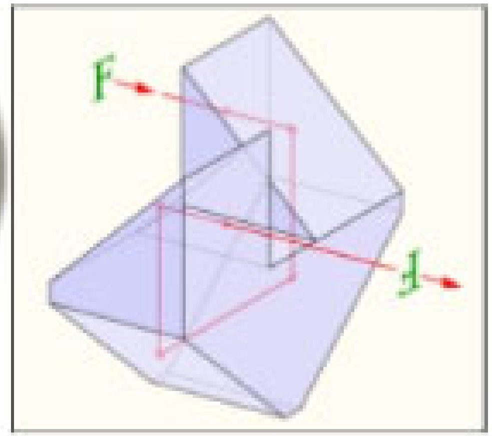
Figure 1: Wavelength-dependent group delay dispersion of a Gires–Tournois interferometer made of a 5-μm thick silica layer on a high reflector.

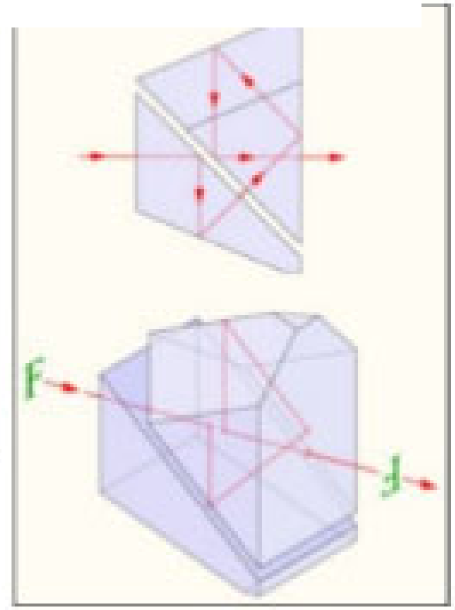
Other Types of Mirrors

- Other types of reflecting device are also called "mirrors".
- **Acoustic mirrors** are passive devices used to reflect and perhaps to focus sound waves. Acoustic mirrors were used for selective detection of sound waves, especially during World War II. They were used for detection of enemy aircraft, prior to the development of radar. Acoustic mirrors are used for remote probing of the atmosphere; they can be used to form a narrow diffraction-limited beam.[51] They can also be used for underwater imaging.
- **Active mirrors** are mirrors that amplify the light they reflect. They are used to make disk lasers.[52] The amplification is typically over a narrow range of wavelengths, and requires an external source of power.
- **Atomic mirrors** are devices which reflect matter waves. Usually, atomic mirrors work at grazing incidence. Such mirrors can be used for atomic interferometry and atomic holography. It has been proposed that they can be used for non-destructive imaging systems with nanometer resolution.[53]
- **Cold mirrors** are dielectric mirrors that reflect the entire visible light spectrum, while efficiently transmitting infrared wavelengths. These are the converse of hot mirrors.
- **Corner reflectors** use three flat mirrors to reflect light back towards its source, they may also be implemented with prisms that reflect using total internal reflection that have no mirror surfaces. They are used for emergency location, and even laser ranging to the Moon.
- **Hot mirrors** reflect infrared light while allowing visible light to pass. These can be used to separate useful light from unneeded infrared to reduce heating of components in an optical device. They can also be used as dichroic beamsplitters. (Hot mirrors are the converse of cold mirrors.)
- **Metallic reflectors** are used to reflect infrared light (such as in space heaters or microwaves).
- **Non-reversing mirrors** are mirrors that provide a non-reversed image of their subjects.
- **X-ray mirrors** produce specular reflection of X-rays. All known types work only at angles near grazing incidence, and only a small fraction of the rays are reflected.[54] See also X-ray optics.

Reflecting Prisms





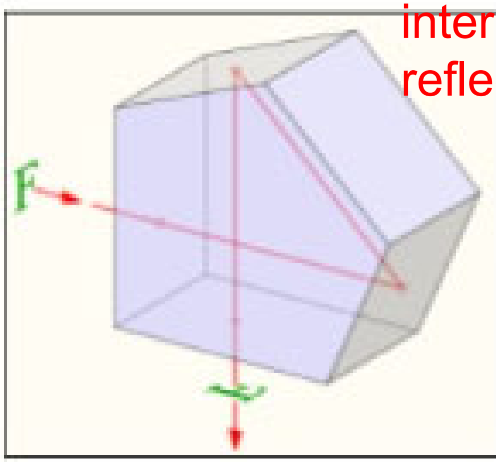


Porro Prism

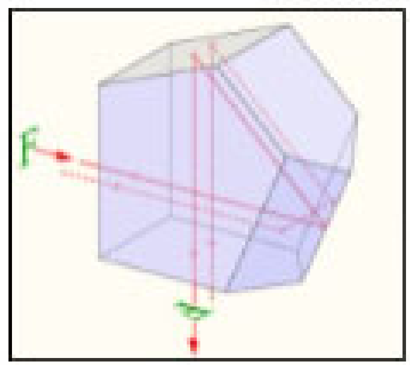
Total internal reflection

Porro-Abbe Prism

Schmidt-Pechan Prism



Penta Prism



Roof Penta Prism

20

Using prism as mirror. Using total reflection to avoid dispersion

alter the direction of light beams, to offset the beam, and to rotate or flip images

W.Wang

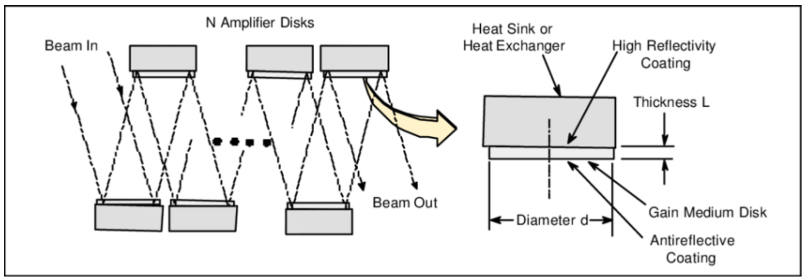
Acoustic Mirror

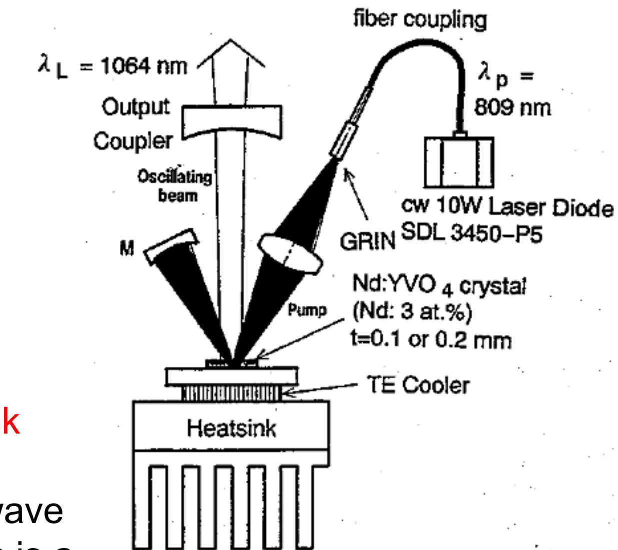
- WWI rconnasant acoustic mirror built as early warning system for aircraft attacks
- The British built an impressive system of acoustic mirrors for coastline defense along the English Channel before World War II. These mirrors consisted of large structures built of concrete. There were several different shapes as the British experimented with the best design, but each shape reflected sound waves and focused them in a particular area where a microphone or human listener could be located.

Photo by flickr user tobyct used under Creative Commons License

Google map

Active Mirror





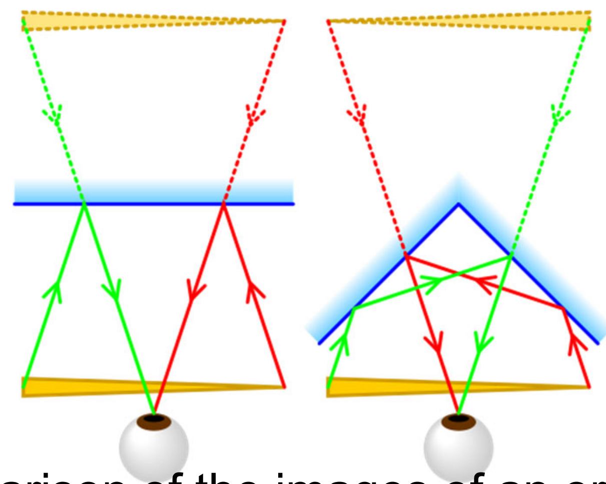
One type of solid-state laser designed for good power scaling is the disk laser (or "active mirror"). Such lasers are believed to be scalable to a power of several kilowatts from a single active element in continuous-wave operation. Perhaps, the expectations for power scalability of disk lasers is a little bit exaggerated: some of publications in favor of disk laser just repeat each other; compare, for example and these articles differ with only titles[neutrality is disputed].

Solidstate laser
Nd or Yb laser

Amplified spontaneous emission, overheating and round-trip loss seem to be the most important processes that limit the power of disk lasers. For future power scaling, the reduction of the round-trip loss and/or combining of several active elements is required.

Non- reversing Mirror

A non-reversing mirror (sometimes marketed as a true mirror) is a mirror that presents its subject as it would be seen from the mirror. A non-reversing mirror can be made by connecting two regular mirrors at their edges at a 90 degree angle. If the join is positioned so that it is vertical, an observer looking into the angle will see a non-reversed image. This can be seen in public toilets when there are mirrors on two walls which are at right angles. Looking towards the corner, such an image is visible. The problem with this type of non-reversing mirror is that there is a big line down the middle interrupting the image. However, if first surface mirrors are used, and care is taken to set the angle to exactly 90 degrees, the join can be made invisible. (wkipedia)



Comparison of the images of an ordinary mirror (left) and the first type of non-reversing mirror (right)

ordinary mirror (left) and two perpendicular mirrors forming the first type of non-reversing mirror (right)

One-Way Mirror

The secret is that it doesn't. A one-way mirror has a reflective coating applied in a very thin, sparse layer -- so thin that it's called a **half-silvered** surface. The name half-silvered comes from the fact that the reflective molecules coat the glass so sparsely that only about half the molecules needed to make the glass an opaque mirror are applied. At the molecular level, there are reflective molecules speckled all over the glass in an even film but only half of the glass is covered. (Howstuff works)

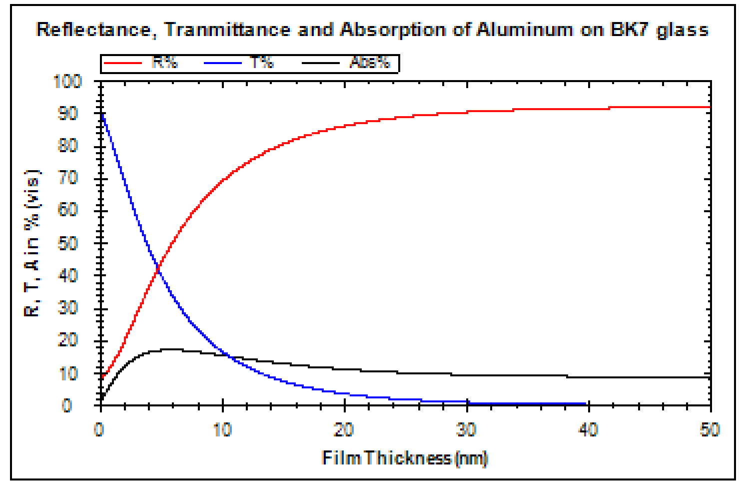
Toilette with mirrored walls

Airport security

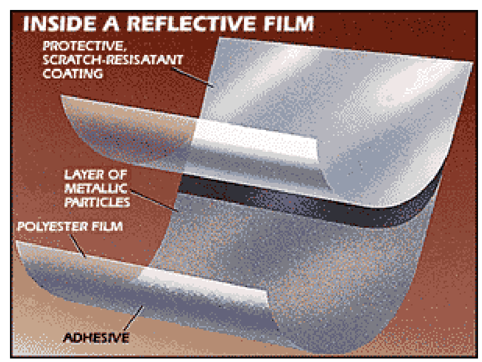
One Way Mirror

The glass is coated with, or has encased within, a thin and almost-transparent layer of metal (usually aluminum). The result is a mirrored surface that reflects some light and is penetrated by the rest.

A true one-way mirror does not, and cannot, exist.[2] Light always passes exactly equally in both directions. However, when one side is brightly lit and the other kept dark, the darker side becomes difficult to see from the brightly lit side because it is masked by the much brighter reflection of the lit side. It may be possible to achieve something similar by combining an optical isolator layer with a traditional one-way mirror, which would prevent light coming from one direction. (Wikipedia)



The optical properties of the mirror can be tuned by changing the thickness of the reflecting layer.



Solarfilmco One-Way Mirror Window Films

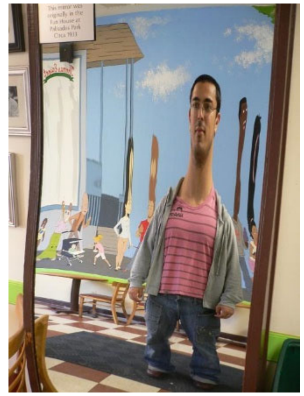
Example o with and without mirror window film

W.Wang

Other neat Mirror Tricks

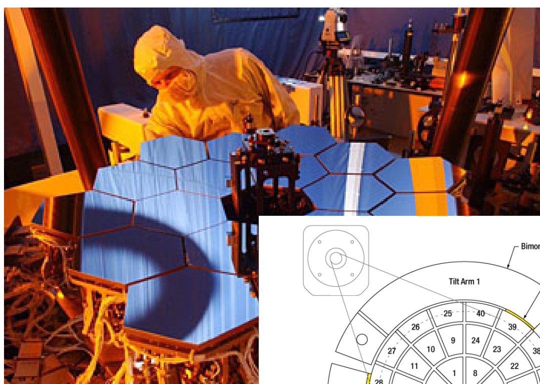
Perpendicular mirrors

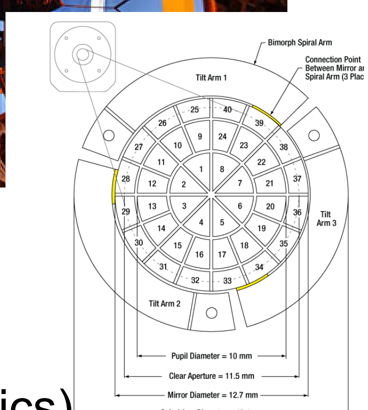
One way mirror



Funny mirror

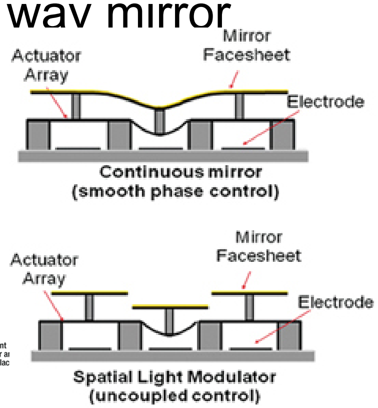
superpositi





Deformable mirrors
(adaptive optics)

w.wang



DLP

Cylindrical mirror

Design project 1

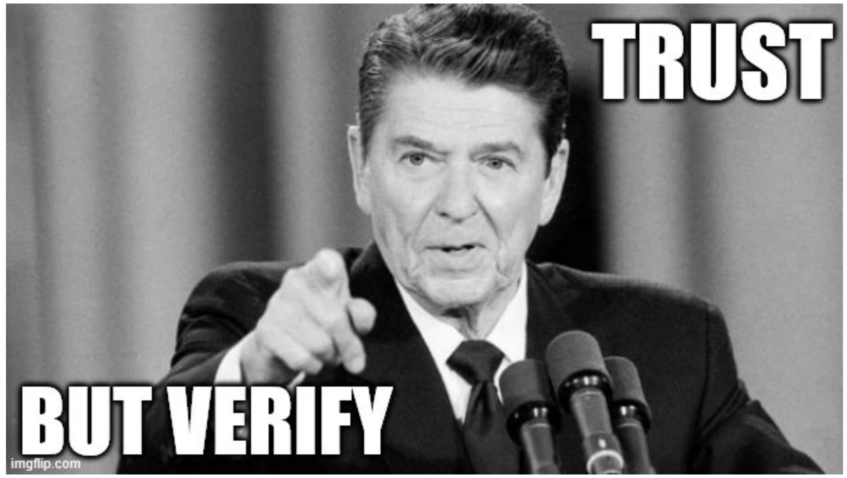
- Design a mirror system to focus, diverge or collimate an incident beam or create some interesting magic tricks or illusion or something you can think of. Please use ray tracing technique to illustrate your concept. (if you have absolute no idea what to do, let me know. I have ideas I like you to try)
- Use single or combination of mirrors to create the design. Grades depending on your creativity and effort you put into your design.
- Show all calculation and design.
- Please follow the Memo requirement instruction for the Memo preparation: http://courses.washington.edu/me557/optics/memo.doc
- Due on 3/20/23 Monday (powerpoint, memo and hardware)

Summary

- Introduction to light
  – Some basic concept of particle and wave behavior
  - quantum theory
- Ray optics
  - terminology
  - law of reflection (introduction and derivation)
  - Different optical reflective surfaces and structure and application (e.g. flat mirror, curve mirrors, one way mirror, prism, active mirror etc.)

Quote of the week

old Russian proverb:



Proverb rhymes in Russian – Doveryai.

It wasn't Reagan who came up with it, but Reagan learned of the Russian proverb when he was preparing for talks with Soviet leader Mikhail Gorbachev. Reagan's adviser on Russia's affairs, Suzanne Massie, suggested the president learn some Russian proverbs to amuse his counterpart. Turns out, Reagan liked "trust, but verify" the best.. **Reagan's favorite cliché. IMF treaty.**

Week 4

- Course Website: http://courses.washington.edu/me557/optics
- Reading Materials:
  - Week 3 and 4 reading materials can be found:
    http://courses.washington.edu/me557/readings/
- **Design project 1 is due 4/1. Prepare a 5 minutes PowerPoint presentation for the third hour and also send me an electronic copy of the PowerPoint slides and memo for the project (send me the PPT and memos electronically to abong@uw.edu).**
- First part of the Lab 1 starts this week (after the lecture)
  - Do problem 1 in assignment before doing lab 1.
- **Monday sometimes might be three hours if we need to finish up the lecture that week, but most of the time is 2 hours.**
- **Special Career presentation 3/25 (the week I will be away for conference)**
- **HW1 assigned due on 4/1**
- Useful website: http://refractiveindex.info/?shelf=organic&book=polycarbonate&page=Sultanova

MICRO TECH LAB
DEPARTMENT OF MECHANICAL ENGINEERING
UNIVERSITY OF WASHINGTON

Design project 1

- Design a mirror system to focus, diverge or collimate an incident beam or create some interesting magic tricks or illusion or something you can think of. Please use ray tracing technique to illustrate your concept. (if you have absolute no idea what to do, let me know. I have ideas I like you to try)
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- Due on 3/29 Monday (PowerPoint, memo and hardware)

This Week

- Ray optics

  - law of refraction (introduction and derivation)

  - Lenses (derivation of different refractive surface and structure)

  - Focal length, Diopter, Effective focal length, Numeric aperture, f-number, Acceptance angle, Depth of Field, circle of confusion, Depth of Focus

  - Matrix optics

- Non Ray Optics

  - Beam width

  - Performance factors

Recap

Geometric Optics

- Geometric optics is also called ray optics. Light travels in the form of rays. Ray optics only concern with the location and direction of light rays.
- Geometric optics completely ignore the finiteness of the wavelength (independent of λ)

Most of what we need to know about geometrical optics can be summarized in two rules:

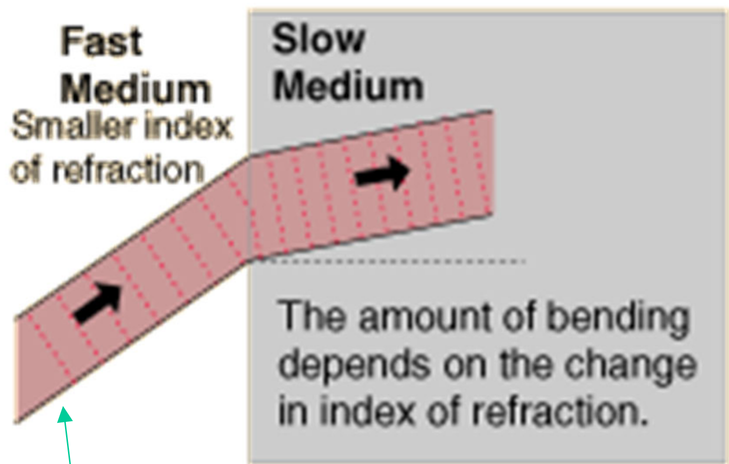
1) the laws of reflection : $|\theta_r| = |\theta_i|$

2) The law of refraction: Snell's law, or the law of refraction: $n_i \sin\theta_i = n_t \sin\theta_t$.

Recap

Index of Refraction

In a material medium the effective speed of light is slower and is usually stated in terms of the index of refraction of the medium. The index of refraction is defined as the speed of light in vacuum divided by the speed of light in the medium.



HyperPhysics

Photon In first matter

Photon in second matter

$$n = \frac{C_o}{C}$$

$$Speed\ of\ light\ in\ a\ medium, v = \frac{1}{\sqrt{\mu\epsilon}}$$

$$Speed\ of\ light\ in\ a\ vacuum, c = \frac{1}{\sqrt{\mu_o\epsilon_o}}$$

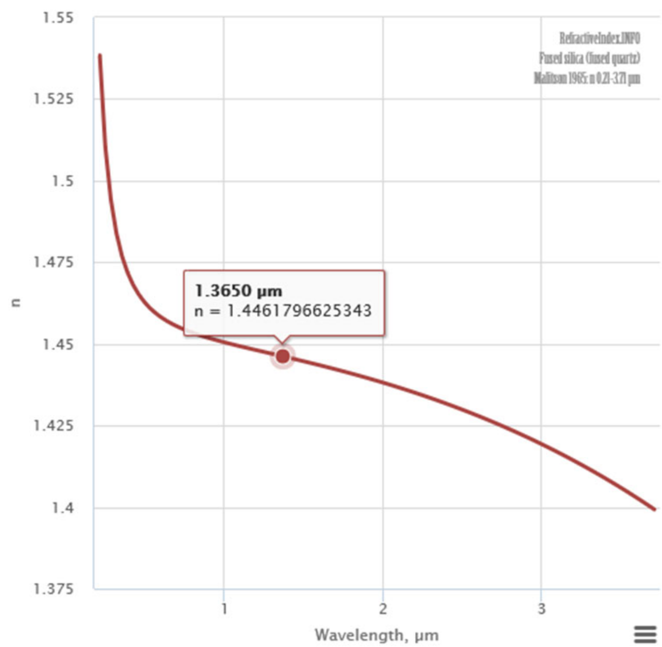
$$Refractive\ index, n = \frac{\sqrt{\mu\epsilon}}{\sqrt{\mu_o\epsilon_o}} = \sqrt{\mu_r\epsilon_r}$$

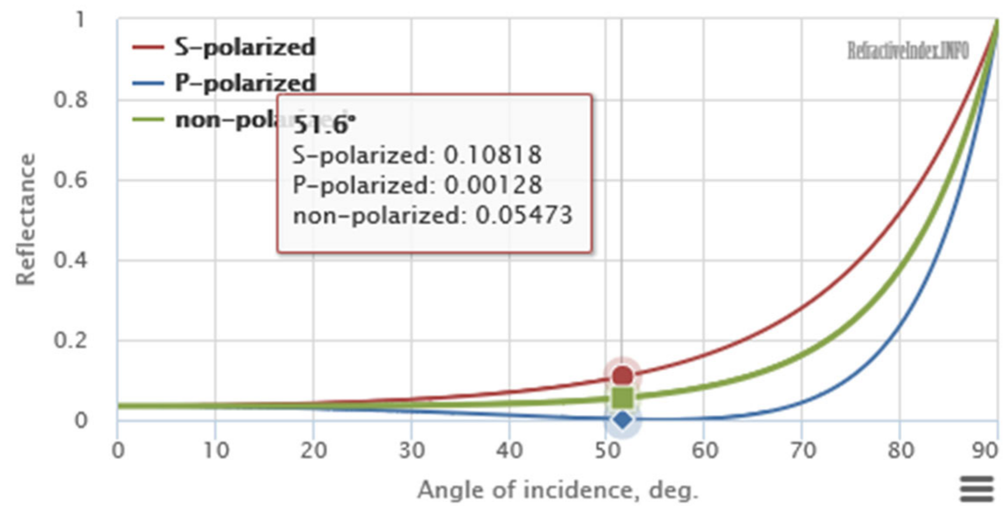
The indices of refraction of some common substances

Vacuum	1.000	Ethyl alcohol	1.362
Air	1.000277	Glycerine	1.473
Water	4/3	Ice	1.31
Carbon disulfide	1.63	Polystyrene	1.59
Methylene iodide	1.74	Crown glass	1.50-1.62
Diamond	2.417	Flint glass	1.57-1.75

The values given are approximate and do not account for the small variation of index with light wavelength which is called dispersion (n = a function of wavelength).

Optical constants of Fused silica (fused quartz)





http://refractiveindex.info/?shelf=organic&book=polycarbonate&page=Sultanova

Recall

Geometric Optics

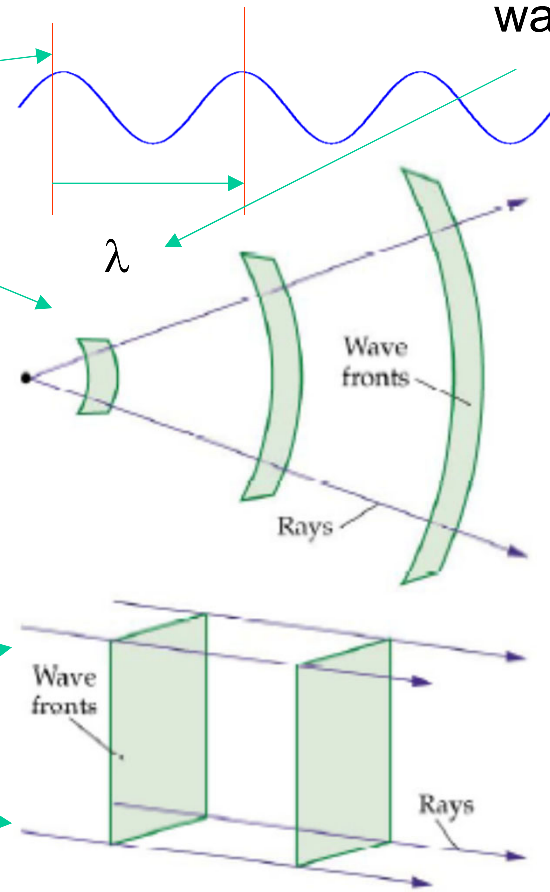
Wave front = wave crest

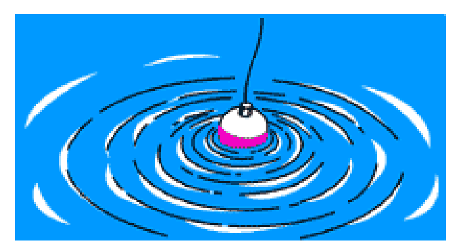
wavelength

To describe the light as ray, we need to borrow some of the properties from wave equation:

Wavefronts: a surface passing through points of a wave that have the same phase and amplitude

Rays (wave vector): a ray describe the direction of the wave propagation. A ray is a vector perpendicular to the wavefront



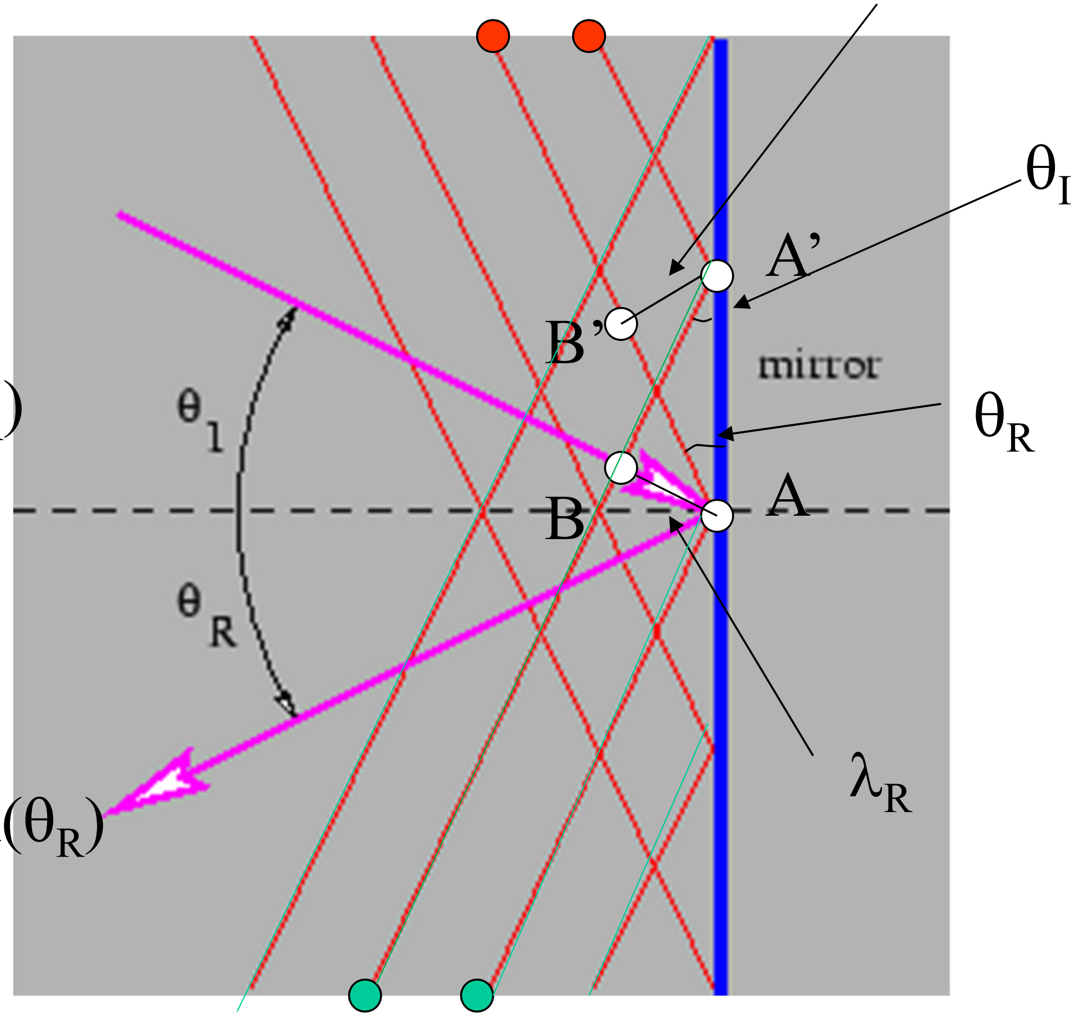


Earlier

Law of Reflection λ_I

λ_I=AB
AA’ = AB/sin(θ_I)

λ_R=A’B’
AA’ = A’B’/sin(θ_R)



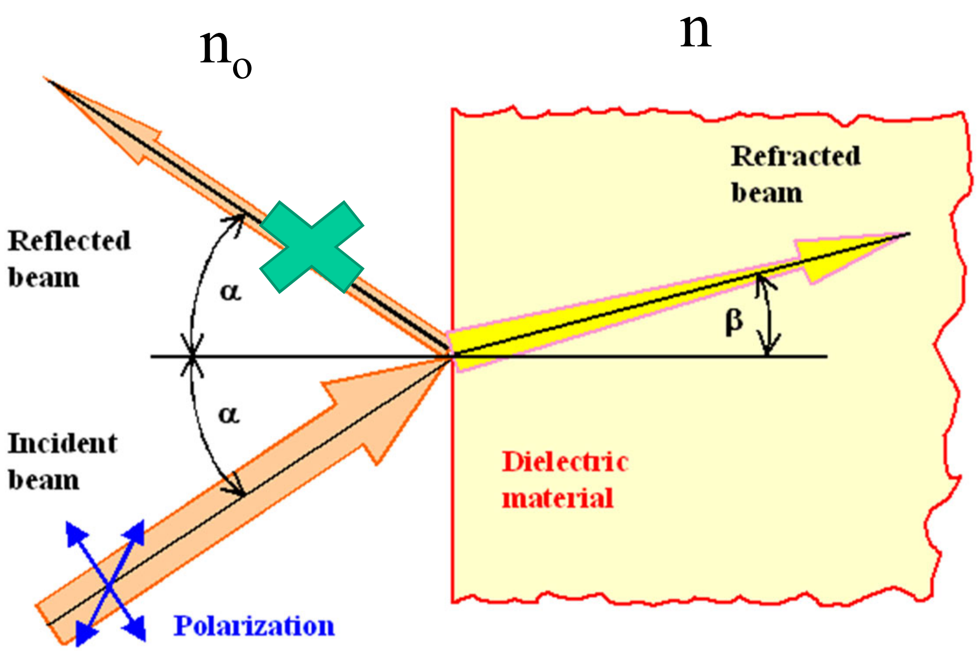
Since $\lambda_I = \lambda_R$
(elastic collision)
Then
$\lambda_I \sin(\theta_I) = \lambda_R \sin(\theta_R)$

So

$$\boxed{\theta_I = \theta_R}$$

W. Wang

Law of Refraction



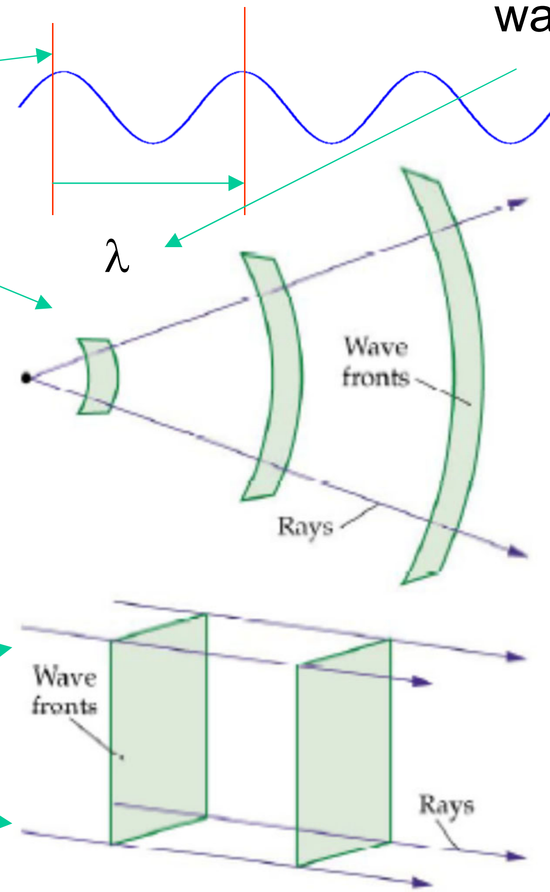
Snell's Law: $n_o \sin\alpha = n \sin\beta$

Recall

Geometric Optics

Wave front = wave crest

wavelength



To describe the light as ray, we need to borrow some of the properties from wave equation:

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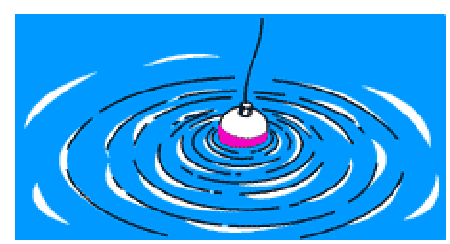
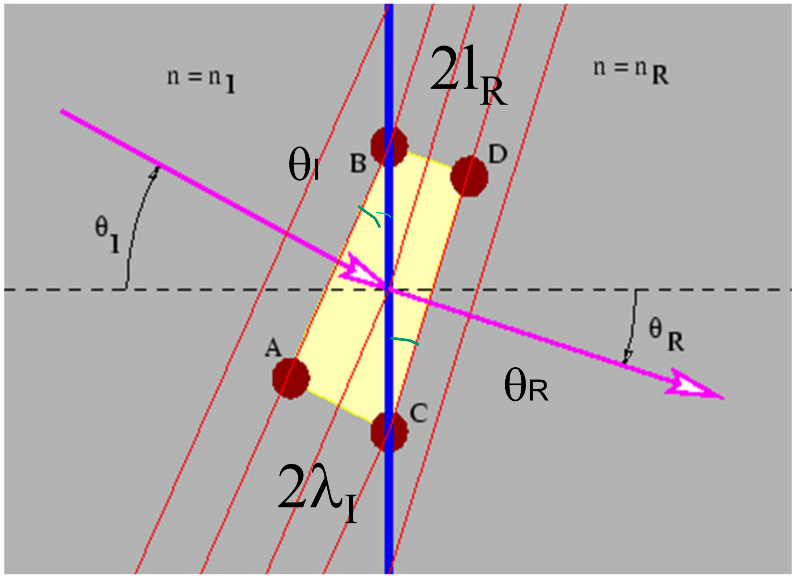


Law of Refraction



Since $n_R > n_I$, the speed of light in the right-hand medium (V_R) is less than in the left-hand medium (V_I). The frequency (f) of the wave packet doesn't change as it passes through the interface, so the wavelength (λ_R) of the light on the right side is less than the wavelength on the left side (λ_I). Recall -> $V(\lambda) = C_o / n(\lambda) = \lambda f / n(\lambda)$, *where* C_o, λ *are speed and wavelength in vacuum.*

The side AC is equal to the side BC times $\sin\theta_I$. However, AC is also equal to $2\lambda_I$, or twice the wavelength of the wave to the left of the interface. Similar reasoning shows that $2\lambda_R$, twice the wavelength to the right of the interface, equals BC times $\sin\theta_R$. Since the interval BC is common to both triangles, we easily see that

$$\frac{\lambda_I}{\lambda_R} = \frac{\sin\theta_I}{\sin\theta_R}$$

Since $\lambda_R = C_R/f$ *and* $\lambda_I = C_I/f$ => $\boxed{n_I \sin\theta_I = n_R \sin\theta_R}$

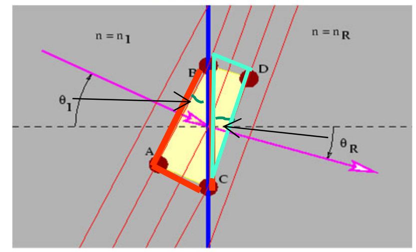
W. Wang

Wavefront Continuity

Reflection is ignored here!!

$$\boxed{AC = 2\lambda_I = BC\sin\theta_I} \Rightarrow BC = \frac{2\lambda_I}{\sin\theta_I}$$

$$\boxed{BD = 2\lambda_R = BC\sin\theta_R} \Rightarrow BC = \frac{2\lambda_R}{\sin\theta_R}$$



— (1)

Look at these three wave fronts

Therefore

$$\frac{2\lambda_I}{\sin\theta_I} = \frac{2\lambda_R}{\sin\theta_R}$$

$$\Rightarrow \boxed{\frac{\lambda_I}{\lambda_R} = \frac{\sin\theta_I}{\sin\theta_R}}$$

$n_R > n_I$

$\lambda_R < \lambda_I$

Since

$$\lambda_I = \frac{C_I}{f} = \frac{C_0}{n_I f}$$

$$\lambda_R = \frac{C_R}{f} = \frac{C_0}{n_R f} \quad \text{— (2)}$$

sub (2) into (1)

$$\Rightarrow \frac{\frac{\cancel{C_0}}{n_I \cancel{f}}}{\frac{\cancel{C_0}}{n_R \cancel{f}}} = \frac{\sin\theta_I}{\sin\theta_R} \Rightarrow \boxed{n_I \sin\theta_I = n_R \sin\theta_R}$$

Law of refraction

Recall

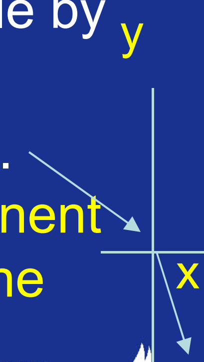
Corpuscular Model

The corpuscular model is the simplest model of light. According to the theory, a **luminous body emits stream of particles in all direction**. Issac Newton, in this book Opticks also wrote," Are not the ray of light every small bodies emitted from shinning substance?"

Based on this, light is assumed to be **consisted of very small particles so that when two light beams overlap**, a **collision between the two particles rarely occurs**. Using this model, one can explain the laws of reflection and refraction (Snell's law):

-Reflection law follows considering the elastic reflection of a particle by plane surface.

- Refraction law assume that the motion is confined to the xy plane. The trajectory of particle is determined by conservation of x component momentum (component parallel to the interface, perpendicular to the direction of propagation)



Recall

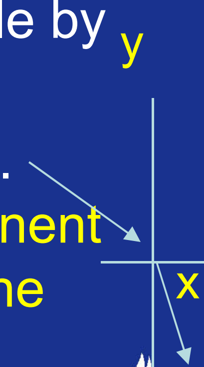
Corpuscular Model

The corpuscular model is the simplest model of light. According to the theory, a **luminous body emits stream of particles in all direction**. Issac Newton, in this book Opticks also wrote,” Are not the ray of light every small bodies emitted from shinning substance?”

Based on this, light is assumed to be **consisted of very small particles so that when two light beams overlap**, a **collision between the two particles rarely occurs**. Using this model, one can explain the laws of reflection and refraction (Snell’s law):

-Reflection law follows considering the elastic reflection of a particle by plane surface.

- Refraction law assume that the motion is confined to the xy plane. The trajectory of particle is determined by conservation of x component momentum (component parallel to the interface, perpendicular to the direction of propagation)



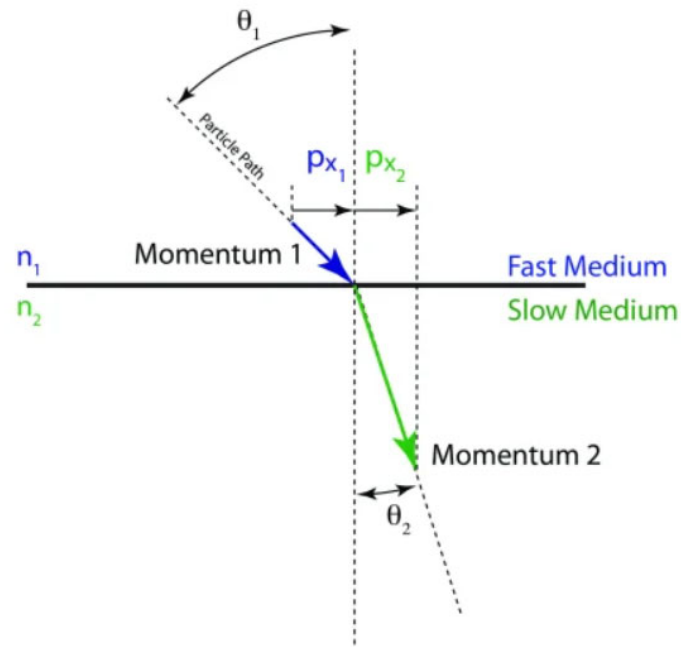
Conservation of Momentum

Going from Maxwell's equations for classical fields to photons keeps the same mathematical form for the transverse components for the k-vectors, but now interprets them in a different manner. Where before there was a requirement for phase-matching the classical waves at the interface, in the photon picture the transverse k-vector becomes the transverse momentum through de Broglie's equation

$$p = \hbar k$$

Therefore, continuity of the transverse k-vector is interpreted as conservation of transverse momentum of the photon across the interface. In the figure the second medium is denser with a larger refractive index n2 > n1. Hence, the momentum of the photon in the second medium is larger while keeping the transverse momentum projection the same

$$n_I sin\theta_I = n_R sin\theta_R$$



There are many ways to solve the law of reflection and refraction as you will see later using wave equations as well

Conservation of Energy and Momentum

$KE_o + Pe_o = KE_f + Pe_f$ Conservation of energy)

$P_o = P_f$ (Conservation of momentum)

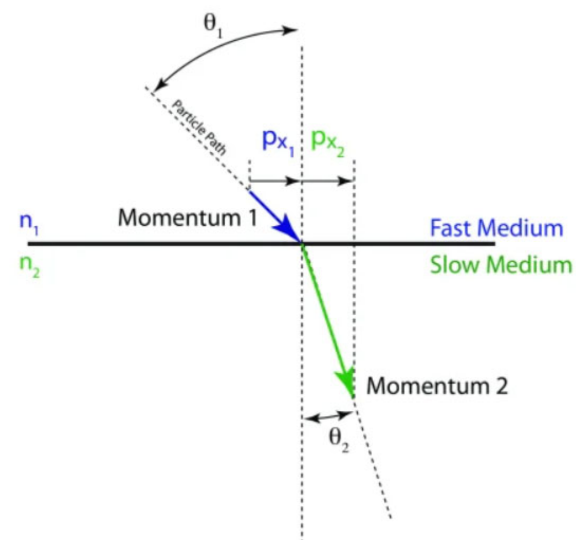
$KE = ½ mV^2$
$P = mV$

$V = f\lambda$ (speed of light)

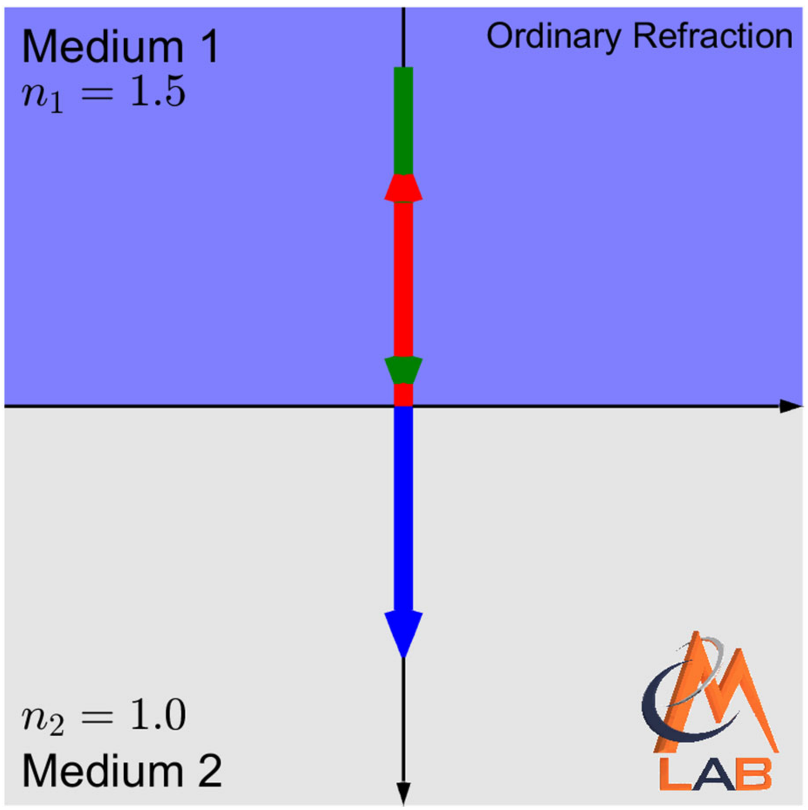
$\lambda_o = 2\pi/k_o$ or $\lambda_f = 2\pi/k_f$ (wavelength in term of wave vector)

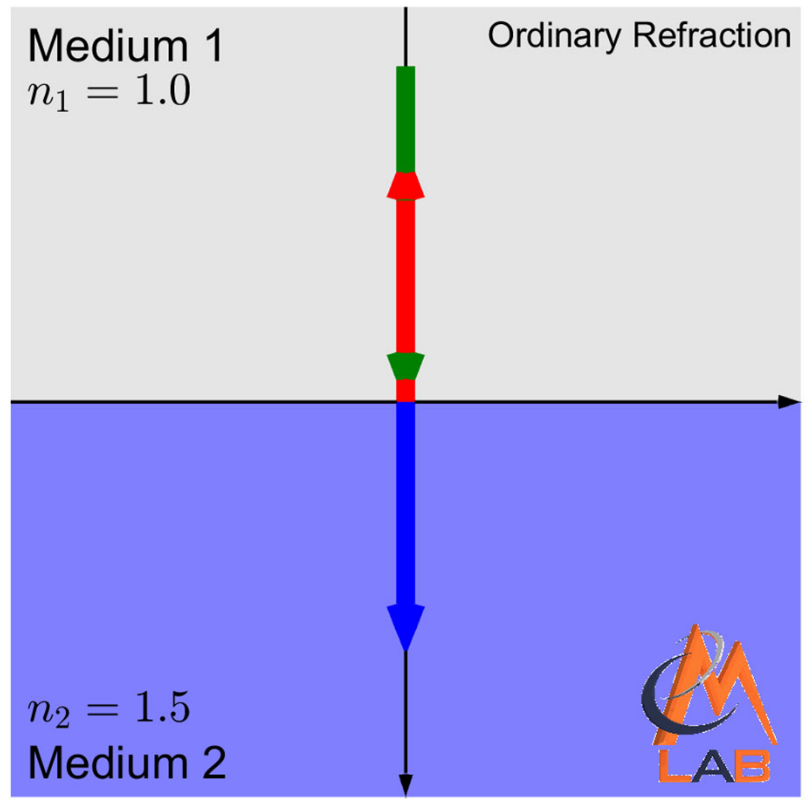
$k = k_o n\sin\theta$ (wave vector or propagation constant)

Use $P = mV \Rightarrow 2\pi f/k_o = 2\pi f/k_f \Rightarrow k_o = k_f$
Or use both energy and momentum in horizontal direction because that's only direction momentum is conserved no vertical we will get $n_I \sin\theta_I = n_R \sin\theta_R$



Refraction/Reflection at different interfaces



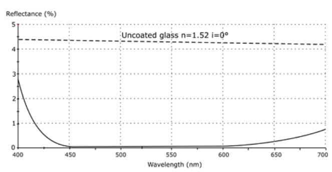


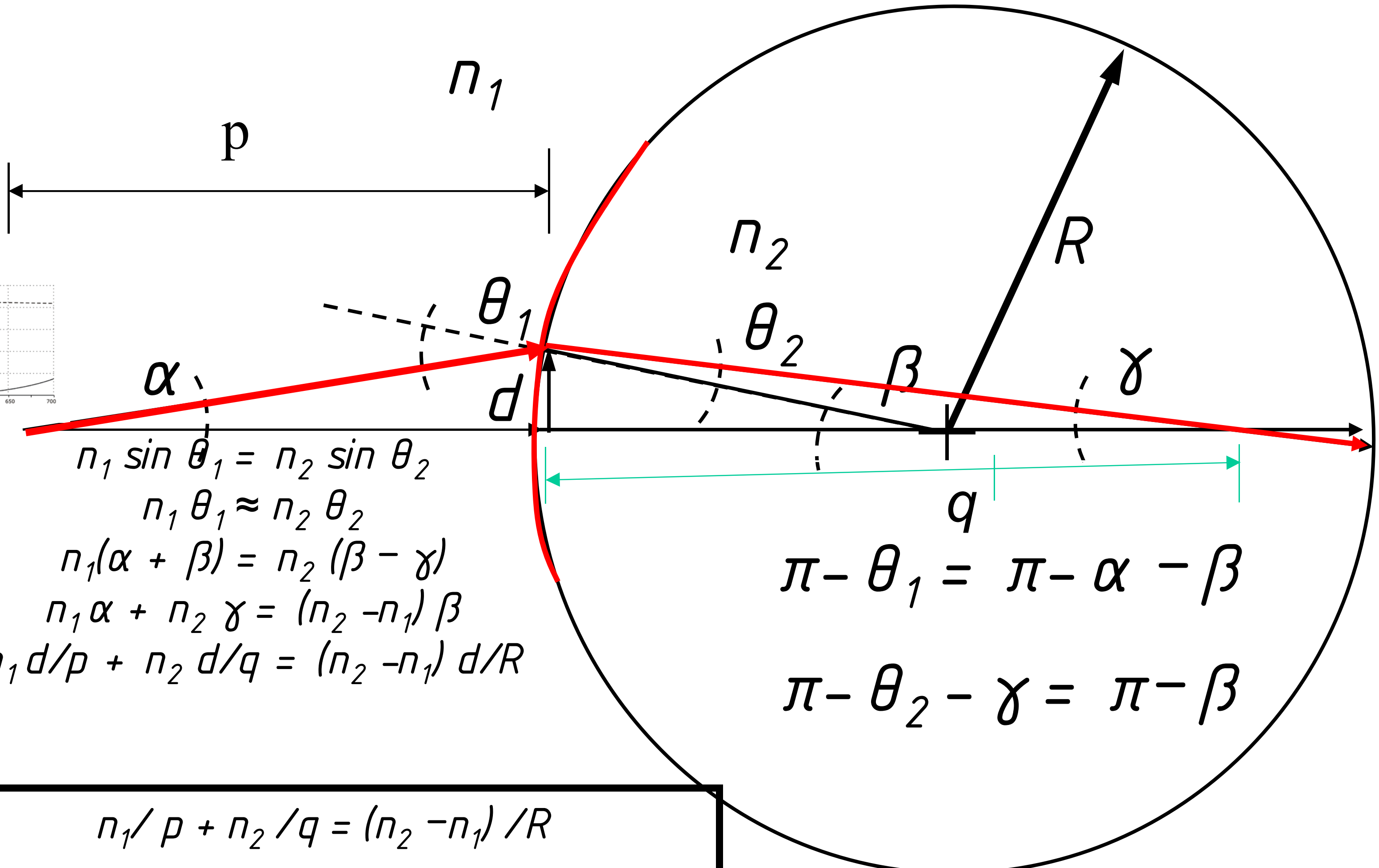
$n_I sin\theta_I = n_R sin\theta_R$

Recap

Spherical Boundary and Lenses

Refraction — *Spherical Surface*





$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$n_1\theta_1 \approx n_2\theta_2$$

$$n_1(\alpha + \beta) = n_2(\beta - \gamma)$$

$$n_1\alpha + n_2\gamma = (n_2 - n_1)\beta$$

$$n_1 d/p + n_2 d/q = (n_2 - n_1)\, d/R$$

$$\pi - \theta_1 = \pi - \alpha - \beta$$

$$\pi - \theta_2 - \gamma = \pi - \beta$$

$$\boxed{n_1/p + n_2/q = (n_2 - n_1)/R}$$

Curve surface refraction formula

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curve refraction surface formula

page 31

(law of refraction) $n_1 \sin\theta_1 = n_2 \sin\theta_2$

p- object length
q - image position

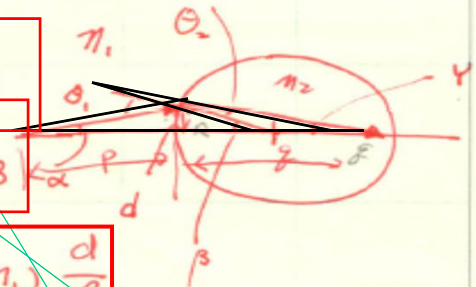
paraxial approximation $\sin\theta_1 \simeq \theta_1$, $\sin\theta_2 \simeq \theta_2$

$n_1 \theta_1 \simeq n_2 \theta_2$

supplementary angles $\boxed{n_1(\alpha+\beta) = n_2(\beta-\gamma)}$

$\boxed{n_1 \alpha + n_2 \gamma = (n_2 - n_1)\beta}$

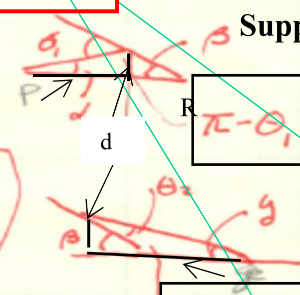
$\boxed{n_1 \frac{d}{p} + n_2 \frac{d}{q} = (n_2 - n_1)\frac{d}{R}}$



$$\boxed{\frac{n_1}{p} + \frac{n_2}{q} = \frac{(n_2 - n_1)}{R}}$$

Curve surface refraction formula

Supplementary Angles



$\boxed{\pi - \theta_1 = \pi - \alpha - \beta}$

$\Downarrow$

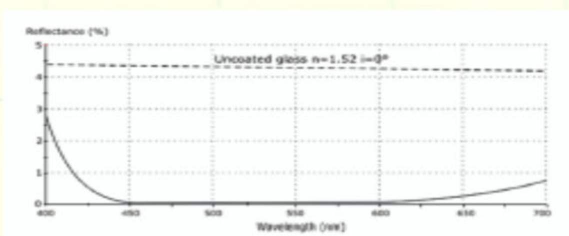
$\boxed{\theta_1 = \alpha + \beta}$

$\boxed{\pi - \beta = \pi - \theta_2 - \gamma}$

$\Downarrow$

$\boxed{\theta_2 = \beta - \gamma}$

reflection is ignore (assume small) (compare to refraction true if air to glass actual reflective Intensity < 4%)



Biconvex spherical lens

We use curve refraction surface formula

$$1/p_1 - n/q_1 = (n-1)/R_1$$

$$n/p_1 + 1/q_2 = (1-n)/R_2$$

and

Paraxial approximation
Rays travel close to optical axis
sinθ ~ θm and thin lens approximation

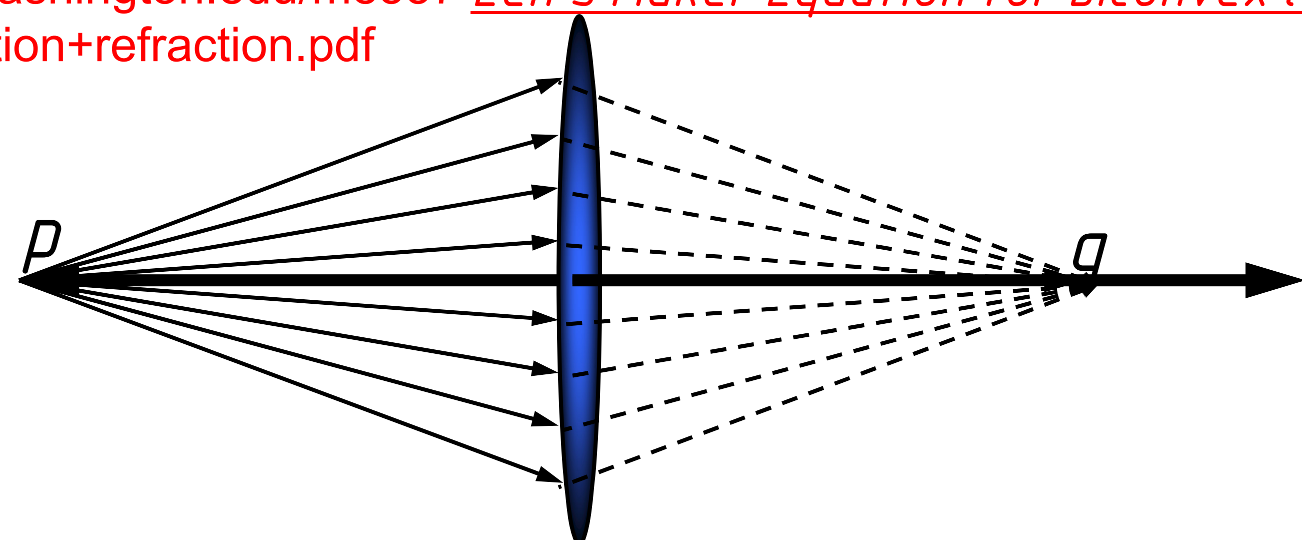
We get $1/p + 1/q = (n-1)\,(1/R_1 - 1/R_2)$

$$1/f = (n-1)\,(1/R_1 - 1/R_2)$$

$$1/p + 1/q = 1/f$$

Please read the hand written derivation for more detail:

http://courses.washington.edu/me557/readings/reflection+refraction.pdf *Len's Maker Equation for biconvex lens*



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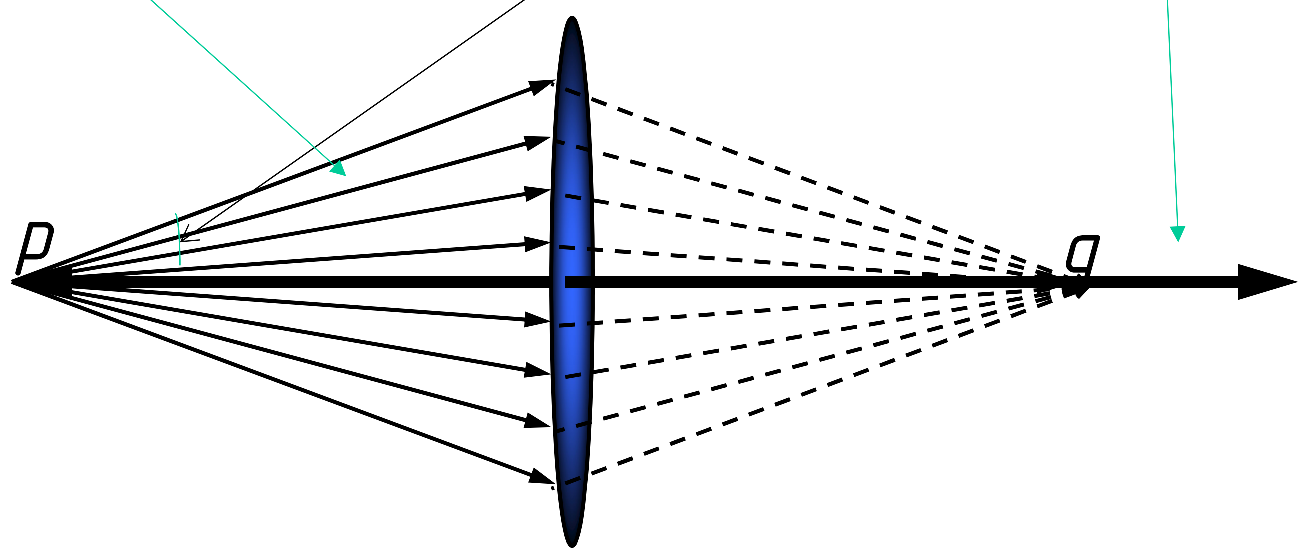
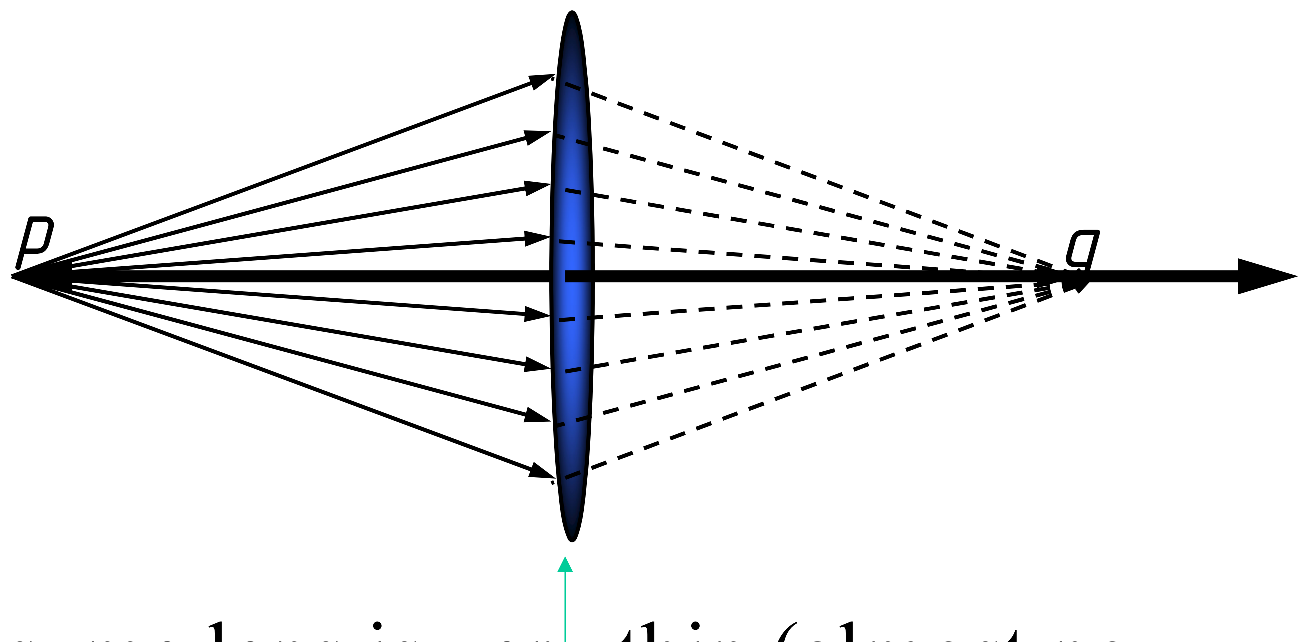
Paraxial Approximation

- Lens is much larger than incoming rays
- Ray makes small angles with optical axis

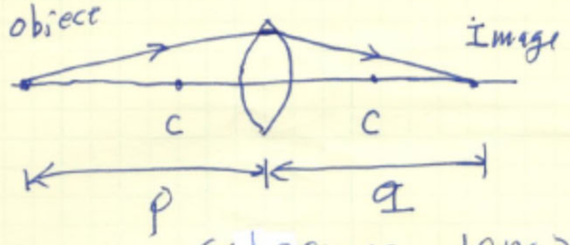
$$\sin\theta \sim \theta$$



Thin Lens Approximation



- Assume lens is very thin (almost no thickness), therefore we can ignore ray propagating inside the lens.



p= object length
q = image position

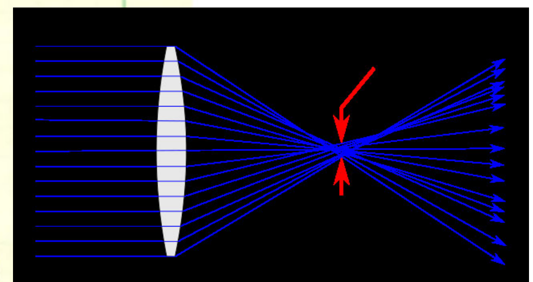
(biconvex lens)

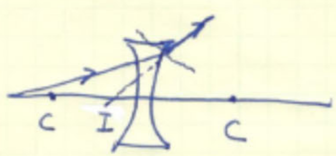
Thin lens approximation:

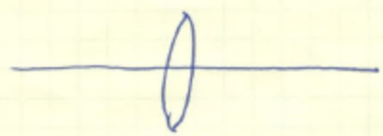
- Lens is assum very thin so that we can ignore the ray propagating inside the lens

bend only once

(assume) (actual)



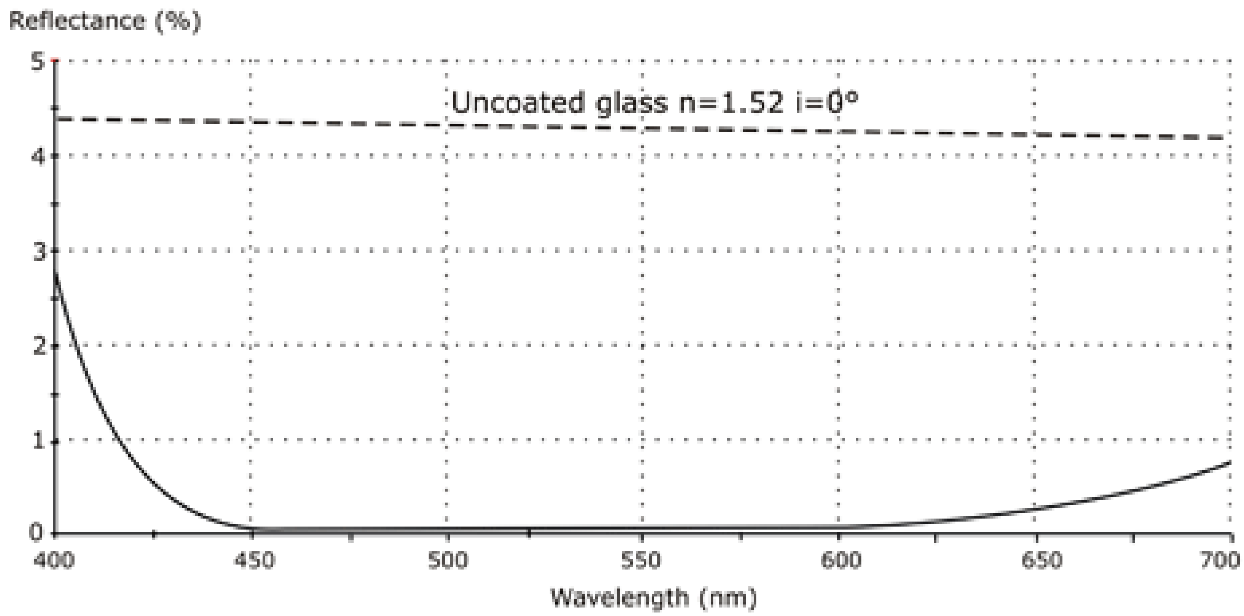




(biconcave lens)

Virtual image real object

Reflection is ignored



- Assume reflection between air and glass is small (~ 4% in visible range)

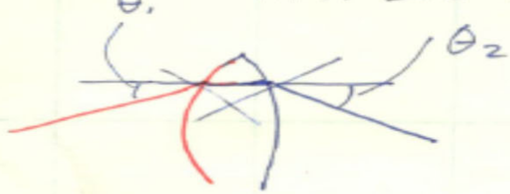
Derivation of Lens Equations

- Please read the hand written lecture notes on reflection and refraction in week1 for lens equation derivation:

depts.Washington.edu/me557/readings/reflection+refraction.pdf

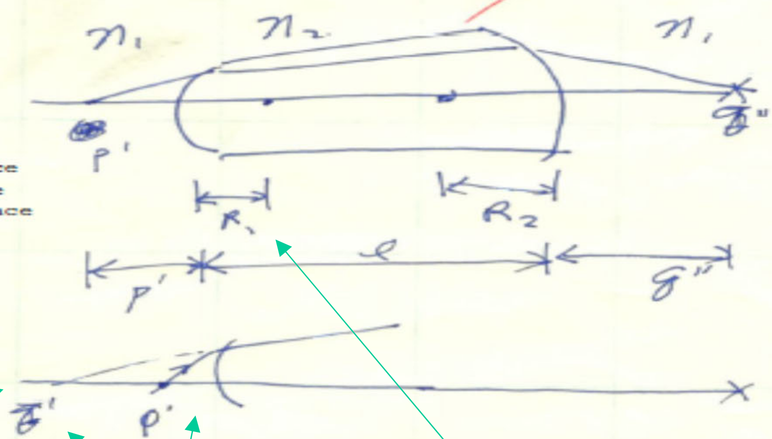
6. Thin Lens derivation

page 34



assume lens here

here 2 different spherical surface



p'- object length of first surface
q' - image position of first surface
p"- object length of second surface
q" - image position of second surface

image form outside ~~spherical~~ sphere

Very large incident angle so virtual image is formed

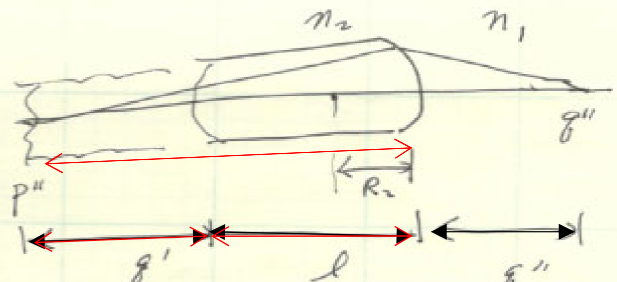
From previous page we know

$$\boxed{\frac{n_1}{p'} + \frac{n_2}{q'} = \frac{n_2 - n_1}{R_1}}$$

sub $n_1 = 1.0$ for air

$$\boxed{\frac{1}{p'} - \frac{n_2}{q'} = \frac{n_2 - 1}{R_1}} \quad \text{— (1)}$$

$q' = -q'$ (negative ~~image~~, image same side as object



since image from front side of lens is object

$$\boxed{p'' = q' + l}$$

q - back side Lens

recall

$$\frac{n_1}{p'} + \frac{n_2}{q'} = \frac{n_2 - n_1}{R_1}$$

sub $n_1 = 1.0$ for air

$$\boxed{\frac{1}{p'} - \frac{n_2}{q'} = \frac{n_2 - 1}{R_1}} \quad — \ (1)$$

$q' = -q'$ (negative ~~image~~, image same side as object

for second surface

$$\frac{n_2}{q'+l} + \frac{n_1}{q''} = \frac{n_1 - n_2}{R_2}$$

sub $n_1 = 1$ & $l \Rightarrow 0$ for thin lens approx.

$$\boxed{\frac{n_2}{q'} + \frac{1}{q''} = \frac{1 - n_2}{R_2}} \quad — \ (2)$$

sub (1) into (2)

$$\frac{1-n_2}{R_1} + \frac{1}{p'} + \frac{1}{q''} = \frac{1 - n_2}{R_2}$$

$$\frac{1}{p'} + \frac{1}{q''} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

Calling original object distance simply $p' = p$
& final image distance simply $q'' = q$

calling original object distance $p' = p$
& final image distance $q'' = q$ we get,

$$\boxed{\frac{1}{p}+\frac{1}{q}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)} \quad (3)$$

Recall

If $p = \infty$ ← Incident beam is collimated

$p=\infty$ $\quad$ $q=f$

$q = f$

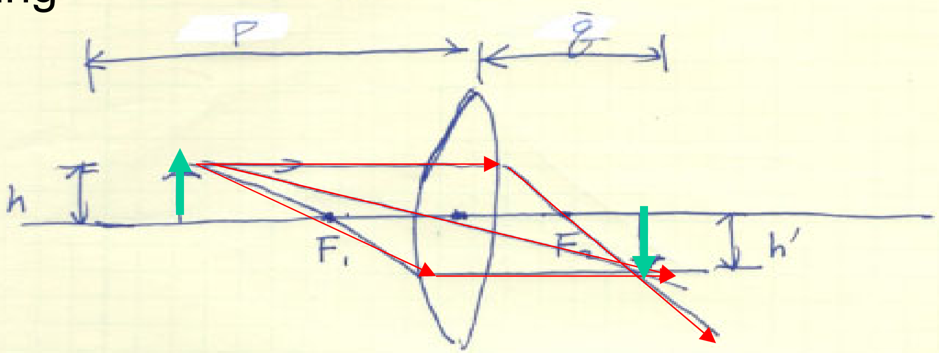
focal length

sub into (3)

$$\boxed{\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)} \quad (4)$$

$$\boxed{\frac{1}{p}+\frac{1}{q}=\frac{1}{f}}$$

Ray tracing

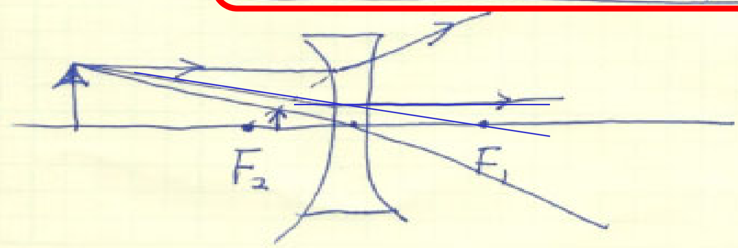


How to find image :

all base on Law of refraction

- parallel ray go thru lens intersects Focal pt on the opposite side
- ray intersects focal pt on the front side & become parallel ray
- ray go thru center of the lens

$$\boxed{m = \frac{h'}{h} = -\frac{q}{p}}$$



Thin Lens (Relat f, p, q to m)

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q}$$

$$\frac{1}{f} = \frac{q+p}{pq}$$

since $m = \frac{q}{p} \Rightarrow q = pm$

$$f = \frac{pq}{p+q} = \frac{p^2 m}{p(1+m)} = \frac{pm}{1+m} \quad — ①$$

$$= \frac{pm(1+m)}{(1+m)^2}$$

$$= \frac{pm(1+\frac{q}{p})}{(1+m)^2}$$

$$= \frac{m(p+q)}{(1+m)^2} \quad — ②$$

$$= \frac{(p+q)}{m+2+\frac{1}{m}} \quad — ③$$

Combin eq ① & ②

$$\frac{pm}{1+m} = \frac{m(p+q)}{(1+m)^2} \Rightarrow p(1+m) = (p+q) — ④$$

Image plane and Focal point

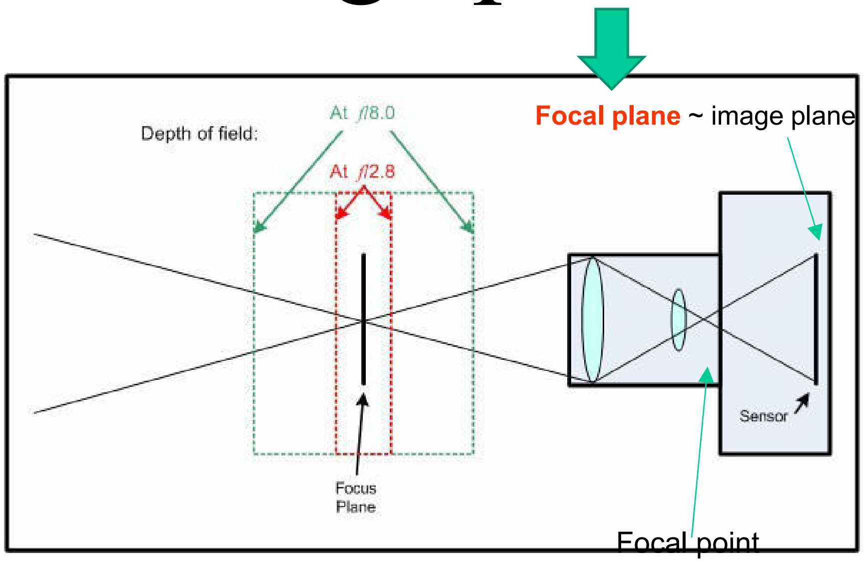
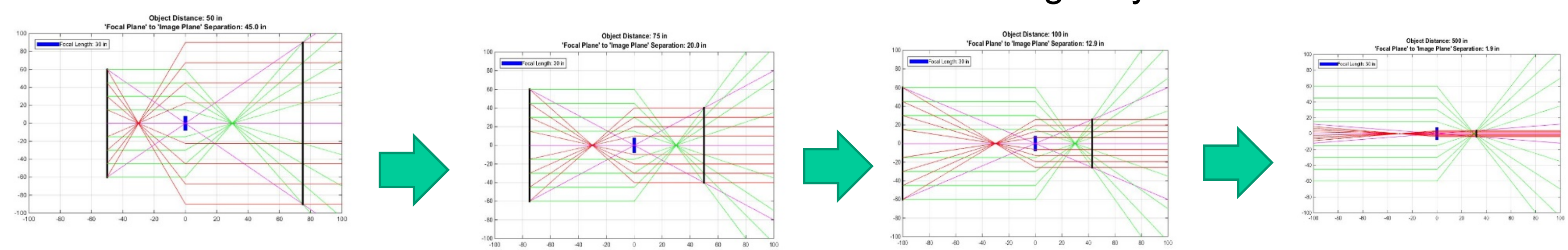


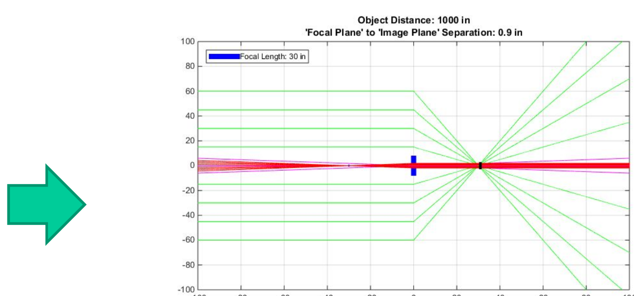
Image plane is same as focal plane but not the same as focal point

Image plane is where we can see the Focused image not at focal point!

Green- focal rays
Red- image rays



Example of an image plane getting closer and closer to focal point when object is moving further and further away



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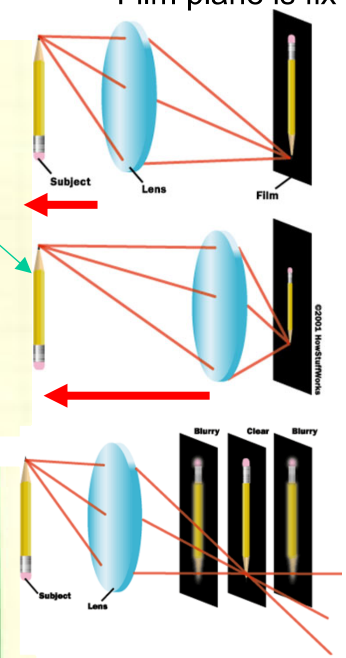
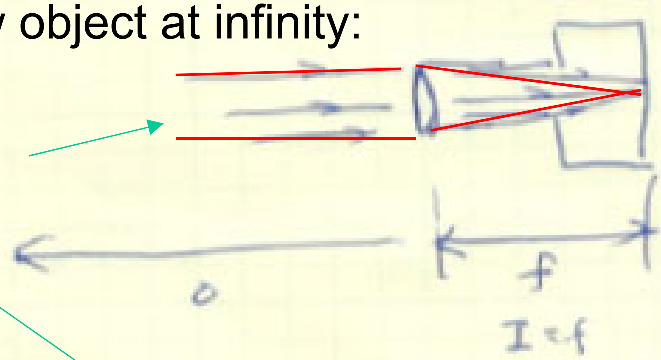
Difference between Camera Lens and Our Eye

Assume we have a camera with **50mm fixed (prime)** lens and we like to find out how far the lens needs to move from film plan when the object moves from infinity to 100cm away from lens

Film plane is fix

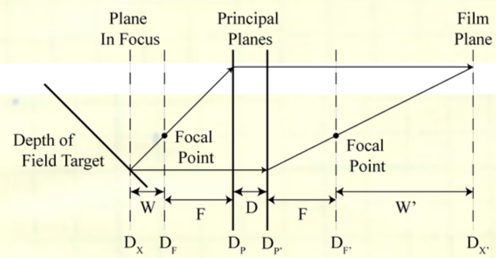


We know object at infinity:

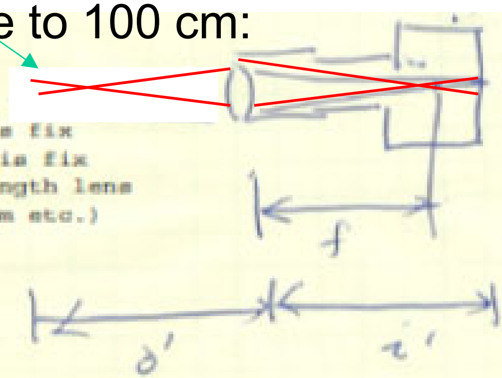


intersecting lines from the object top and bottom point

$$\frac{1}{f} = \frac{1}{i} + \frac{1}{o}$$



When object move to 100 cm:



Lens curvature is fix
so focal length is fix
For fix focal length lens
(e.g. 50mm, 100mm etc.)
not zoom lens

$$\frac{1}{f} = \frac{1}{i'} + \frac{1}{o'}$$

Moving imaging plane

– focal length doesn't move (because same length)
– image length move

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$$\frac{1}{f} = \frac{1}{o'} + \frac{1}{i'}$$

$$\frac{1}{5} = \frac{1}{100} + \frac{1}{i'}$$

$$\frac{19}{100} = \frac{1}{i} \qquad i' = 5.26 \text{ (cm)}$$

$$i' - f = 0.26 \text{ (cm)}$$

o' now $= 100$ cm

where $i' = ?$

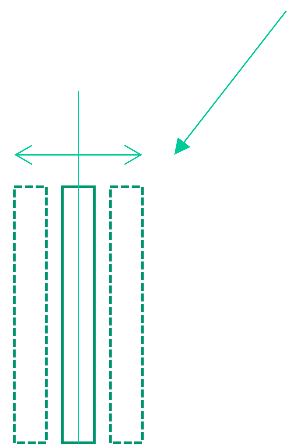
how far lens moves ?

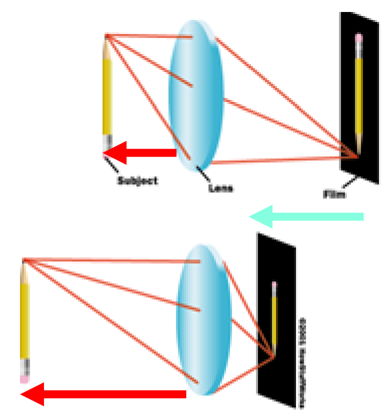
$$\begin{array}{r} 5.26 \\ 19\overline{)100} \\ 95 \\ \hline 50 \\ 38 \\ \hline 120 \\ 114 \\ \hline 60 \end{array}$$

image length move when object moves

Special Camera
(Lens doesn’t move, image “film” plane moves)

Contax AX
- Lens doesn’t move
- Film plane moves





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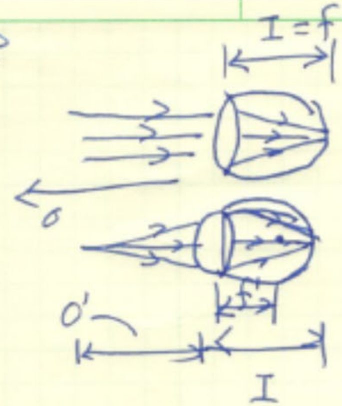
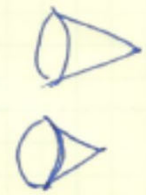
eye



- Image length doesn't change
- focal length change



our eye lens' curvature change
so focal length change

If object moves closer, how does eye adjust to see image clearly

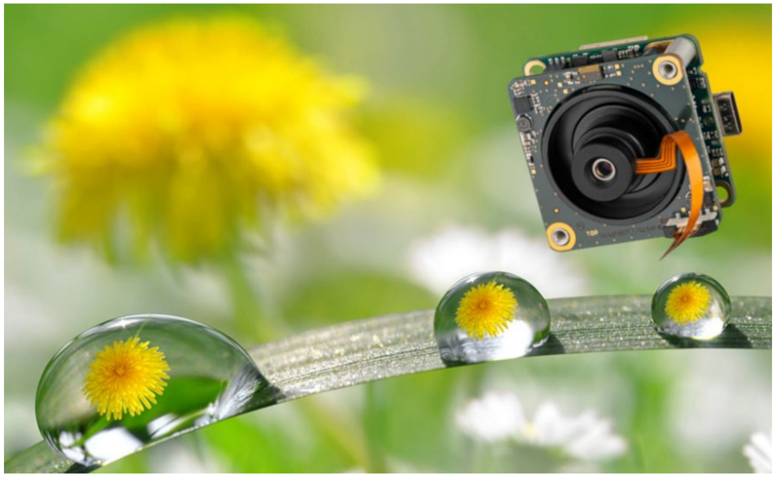
far sight

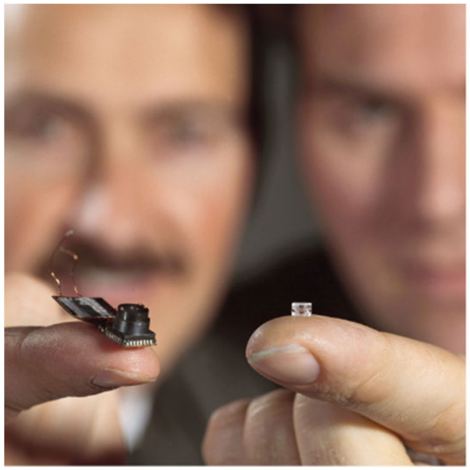
near sight

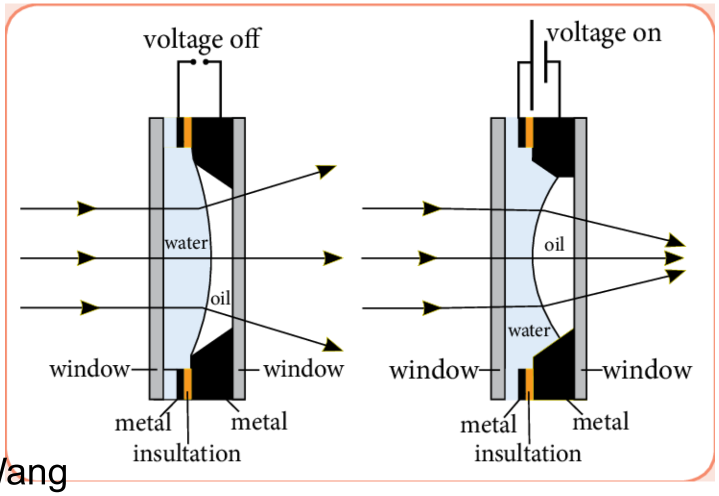
For eye image plane is fixed so we change Lens curvature change to change the focal length

Dial Vision

Electro wet Lens







The contact angle of a two-fluid interface with a solid surface is determined by the balance of the forces at the contact point. In electrowetting the balance of forces at the contact point is modified by the application of a voltage between a conducting fluid and the solid surface.

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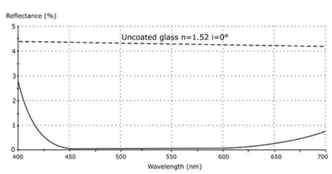
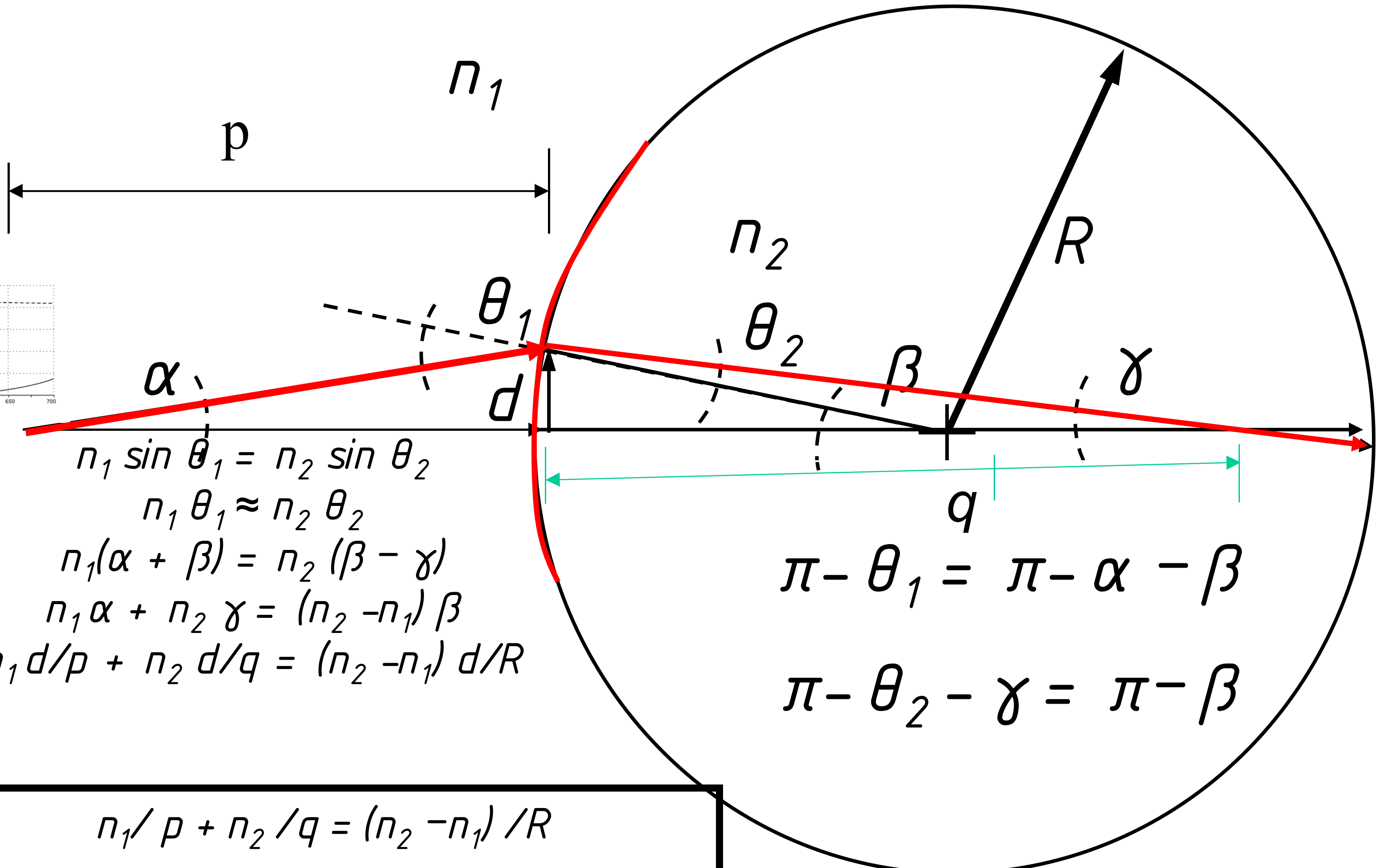
Recap

Spherical Boundary and Lenses

Refraction

Spherical Surface





$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$n_1\theta_1 \approx n_2\theta_2$$

$$n_1(\alpha+\beta) = n_2(\beta-\gamma)$$

$$n_1\alpha + n_2\gamma = (n_2-n_1)\beta$$

$$n_1 d/p + n_2 d/q = (n_2-n_1)\, d/R$$

$$\pi-\theta_1 = \pi-\alpha-\beta$$

$$\pi-\theta_2-\gamma = \pi-\beta$$

$$\boxed{n_1/p + n_2/q = (n_2-n_1)/R}$$

Curve surface refraction formula

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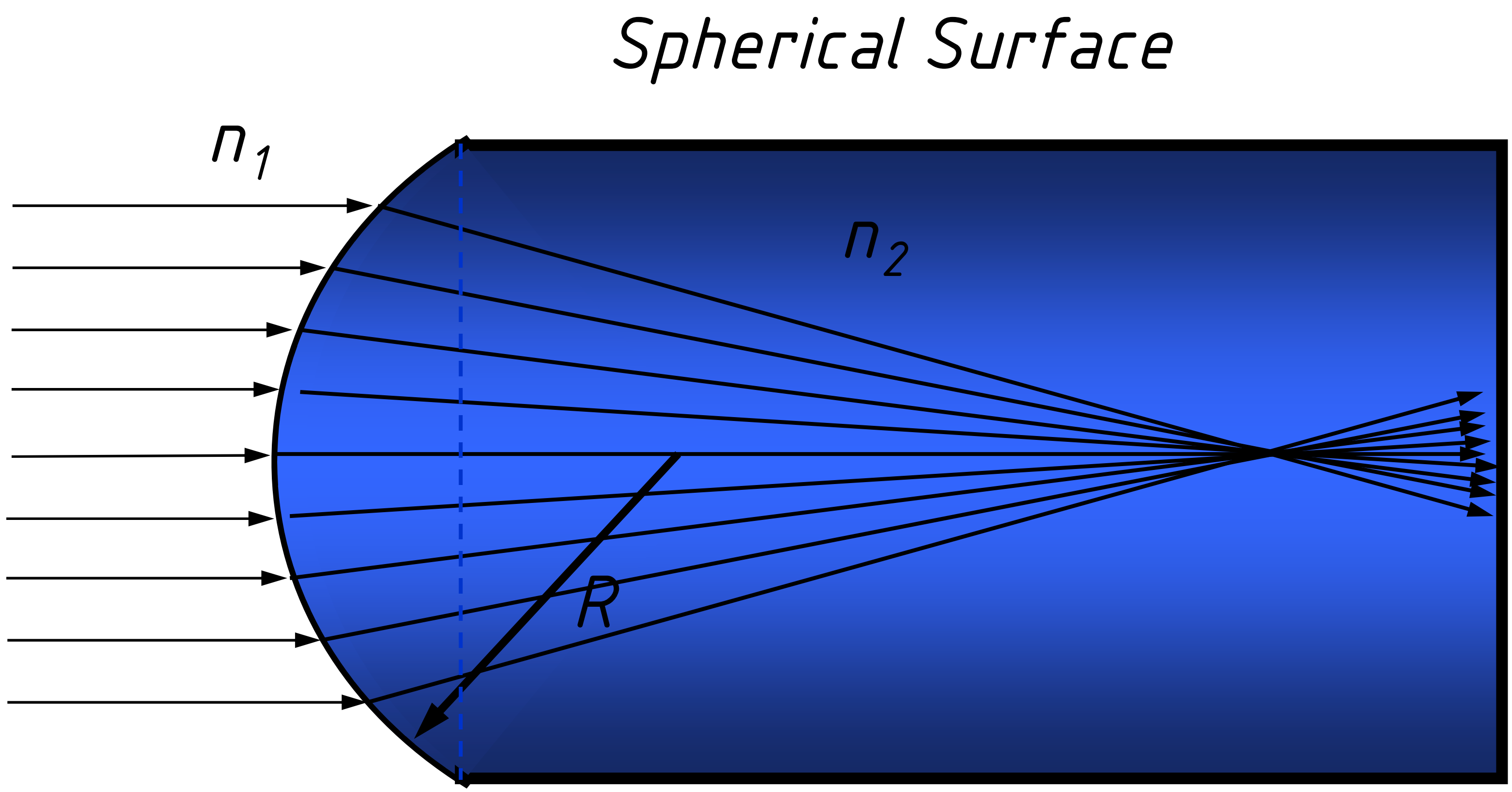
Lens bounded by spherical surface and flat surface
Refraction



$n_1/p + n_2/q = (n_2 - n_1)/R$

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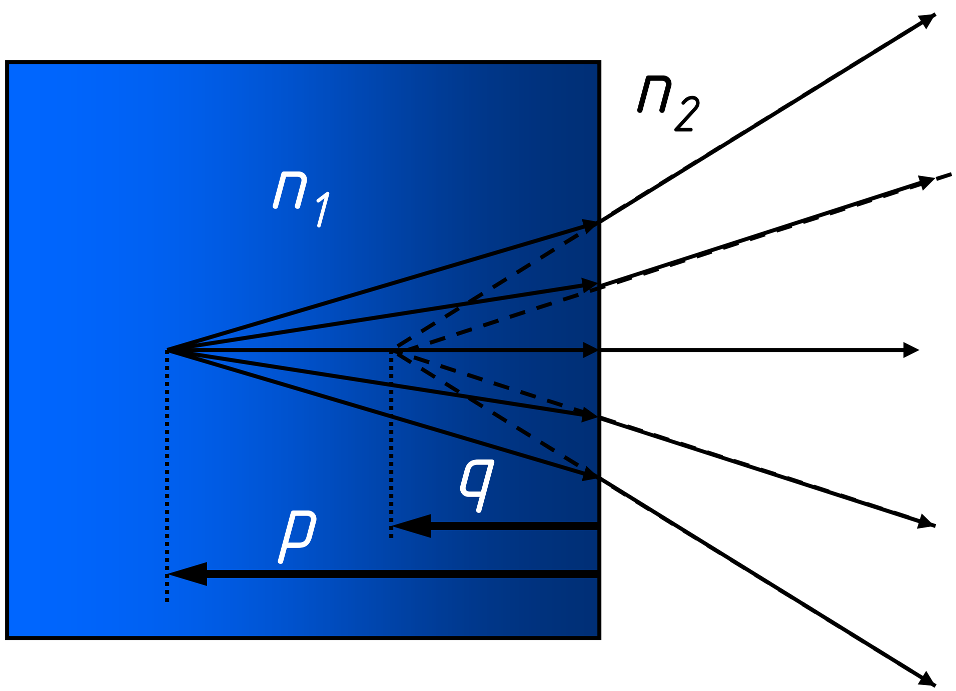
Flat Refracting Surface



$n_1/p + n_2/q = 0$

R> > >

$q = -(n_2/n_1)p$

f= infinity

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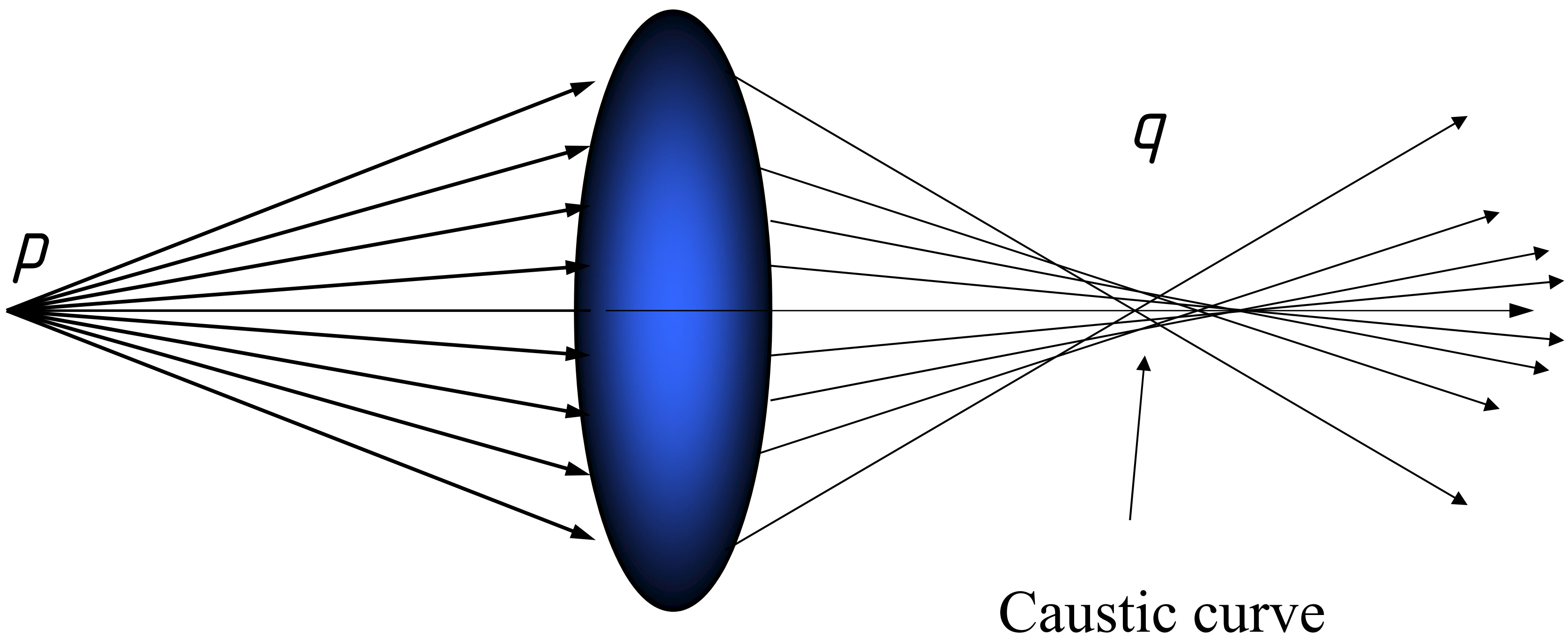
Thick Lens



Nonparaxial rays do not meet at the paraxial focus.

Biconvex spherical lens

We use curve surface refractive formula

$$1/p_1 - n/q_1 = (n-1)/R_1$$

$$n/p_1 + 1/q_2 = (1-n)/R_2$$

and

Paraxial approximation
Rays travel close to optical axis
$\sin\theta \sim \theta$m and thin lens approximation

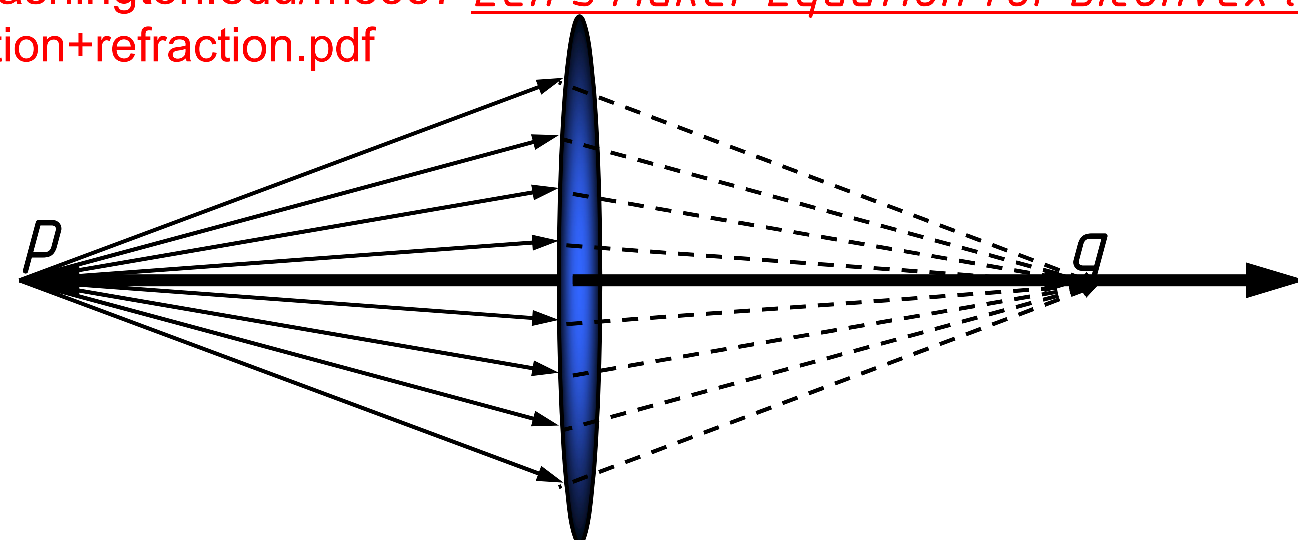
We get $1/p + 1/q = (n-1)\,(1/R_1 - 1/R_2)$

$$1/f = (n-1)\,(1/R_1 - 1/R_2)$$

$$1/p + 1/q = 1/f$$

Please read the hand written derivation for more detail:

http://courses.washington.edu/me557/readings/reflection+refraction.pdf *Len's Maker Equation for biconvex lens*



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Thin Lens

The Thin Lens Equation:

$$1/f = 1/p + 1/q$$

Lateral Magnification:

$$M = h'/h$$
$$M = -q/p$$

Focal length in terms of M:

$$f = pM/(1+M)$$
$$= M(p+q)/(1+M)^2$$
$$= (p+q)/(M+2+1/M)$$

Please see hand written handout for derivation

Object to image distance in terms of M:

$$p(1+M) = (p+q)$$

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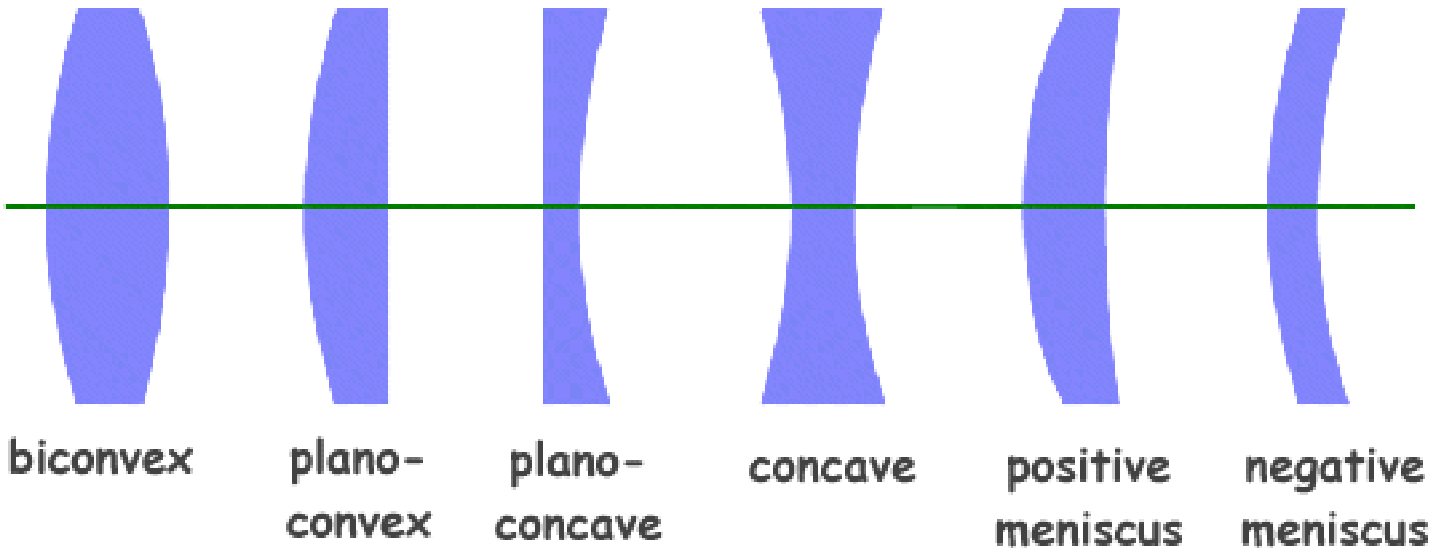
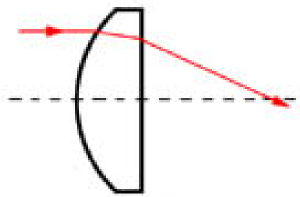
Different Types of Lenses

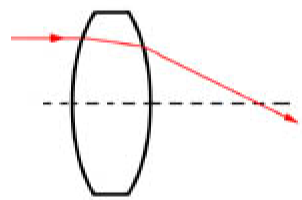


Double
concave

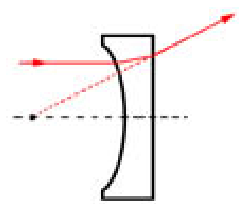
Plano-convex



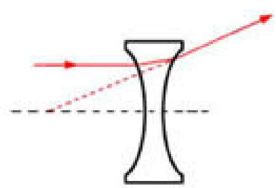
Double-convex



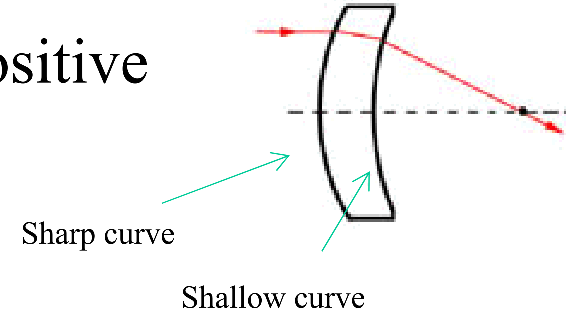
Plano-concave



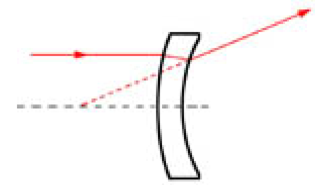
Double concave



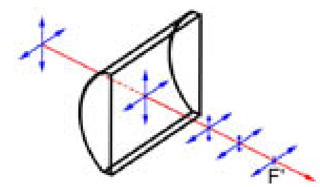
Meniscus positive



Meniscus negative

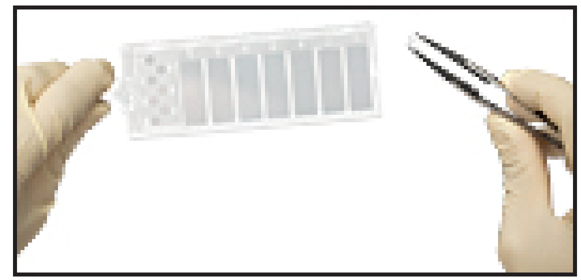


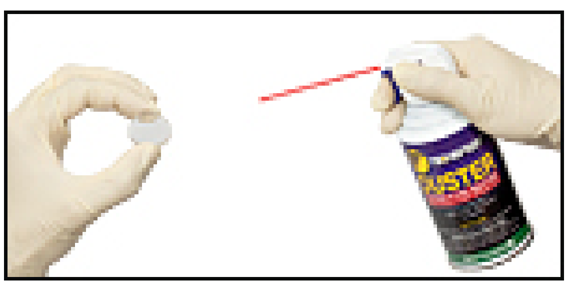
cylindrical



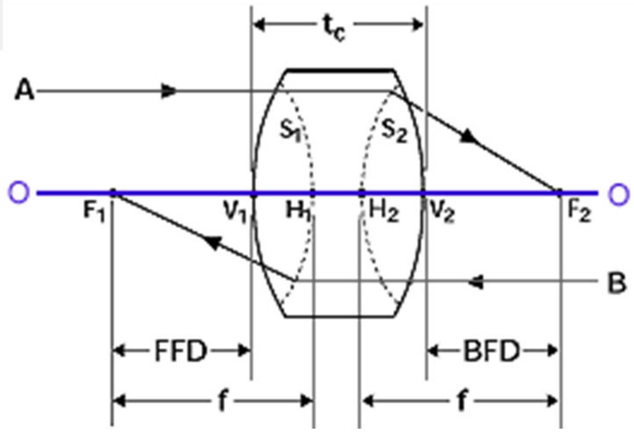
Don’t clean the lens with lens tissue paper!

Cleaning Procedure





Lens Terminology



The following definitions refer to the singlet lens diagram shown left. In the paraxial limit, however, *any* optical system can be reduced to the specification of the positions of the principal

F1
Front Focal Point: Common focal point of all rays B parallel to optical axis and travelling from right to left.
V1
Front Vertex: Intersection of the first surface with the optical axis.
S1
First Principal Surface: A surface defined by the intersection of incoming rays B with the corresponding outgoing rays focusing at F1.
H1
First Principal Point: Intersection of the first principal surface with the optical axis.
f
Effective Focal Length: The axial distance from the principal points to their respective focal points. This will be the same on both sides of the system provided the system begins and ends in a medium of the same index.
FFD
Front Focal Distance: The distance from the front vertex to the front focal point
F2
Back Focal Point: Common focal point of all rays A parallel to optical axis and travelling from left to right.
V2
Back Vertex: Intersection of the last surface with the optical axis.
S2
Second Principal Surface: A surface defined by the intersection of rays A with the outgoing rays focusing at F2.
H2
Second Principal Point: Intersection of the second principal surface with the optical axis.
BFD
Back Focal Distance: The distance from the back vertex to the back focal point.

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Principle Focal Length

- Principal focal length of a lens is determined by the index of refraction of the glass, the radii of curvature of the surfaces, and the medium in which the lens resides.

Diopter

- A dioptre (uk), or diopter (us), is a unit of measurement of the optical power of a lens or curved mirror, which is equal to the reciprocal of the focal length measured in meters (that is, 1/meters). It is thus a unit of reciprocal length. For example, a 3-dioptre lens brings parallel rays of light to focus at 1⁄3 metre. A flat window has an optical power of zero dioptres, and does not converge or diverge light.

Thick Lens

TYPE	ORIENTATION	Focal Length f	BFD	FFD
GENERAL	$R_1 = R_1$, $R_2 = R_2$	$\left[(n-1)\bullet\left(\frac{1}{R_1}-\frac{1}{R_2}\right)+\frac{t_c(n-1)^2}{nR_1R_2}\right]^{-1}$	$f\bullet\left[1-\frac{t_c(n-1)}{nR_1}\right]$	$f\bullet\left[1+\frac{t_c(n-1)}{nR_2}\right]$
PLANO-CONVEX	$R_1 = R$, $R_2 = \infty$	$\frac{R}{n-1}$	$f\bullet\left[1-\frac{t_c(n-1)}{nR_1}\right]$	f=infinity
PLANO-CONCAVE	$R_1 = -R$, $R_2 = \infty$	$\frac{-R}{n-1}$	$f\bullet\left[1+\frac{t_c(n-1)}{nR_2}\right]$	f= infinity
BI-CONVEX	$R_1 = R$, $R_2 = -R$	$\left[\frac{2(n-1)}{R}-\frac{t_c(n-1)^2}{nR^2}\right]^{-1}$	$f\bullet\left[1-\frac{t_c(n-1)}{nR_1}\right]$	$f\bullet\left[1-\frac{t_c(n-1)}{nR_1}\right]$
BI-CONCAVE	$R_1 = -R$, $R_2 = R$	$-\left[\frac{2(n-1)}{R}-\frac{t_c(n-1)^2}{nR^2}\right]^{-1}$	$f\bullet\left[1+\frac{t_c(n-1)}{nR_2}\right]$	$f\bullet\left[1+\frac{t_c(n-1)}{nR_2}\right]$

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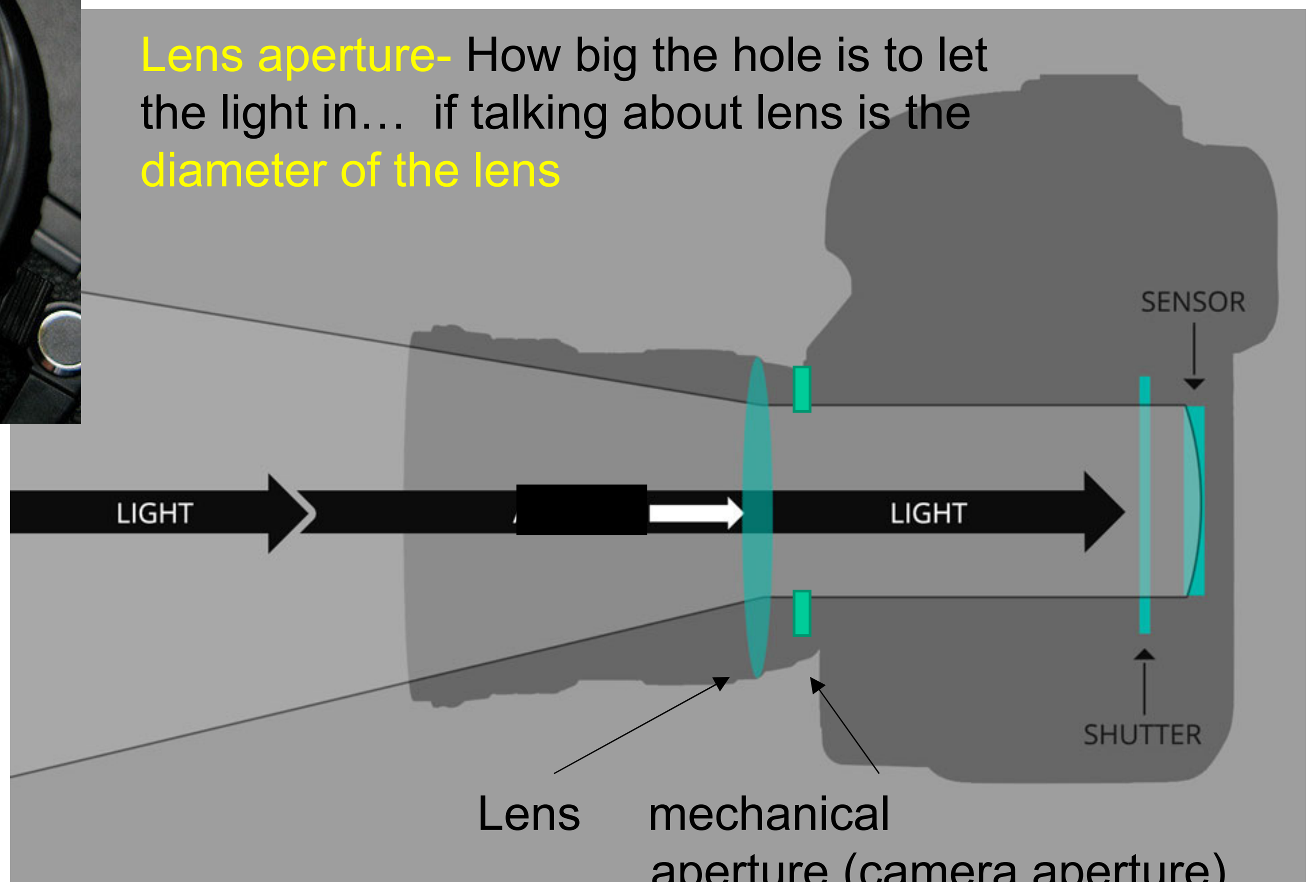
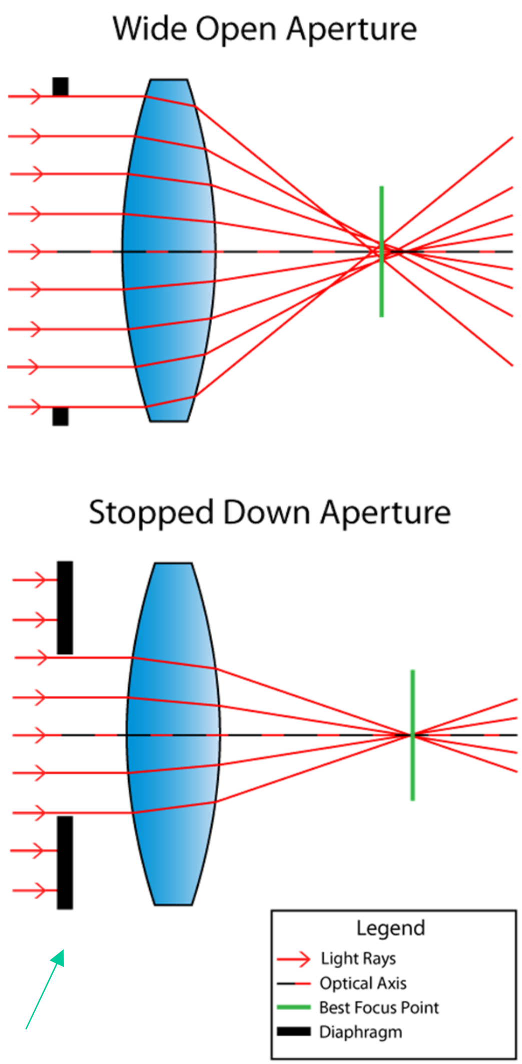
Lens and Aperture

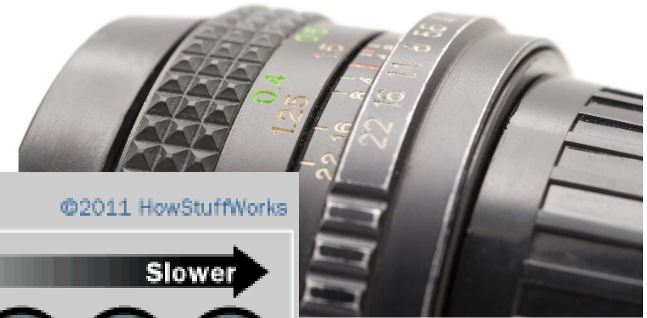
Lens aperture- How big the hole is to let the light in… if talking about lens is the diameter of the lens

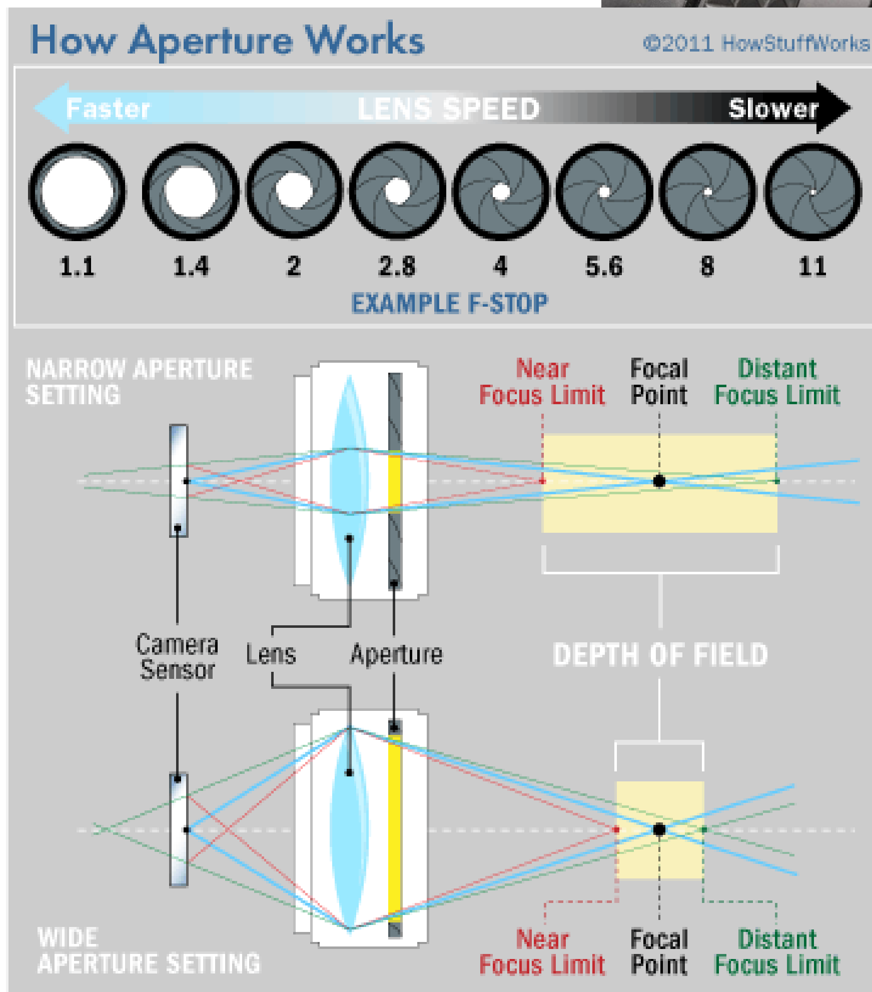


Lens mechanical aperture (camera aperture)

Camera Lens Aperture







Like paraxial effect Make image more clear more focus

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F-number and Numerical Aperture of Lens

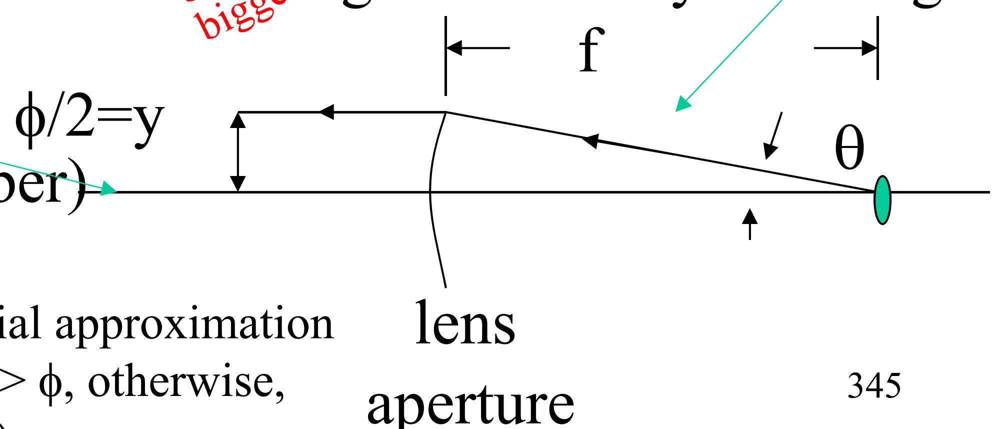
The f-number (focal ratio) is the ratio of the focal length f of the lens to its clear aperture ϕ (effective diameter). The f-number defines the angle of the cone of light leaving the lens which ultimately forms the image. This is important concept when the throughput or light-gathering power of an optical system is critical, such as when focusing light into a monochromator or projecting a high power image.):

$$\text{f-number} = f/\phi$$

brighter the lens, larger the aperture
Longer the focal lens, the harder to get smaller f-number (you need bigger diameter lens)

Numeric aperture is defined as sine of the angle θ made by the marginal ray with the optical axis:

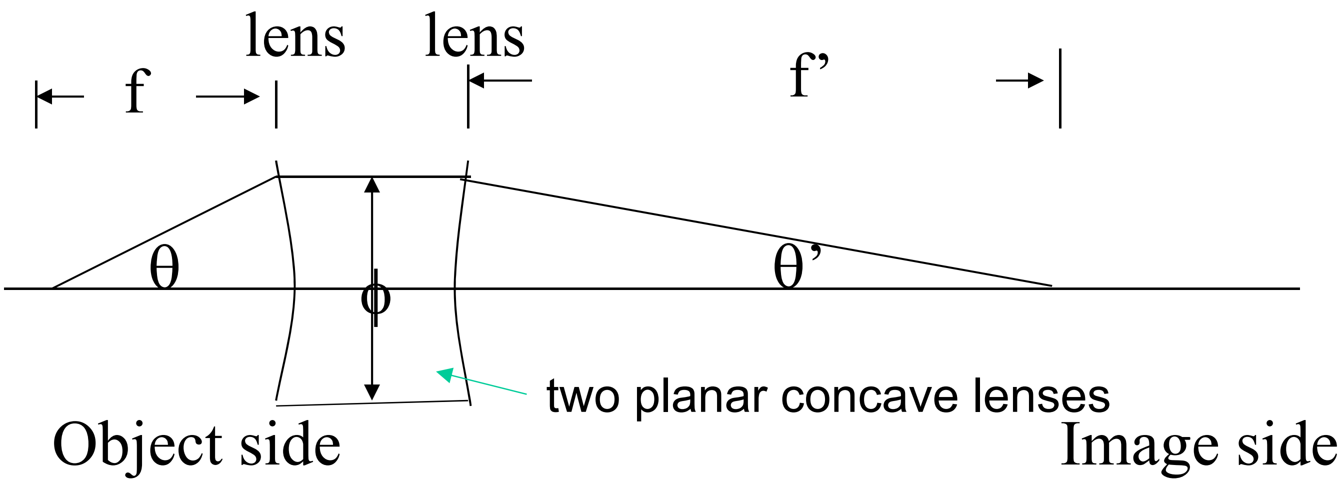
$$NA = \sin\theta \sim \phi/(2f) = 1/(2\text{f-number})$$



Acceptance angle

In the paraxial approximation because f>>> ϕ, otherwise, $\tan\theta = \phi/(2f)$

Numeric Aperture and Magnification



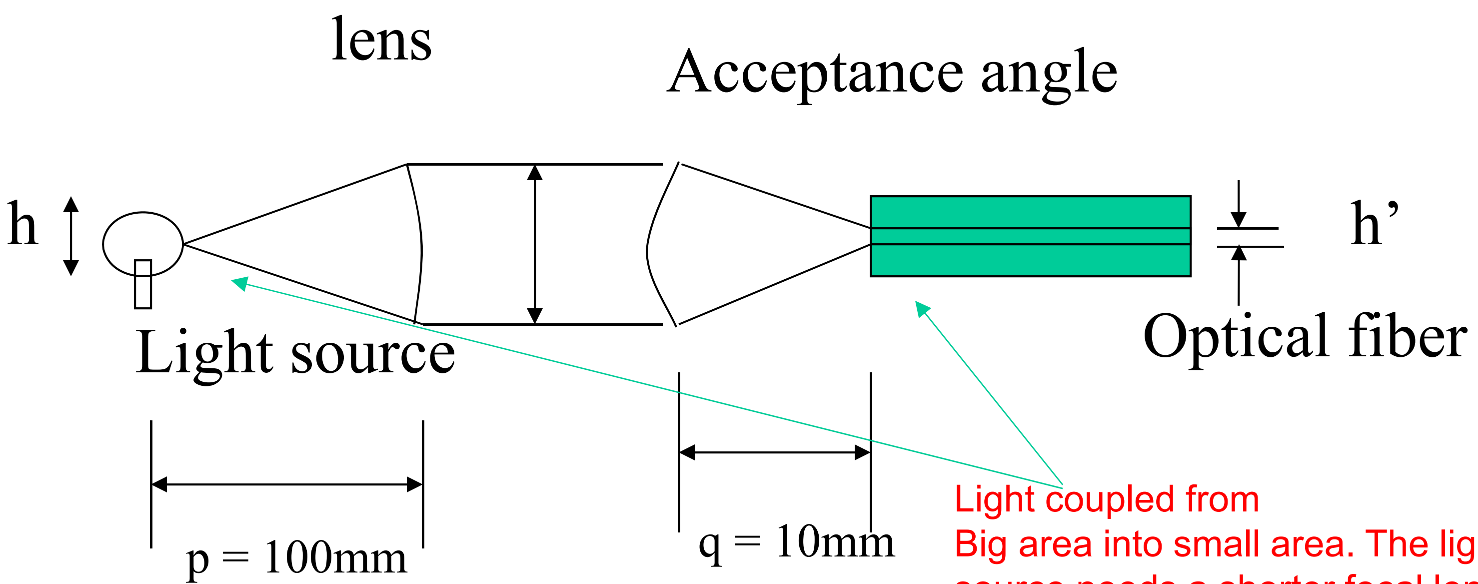
$$NA = \sin\theta = \phi/2f \qquad NA' = \sin\theta' = \phi/2f'$$

$$2f\sin\theta = \phi \qquad 2f'\sin\theta' = \phi$$

$$f'/f = \sin\theta/\sin\theta' = NA/NA' = m$$

Magnification is ratio of the numerical apertures on object and image sides. *This result is useful because it is completely independent of the specifics of optical system*. Use in determining the lens diameter involving aperture constraints.

Coupling light into optical fiber



Light coupled from
Big area into small area. The light source needs a shorter focal length lens and fiber needs a longer focal length lens

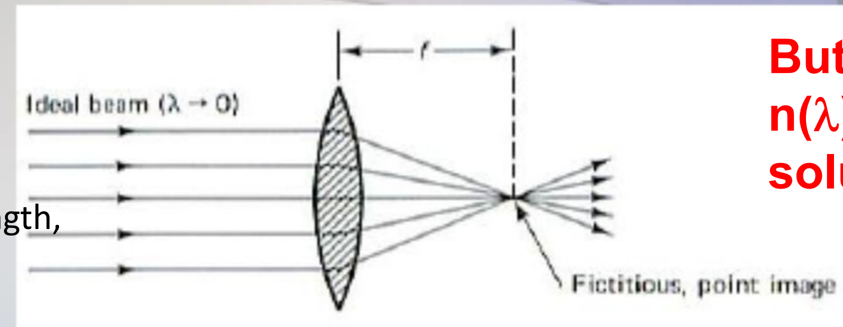
Using lenses' F# and NA for light coupling

Focusability of Light

Multi wavelength
(ray optics)

Ideal situation

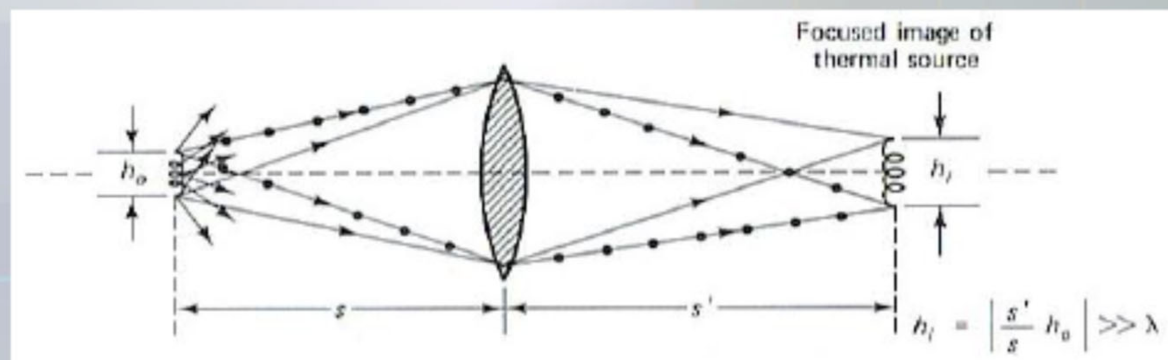
For ray, you can find the estimated focal length, image length and image side based on objective length and size, but not the beam width at focus.



But you can still use $n(\lambda)$ to get the correc solution

Multi wavelength
(EM theory)

Focusing an incoherent light



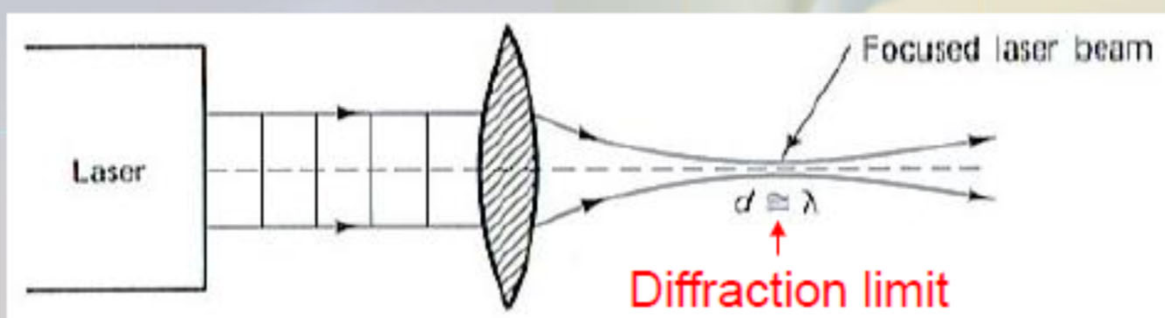
Single wavelength
(EM theory)

Focusing a laser light

$$\phi \sim 1.27\frac{\lambda}{d}$$

Reversibility $\rightarrow$

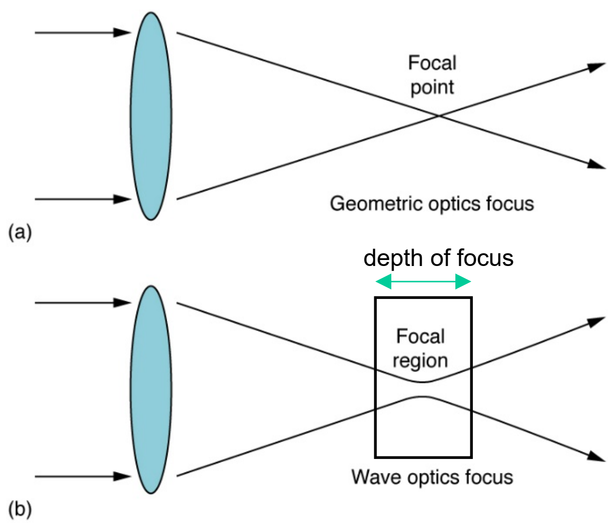
$$d \approx 1.27\frac{\lambda}{\phi} \geq \lambda, \text{ approximately}$$



Beam optics (EM theory)

- Diameter of focus spot: $$2W_o' = \frac{4}{\pi}\lambda(f/\phi);\ f/\phi = F\#$$
- Depth of focus: $$D_{im} = M^2(D_{ob})$$
- Magnification: $$M = \left|\frac{f}{p-f}\right|$$

Above equations derived based on lens properties



Fundamentals of Photonics, B. Saleh, John Wiley& Sons.

p = object length

D_{im} = depth of focus

D_{ob} = depth of field

ϕ = effective diameter = lens diameter

f= focal length

F# = F-number

Beam spot size

$$\frac{W_s}{W_o} = \frac{\lambda R_o}{\pi W_o^2}$$

Relative beam width at focal length based on original beam width at object length

Where W_s = beam spot size at focus, W_o = beam spot size, λ= operating wavelength, R_o = radius of curvature

Image plane and Focal point

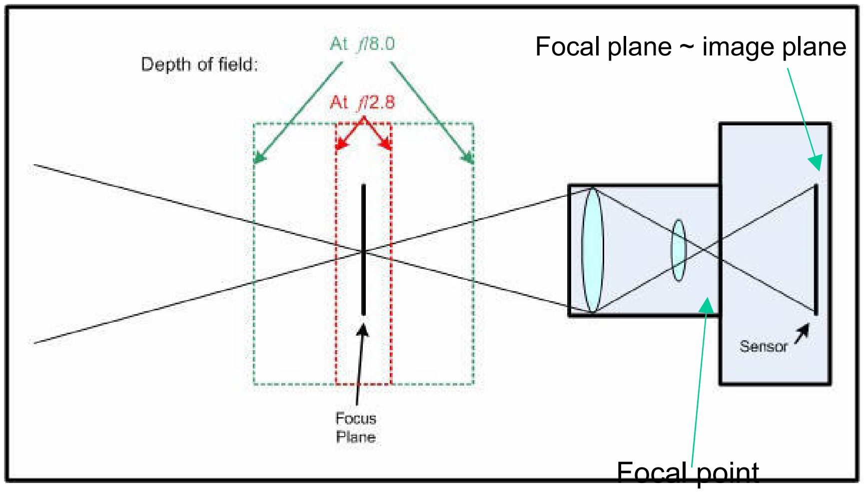
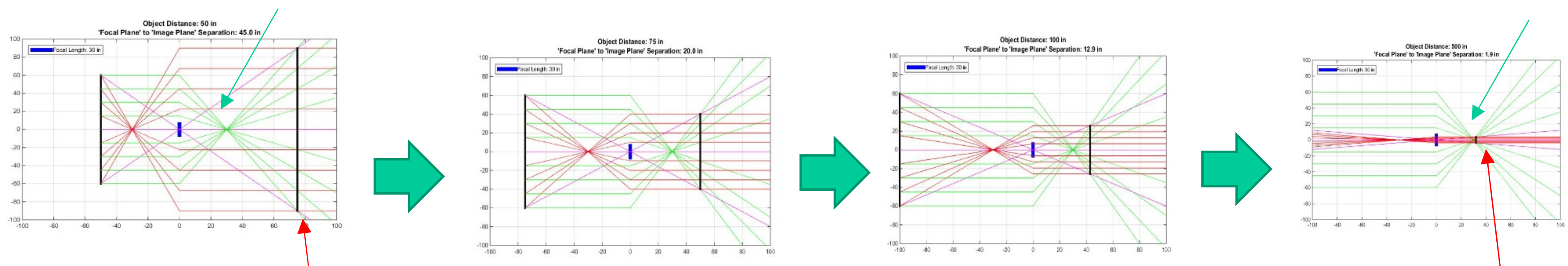
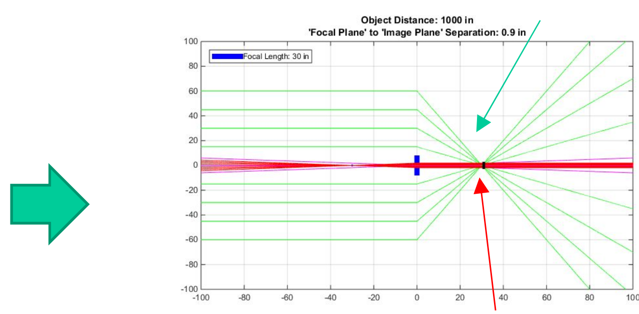


Image plane is same as focal plane
but not the same as focal point

Image plane is where we can see the
Focused image not at focal point!

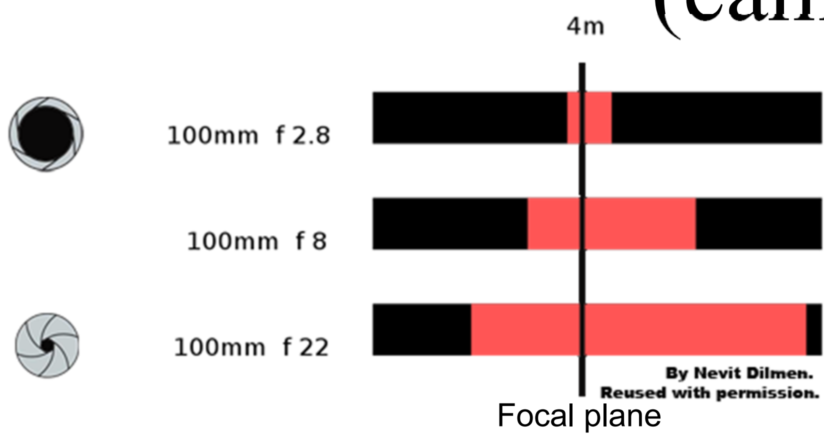


Example of an image plane getting closer and closer to focal point when object is moving further and further away

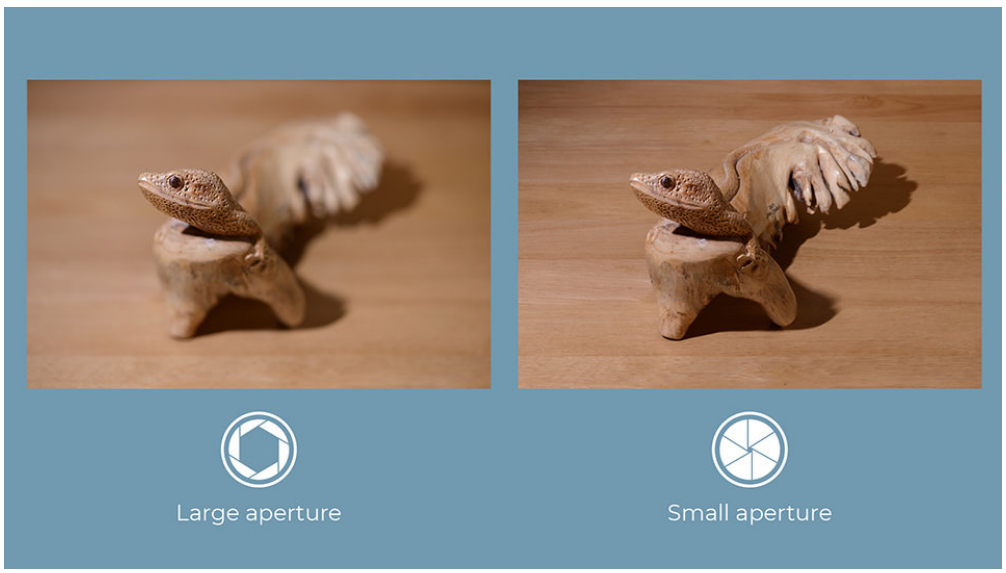


Most of the time, the object we are observing is much longer than focal length… so image length~ focal length

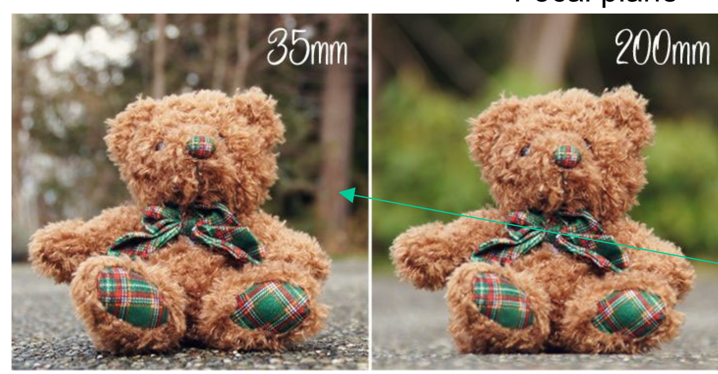
Depth of Field (camera)



Function of focal length and aperture



Same focal length different aperture



bokeh

Same aperture but different focal length

Depth of Focus and Depth of Field (beam optics)

Move your eye in and out or move object in and out

- The depth of focus, D_{im} is the extent of the region around the image plane in which the image will appear to be sharp. This depends on magnification, M.

$$D_{im} = \frac{C^*}{\beta_{ob}} M^2 \quad D \quad = 2M^2CD^2f\#/f^2$$

Using beam at foci

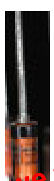
- The depth of field , D_{ob} is the range of distance along the optical axis in which the specimen can move without the image appearing to lose sharpness. This obviously depends on the **resolution of the lens system.**

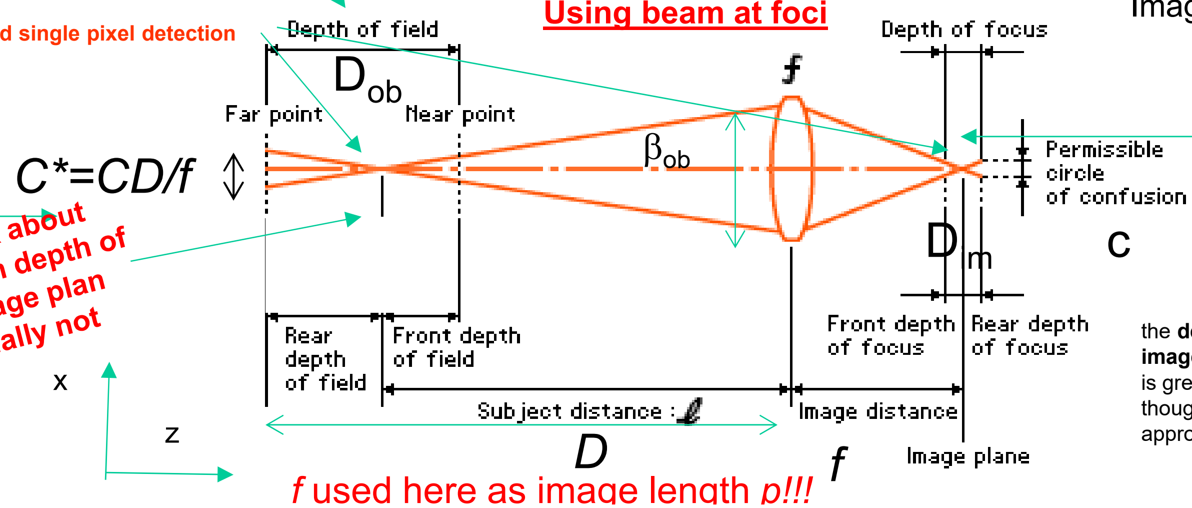
$$D_{ob} = \frac{C^*}{\beta_{ob}} \quad D \quad = 2CD^2f\#/f^2$$

- Both depth of field and depth of focus are strongly **dependent on changes in aperture (hence the semi-angle effective diameter $\beta_{ob}=\phi$) and working distance (C*) = conjugate of circle of confusion.**

Assume point source and single pixel detection

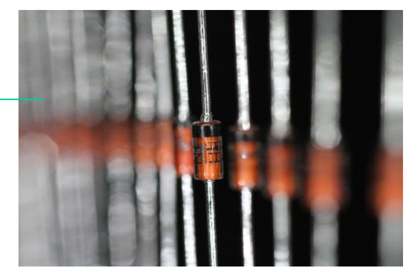
Using beam at foci





Jsually when we talk about camera, we deal with depth of field more since image plan or focal plan is usually not moved.

Image appeared near image plane



the **depth of focus** is symmetrical about the **image plane**; with the second, the **depth of focus** is greater on the far side of the image plane, though in most cases the distances are approximately equal.

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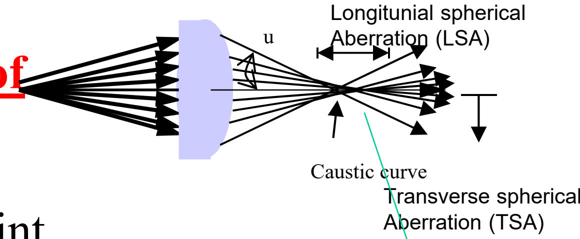
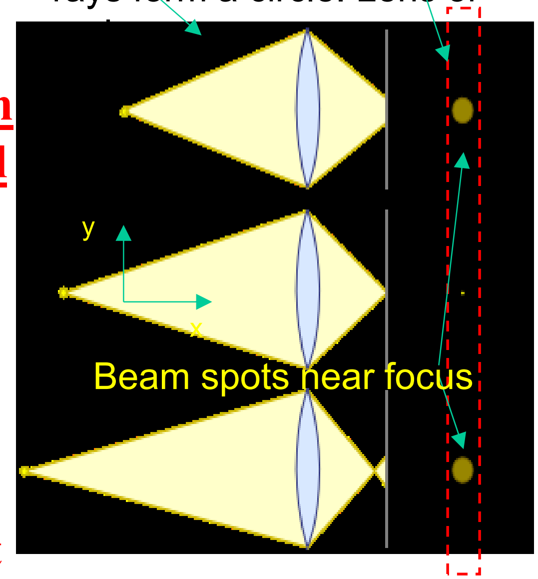
Circle of Confusion

The term *circle of confusion* is applied more generally, **to the size of the out-of-focus spot to which a lens images an object point**.

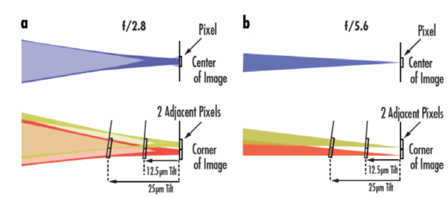


In idealized ray optics, where rays are assumed to converge to a point when perfectly focused, the shape of a defocus blur spot from a lens with a circular aperture is a hard-edged circle of light. A more general blur spot has soft edges due to diffraction and aberrations (Stokseth 1969, 1317; Merklinger 1992, 45–46), and may be non-circular due to the aperture shape. Therefore, the diameter concept needs to be carefully defined in order to be meaningful. **Suitable definitions often use the concept of encircled energy, the fraction of the total optical energy of the spot that is within the specified diameter. Values of the fraction (e.g., 80%, 90%) vary with application.**

In a perfect lens *L*, all the rays pass through a focal point *F*. However at other distances from the lens the rays form a circle. Lens or



In photography, the circle of confusion diameter limit ("CoC") for the final image is often defined as the **largest blur spot that will still be perceived by the human eye as a point**. The circle of confusion (CoC) is used to determine the depth of field, the part of an image that is acceptably sharp; ; the range of object distances over which objects appear sharp is the depth of field ("DoF"). The common criterion for "acceptable sharpness" in the final image (e.g., print, projection screen, or electronic display) is that the blur spot be indistinguishable from a point.



CoC depends on the **resolution of thecamera system.**

Conjugated of Circle of Confusion

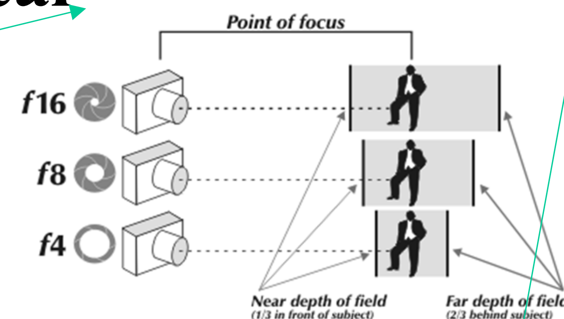
- Backproject the image onto the plane in focus

$q = -mp,\ f = pM/(1+M)$

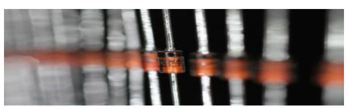
– Depends on *magnification factor: if m=1/4 =>h'/h=* $\boxed{C/C^* = q/p = (1+\cancel{1/4})f/D}$

f used here as image length p (paraxial approx. D is big or object far away)

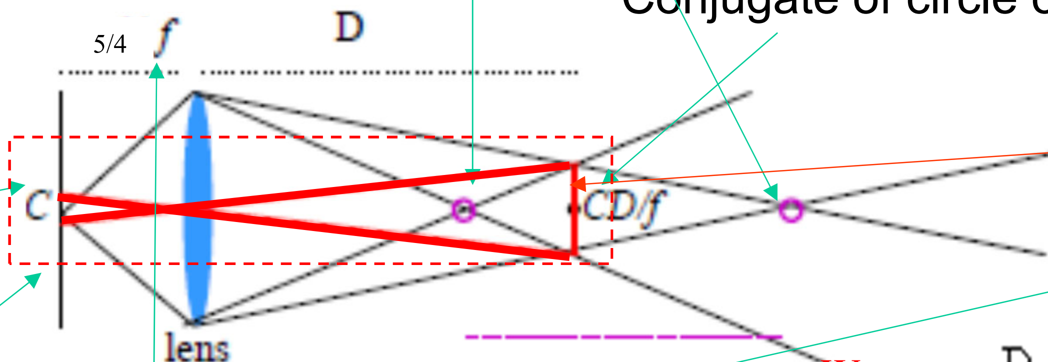
Depth of field D_{ob}, is slightly asymmetrical



Not work for micro lens



Conjugate of circle of confusion c*



$\boxed{\begin{array}{l} C^*/D = C/f \\ C^* = CD/f \end{array}}$

Circle of confusion c

Image plane

We want $D_{ob} = \dfrac{C^*}{\beta_{ob}} D = 2CD^2 f\#/f^2$

D_{ob}

For camera, image plane is really close to the focal plane so focal length is used to approximate the image distance

W. Wang

Hyperfocal Distance

- In optics and photography, **hyperfocal distance** is a distance beyond which all objects can be brought into an "acceptable" focus. There are two commonly used definitions of *hyperfocal distance*, leading to values that differ only slightly:
- *Definition 1:* The hyperfocal distance is the closest distance at which a lens can be focused while keeping objects at infinity acceptably sharp. When the lens is focused at this distance, all objects at distances from half of the hyperfocal distance out to infinity will be acceptably sharp.
- *Definition 2:* The hyperfocal distance is the distance beyond which all objects are acceptably sharp, for a lens focused at infinity.
- The distinction between the two meanings is rarely made, since they have almost identical values. The value computed according to the first definition exceeds that from the second by just one focal length.
- As the hyperfocal distance is the focus distance giving the maximum depth of field, it is the most desirable distance to set the focus of a fixed-focus camera.[1]

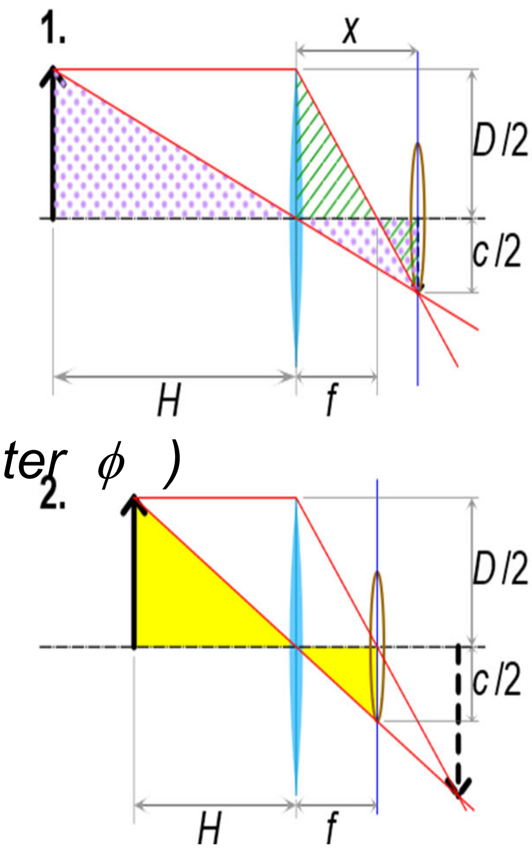
Hyperfocal Distance

Case 1: For the first definition,

$$H = (f^2/f\#c) + f$$

where

H is hyperfocal distance
f is focal length
f# is f-number (f/ϕ for aperture diameter ϕ)
c is the circle of confusion limit



Case 2: For any practical f-number, the added focal length is insignificant in comparison with the first term, so that

$$H = (f^2/f\#c)$$

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(thin lens approximation)

Deriving Depth of Field

- Circle of confusion C, magnification $m=D/f$
- Simplification: $S=f/D$
- Focusing distance D, focal length f, f-number f#, *effective diameter* $\boxed{\beta_{ob}= f/f\#}$
- As usual, similar triangles:

$\boxed{C^*=CD/f}$

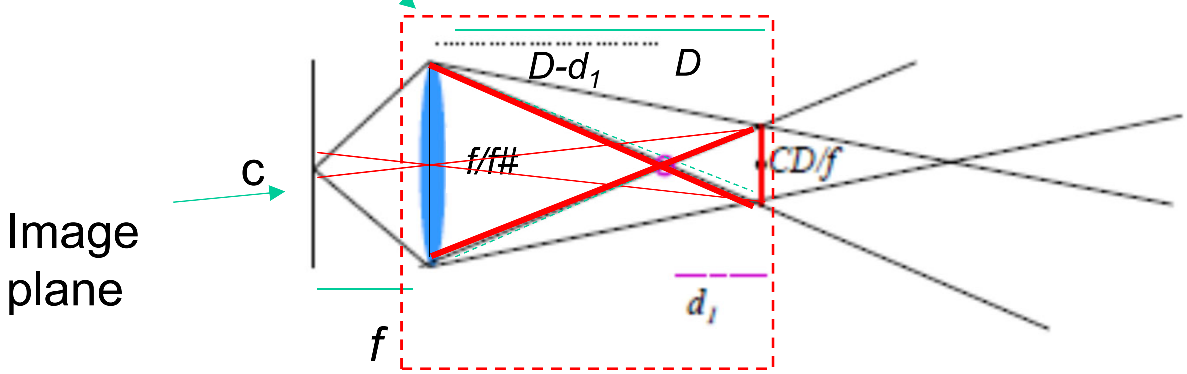
ϕ

$$d_1 / C^* = (D - d_1) / \beta_{ob}$$

Use the two similar triangles, divide adjacent over opposite side we get:

$$\frac{f d_1}{CD} = \frac{D - d_1}{f/(f\#)} \Longrightarrow \frac{f d_1}{CD} + \frac{d_1}{f/(f\#)} = \frac{D}{f/(f\#)}$$

$$\frac{d_1(f^2/(f\#)) + CD}{CDf/(f\#)} = \frac{D}{f/(f\#)} \Longrightarrow d_1 = \frac{CD^2}{f^2/(f\#) + CD} = \frac{f\#\,CD^2}{f^2 + f\#CD}$$



Thie ratio is to show how object can be captured by the aperture of the Lens system

Deriving Depth of Field

$$d_1 = \frac{f\#CD^2}{f^2 + f\#CD}$$

$$d_2 = \frac{2f\#CD^2}{f^2 - f\#CD}$$

The **depth of focus** is greater on the far side of **the image plane,** though in most cases the distances are approximately equal.

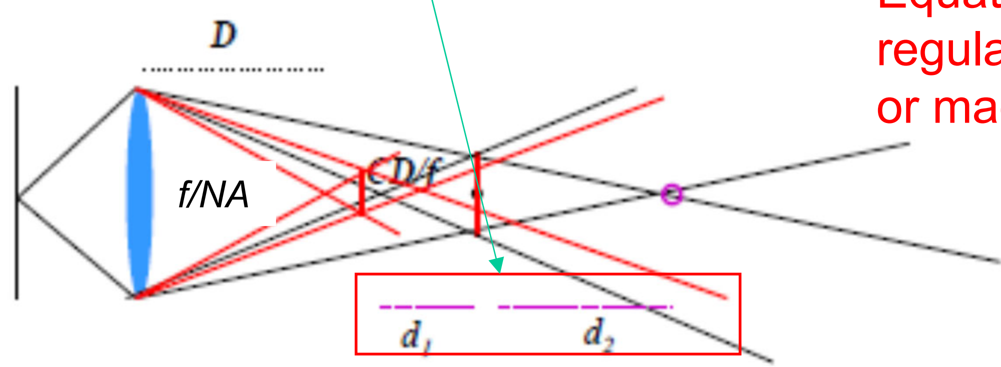
$$D_{ob} = d_1 + d_2 = \frac{2f\#CD^2f^2}{f^4 - f\#^2C^2D^2}$$

$NA^2C^2D^2$ term can often be neglected when DoF is small (conjugate of circle of confusion is smaller than lens aperture) Then

$$D_{ob} = \frac{2f\#CD^2}{f^2}$$

Equation only works for regular lens, not micro or macro lens

Image plane



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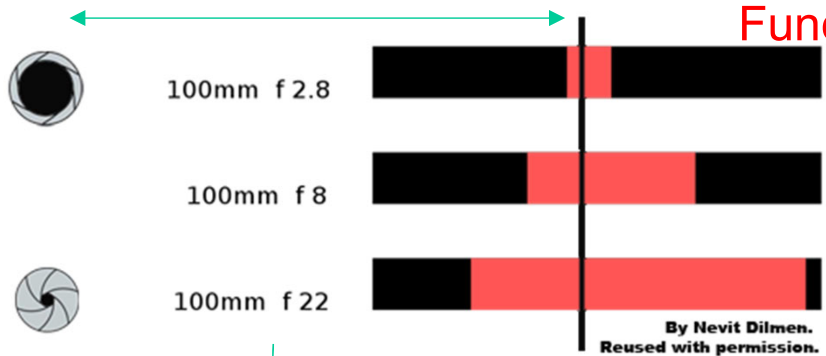
Depth of Field

D

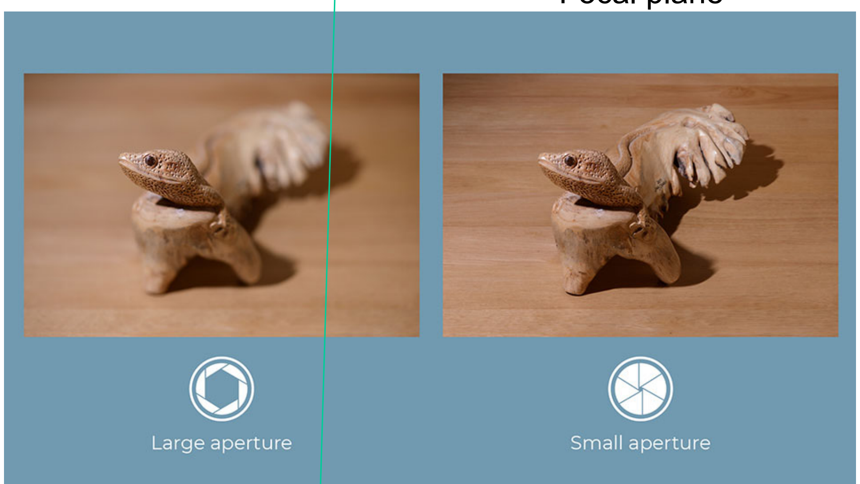
4m



Focal plane

Function of focal length and aperture

Focal plane



Same focal length different aperture

$$D_{ob} = \frac{2f\#CD^2}{f^2}$$ Depth of field

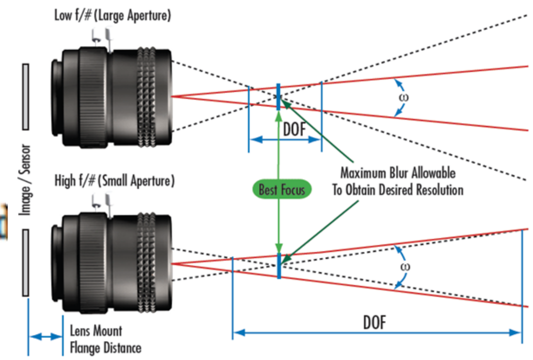
NA = sinθ = ϕ/(2f)=1/(2f-number)

$$D_{ob} = 2CD^2/(\phi f)$$

Same aperture but different focal length

W.Wang

DoF and Aperture



- http://www.juzaphoto.com/eng/articles/depth_of_field

f/2.8

f/16

- Larger the f#, longer the D_{ob}
-Longer the focal length, shorter the D_{ob}

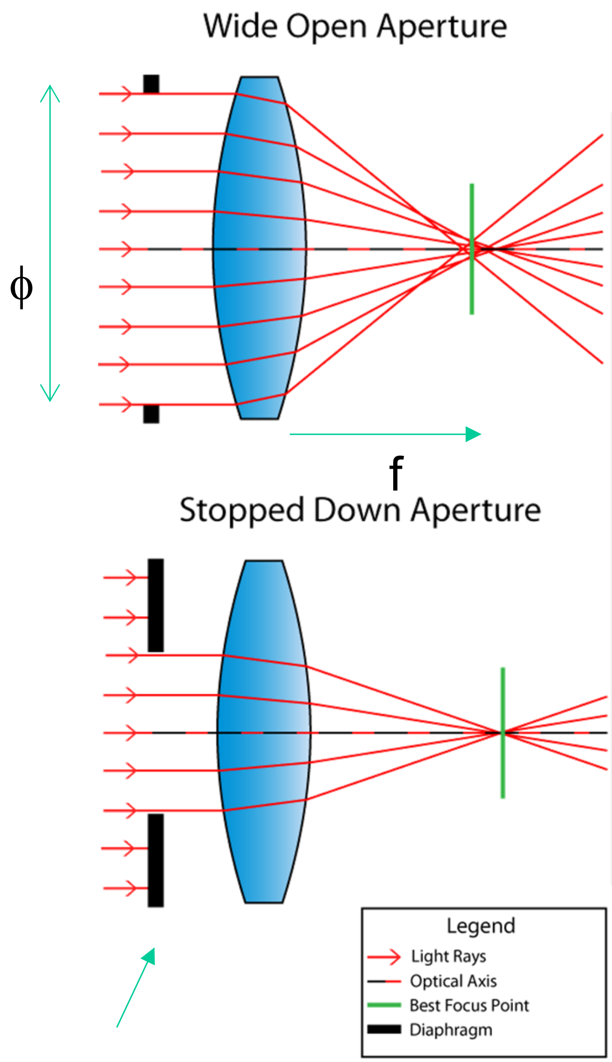
$$D_{ob} = \frac{2f\#CD^2}{f^2}$$ Depth of field

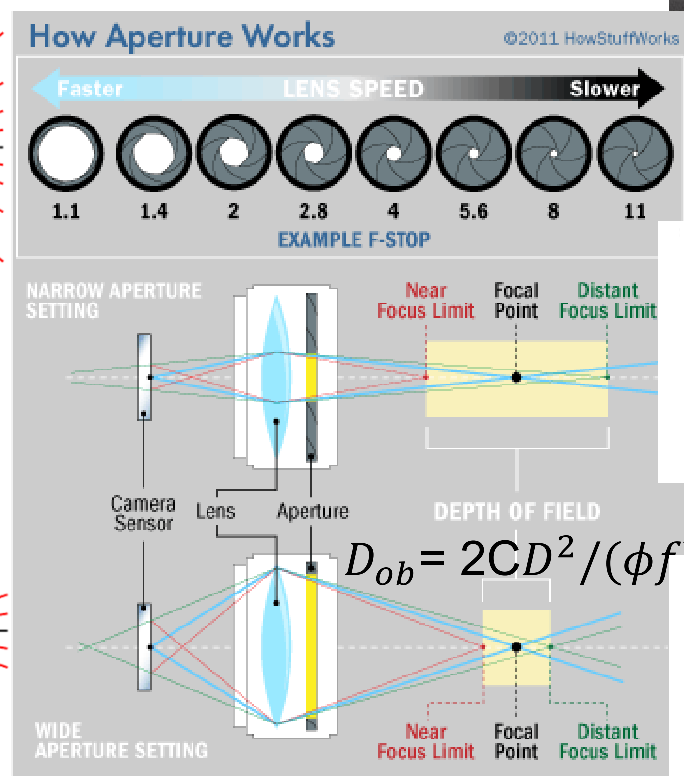
NA = sinθ = ϕ/(2f)=1/(2f-number)

$$D_{ob} = 2CD^2/(\phi f)$$

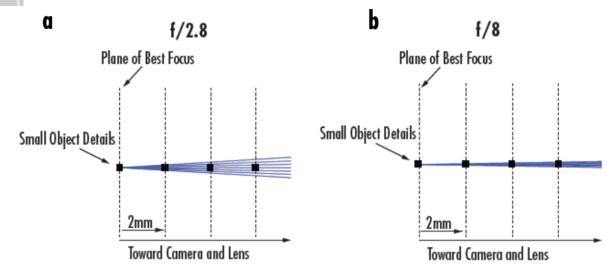
f/2.8 f/32 ← f-number

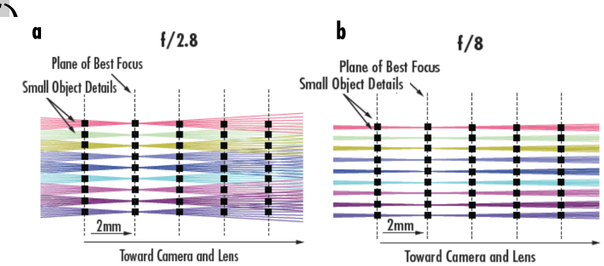
Aperture

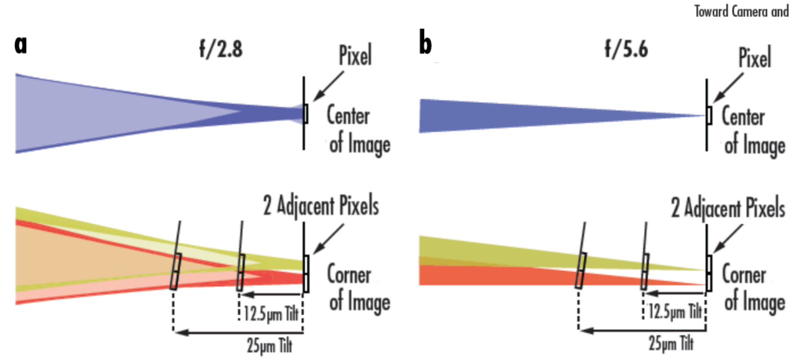




$D_{ob} = 2CD^2/(\phi f)$







Like paraxial effect Make image more clear more focus

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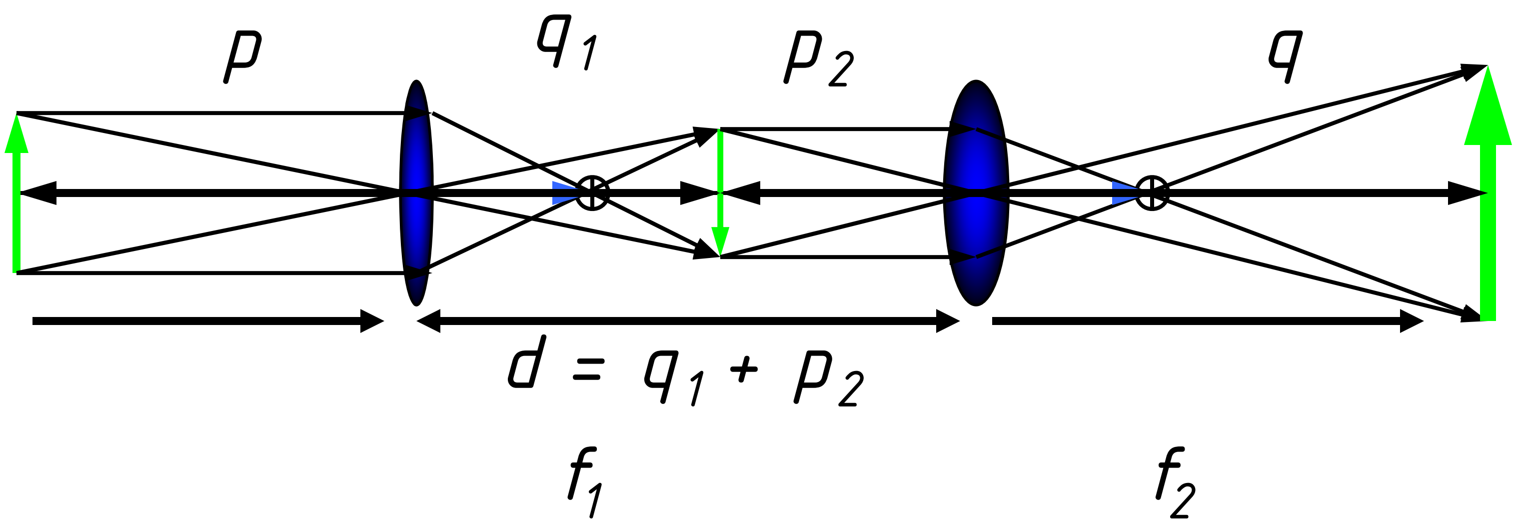
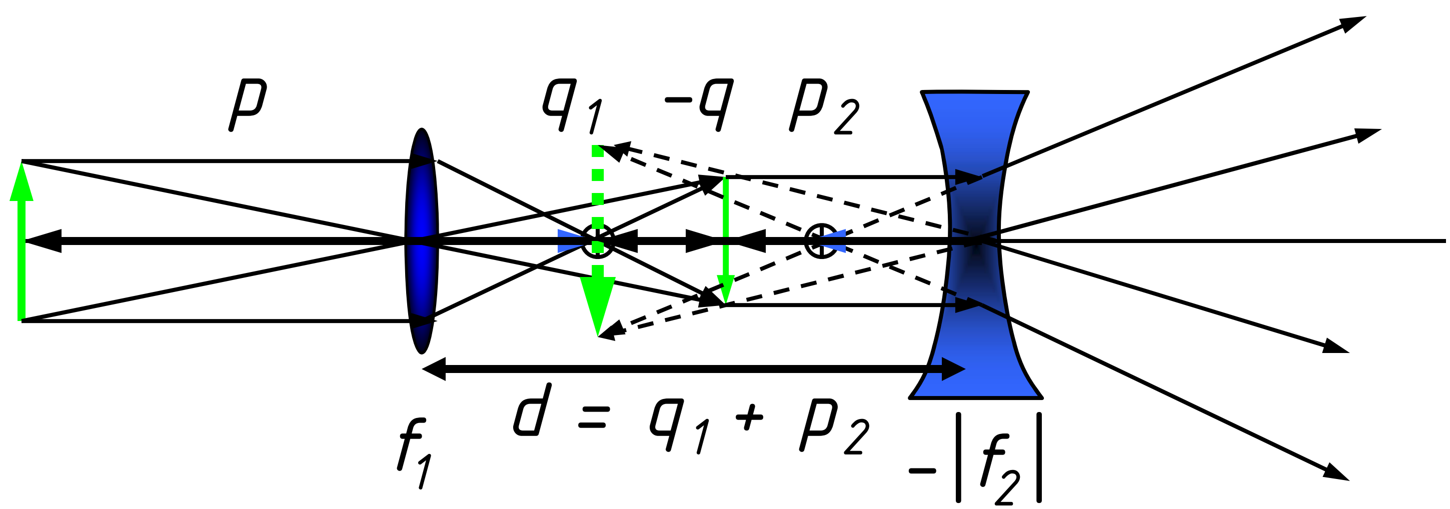
Combinations of Lenses

Mgnification



$$M = M_1 M_2 = (-q_1 / p)(-q / p_2)$$

Combinations of Lenses



$$M = M_1 M_2 = (-q_1/p_1)(q/p_2)$$

W.Wang

Effective Focal Length

The expression for the combination focal length is the same whether lens separation distance (d), defined as the distance between the secondary principal point H_1" of the first (left-hand) lens and the primary principal point H_2 of the second (right-hand) lens, is large or small or whether the focal lengths f_1 and f_2 are positive or negative:

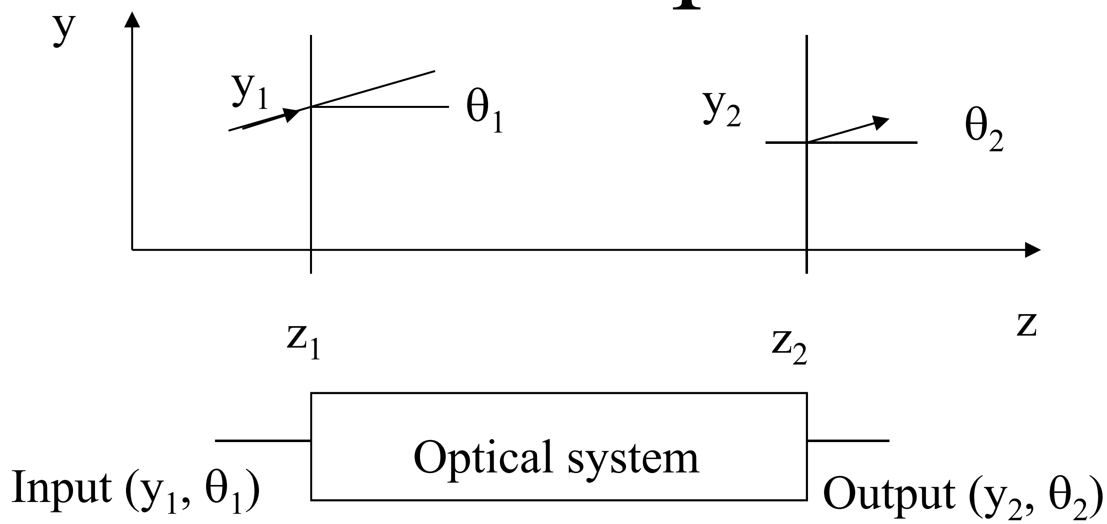
$$f = \frac{f_1 f_2}{f_1 + f_2 - d}$$

This may be more familiar in the form

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}$$

Notice that the formula is symmetric with the lenses (end-for-end rotation of the combination) at constant d.

Matrix Optics



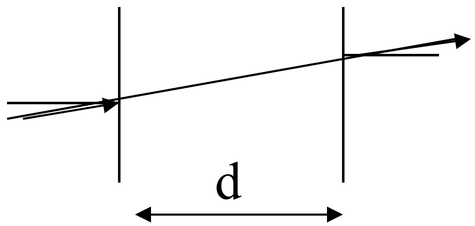
Matrix optics is a technique for tracing paraxial rays. The rays are assumed to travel only within a single plane (as shown in yz plane)
-A ray is described by its position y and its angle θ with respect to the optical axis. These variables are altered as the ray travels through the optical system, where The system can be represent by a transfer function like matrix to represent the relation between (y_2, θ_2) and (y_1, θ_1) as,

$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \underbrace{\begin{bmatrix} A & B \\ C & D \end{bmatrix}}_{M} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix}$$

Where A,B,C,D are elements characterizes the optical system

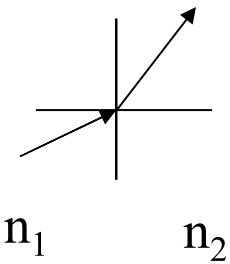
Matrices of Simple Optical Components

Free-space propagation



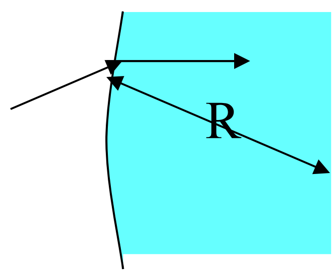
$$M = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$

Refraction at a planar boundary



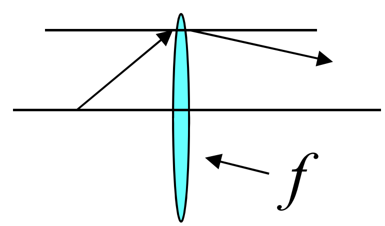
$$M = \begin{bmatrix} 1 & 0 \\ 0 & \dfrac{n_1}{n_2} \end{bmatrix}$$

Refraction at a spherical boundary



$$M = \begin{bmatrix} 1 & 0 \\ \dfrac{-(n_2 - n_1)}{n_2 R} & \dfrac{n_1}{n_2} \end{bmatrix}$$

Transmission through a thin lens

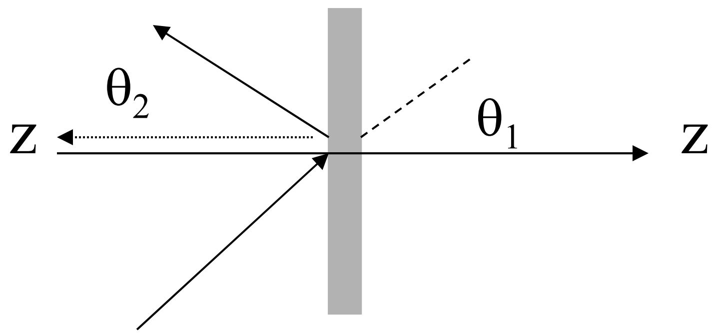


Convex $f > 0$
Concave $f < 0$

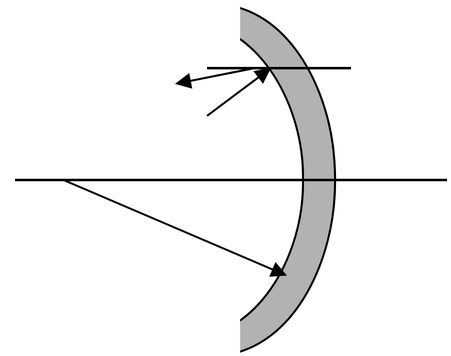
$$M = \begin{bmatrix} 1 & 0 \\ -\dfrac{1}{f} & 1 \end{bmatrix}$$

Matrices of Simple Optical Components

Reflection from a planar mirror



Reflection from a spherical mirror



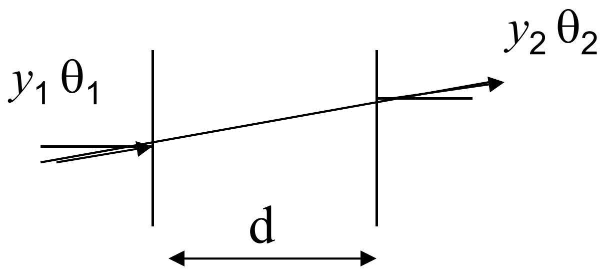
Concave R<0
Convex R>0

$$M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} 1 & 0 \\ \dfrac{2}{R} & 1 \end{bmatrix}$$

Work only for simple refraction and reflection

Free-space propagation



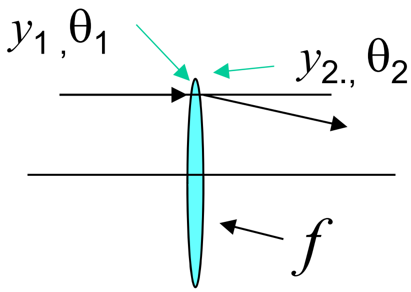
$$M = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix}$$

$y_2 = y_1 + d\,\theta_1$

$\theta_2 = \theta_1$

Transmission through a thin lens



Convex f > 0
Concave f < 0

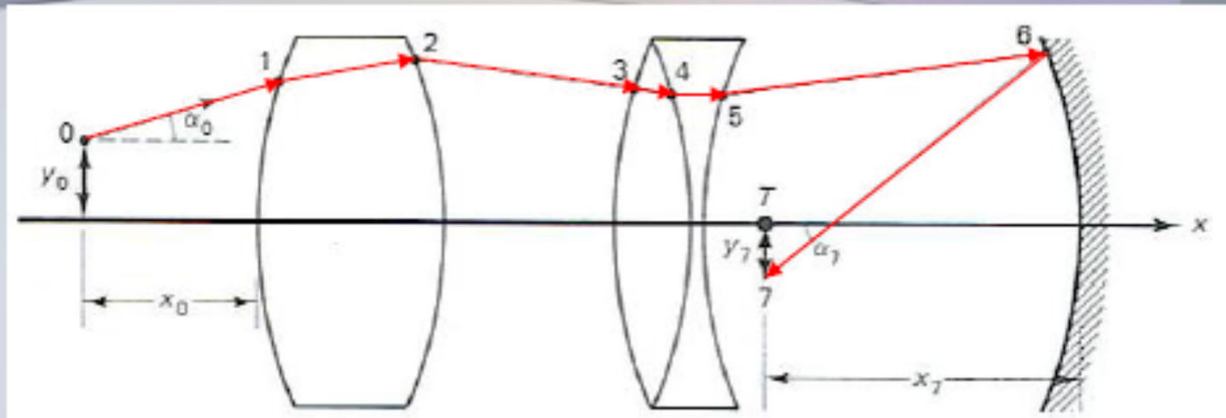
$$M = \begin{bmatrix} 1 & 0 \\ -\dfrac{1}{f} & 1 \end{bmatrix}$$

$$\begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix}$$

$\Rightarrow$

$y_2 = y_1$
$\theta_2 = -y_1/f + \theta_1$

Matrices of Cascaded Optical Components



$n_0 \rightarrow$ M_1 $\rightarrow$ M_2 $\rightarrow$ M_3 ………… M_N $\rightarrow n_f$

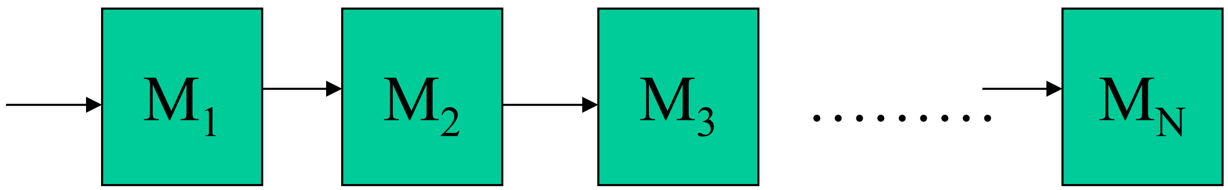
$$\begin{bmatrix} y_0 \\ \theta_0 \end{bmatrix} \qquad \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix} = \mathbf{M}_1 \begin{bmatrix} y_0 \\ \theta_0 \end{bmatrix} \qquad \begin{bmatrix} y_2 \\ \theta_2 \end{bmatrix} = \mathbf{M}_2 \begin{bmatrix} y_1 \\ \theta_1 \end{bmatrix} \qquad \begin{bmatrix} y_N \\ \theta_N \end{bmatrix}$$

$$\begin{bmatrix} y_N \\ \theta_N \end{bmatrix} = \mathbf{M} \begin{bmatrix} y_0 \\ \theta_0 \end{bmatrix} \qquad \mathbf{M} = \mathbf{M}_N \cdots \mathbf{M}_2 \mathbf{M}_1 = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$$

Works particularly well with computer programming

A useful tip: $\boxed{\text{Det } \mathbf{M} = AD - BC = \dfrac{n_o}{n_f}}$

Cascaded Matrices



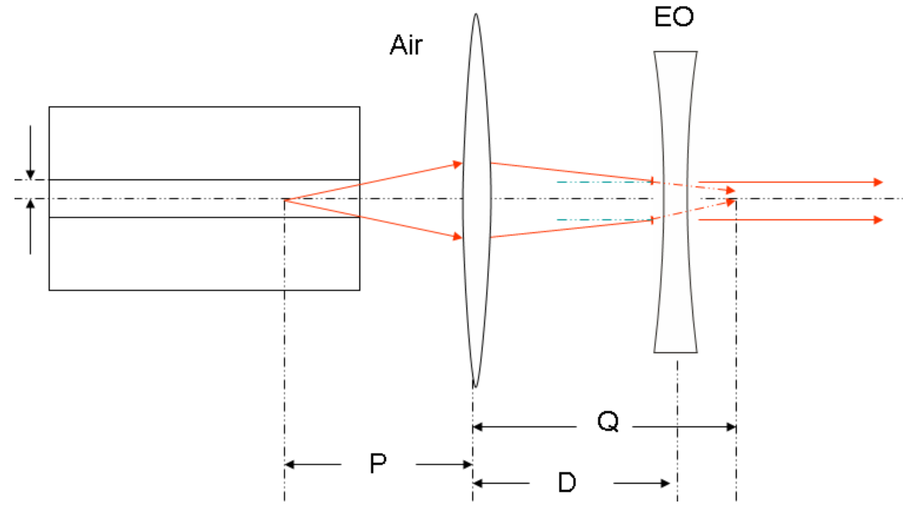
$$M = M_N \bullet \bullet \bullet\bullet\ M_2\, M_1$$

Show opticlab and oslo software

Example of Collimator

Figure 13, 2-D path of light via Convex-Concave lens combination

Assuming both of the lenses are thin lenses, the govern equations for this collimator are:



$$\frac{1}{P}+\frac{1}{Q}=\frac{1}{f_1}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) \quad \langle R_1{:}+\ R_2{:}\ -\rangle$$

$$Q-D=f_2=\frac{1}{(n-1)\left(\frac{1}{R_3}-\frac{1}{R_4}\right)} \quad \langle R_3{:}-\ R_4{:}\ +\rangle$$

There are two more design rules to follow:

1. **H/2>Rcone** , where H the height of the lens, Rcone the radius of the diverged beam
2. **H/W is relatively big to qualify thin lens,** where H the height of the lens, w is the thickness of the lens

‘ Then, by defining the P , D and one of the focal lenghth(or radius), the other focal length will be determined.

Another approach to address the problem is by using ray-transfer matrics. The detail was addressed as following:

$$M=\begin{pmatrix}1 & 0\\ -\dfrac{1}{f_2} & 1\end{pmatrix}*\begin{pmatrix}1 & d_2\\ 0 & 1\end{pmatrix}*\begin{pmatrix}1 & 0\\ -\dfrac{1}{f_1} & 1\end{pmatrix}*\begin{pmatrix}1 & d_1\\ 0 & 1\end{pmatrix}$$

$$=\begin{pmatrix}1 & d_2\\ -\dfrac{1}{f_2} & 1-\dfrac{d_2}{f_2}\end{pmatrix}*\begin{pmatrix}1 & 0\\ -\dfrac{1}{f_1} & 1\end{pmatrix}*\begin{pmatrix}1 & d_1\\ 0 & 1\end{pmatrix}$$

$$=\begin{pmatrix}1-\dfrac{d_2}{f_1} & d_2\\ -\dfrac{1}{f_2}-\dfrac{1}{f_1}(1-\dfrac{d_2}{f_2}) & 1-\dfrac{d_2}{f_2}\end{pmatrix}*\begin{pmatrix}1 & d_1\\ 0 & 1\end{pmatrix}$$

$$=\begin{pmatrix}1-\dfrac{d_2}{f_1} & d_2+d_1(1-\dfrac{d_2}{f_1})\\ -\dfrac{1}{f_2}-\dfrac{1}{f_1}(1-\dfrac{d_2}{f_2}) & 1-\dfrac{d_2}{f_2}-\dfrac{d_1}{f_2}-\dfrac{d_1}{f_1}(1-\dfrac{d_2}{f_2})\end{pmatrix}$$

$$\begin{bmatrix}y_2\\ \theta_2\end{bmatrix}=M*\begin{bmatrix}y_1\\ \theta_1\end{bmatrix}=\begin{pmatrix}1-\dfrac{d_2}{f_1} & d_2+d_1(1-\dfrac{d_2}{f_1})\\ -\dfrac{1}{f_2}-\dfrac{1}{f_1}(1-\dfrac{d_2}{f_2}) & 1-\dfrac{d_2}{f_2}-\dfrac{d_1}{f_2}-\dfrac{d_1}{f_1}(1-\dfrac{d_2}{f_2})\end{pmatrix}*\begin{bmatrix}y_1\\ \theta_1\end{bmatrix}$$

$$=\begin{bmatrix}(1-\dfrac{d_2}{f_1})*y_1+(d_2+d_1(1-\dfrac{d_2}{f_1}))\theta_1\\ (-\dfrac{1}{f_2}-\dfrac{1}{f_1}(1-\dfrac{d_2}{f_2}))y_1+\theta_1(1-\dfrac{d_2}{f_2}-\dfrac{d_1}{f_2}-\dfrac{d_1}{f_1}(1-\dfrac{d_2}{f_2}))\end{bmatrix}$$

To be collimated beam, θ2 must equal 0 for any input angle θ1, which means:
Let y1=0

$$\theta_1(1-\frac{d_2}{f_2}-\frac{d_1}{f_2}-\frac{d_1}{f_1}(1-\frac{d_2}{f_2})=0$$

$$\Rightarrow 1-\frac{d_2}{f_2}-\frac{d_1}{f_2}-\frac{d_1}{f_1}(1-\frac{d_2}{f_2})=0$$

$$\Rightarrow f_1 f_2 - f_1 d_2 - f_1 d_1 - f_2 d_1 + d_1 d_2 = 0$$

$$f=\frac{1}{(n-1)(\frac{1}{R_1}-\frac{1}{R_2})}$$

Lenses and Mrrors

Summary:

- *Lateral Magnification* $M = h'/h = -q/p$
- *Mirror Equation:*

$$1/p + 1/q = 2/R = 1/f$$

- *The object-image relation for a curved surface:*

$$n_1/p + n_2/q = [n_1 - n_2]/R$$

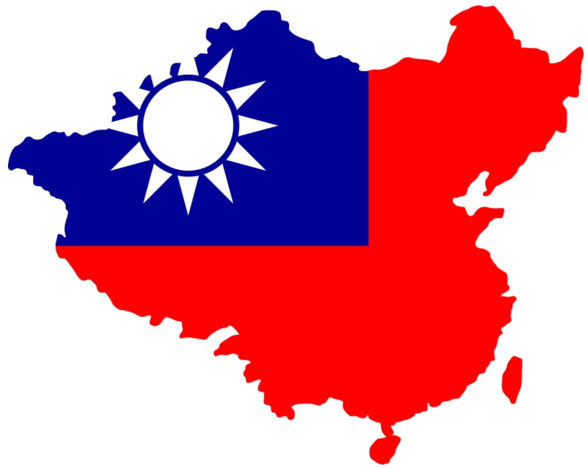
- *Len's Maker equation:*

$$1/f = (n-1)(1/R_1 - 1/R_2)$$

- *Thin Lens Equation:*

$$1/p + 1/q = 1/f$$

Quote of the week



If one can justify invasion of another country because they claim the land is theirs. Than Taiwan should immediate invade China because they believe China is part of Taiwan (ROC).

W.C. Wang

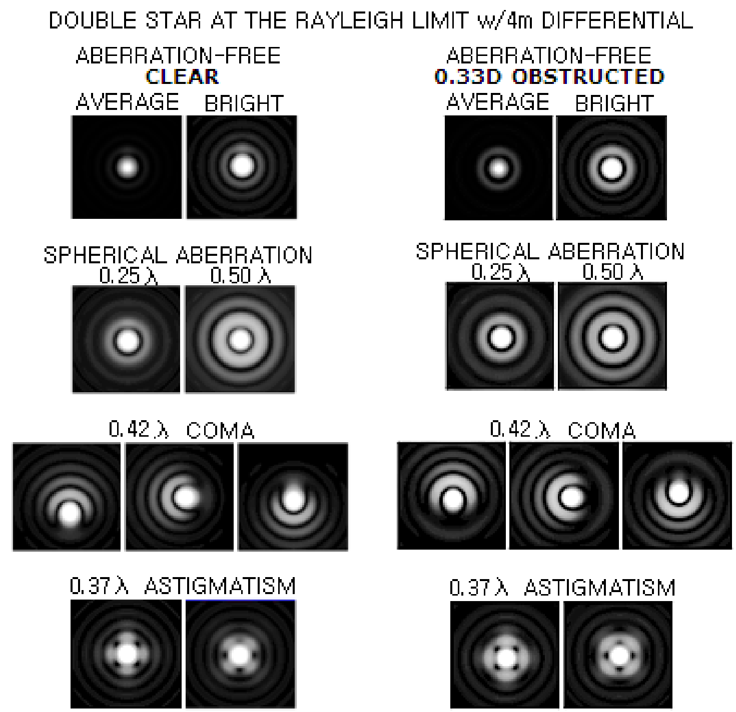
hergemonic

Terminology

- Focal length: distance between the lens and the image sensor when the subject is in focus
- Diopter: 1/ focal length measured in meters
- Effective focal length: focal length of an optical system
- Numeric aperture: NA = $\sin\theta = \phi/(2f) = 1/2$f-number
- Acceptance angle: θ
- f-number: f/ϕ
- Depth of Field: range of distance along the optical axis in which the specimen can move without the image appearing to lose sharpness
- Depth of Focus: region around the image plane in which the image will appear to be sharp

Performance factors

Diffraction effect
Aberrations- Spherical (geometry), chromatic (wavelength)
Astigmatism
Coma
Field curvature
Chromatic distortion



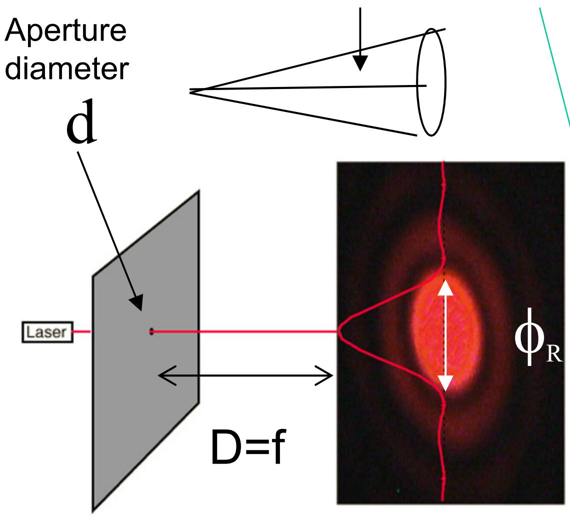
Examples in telescope lens

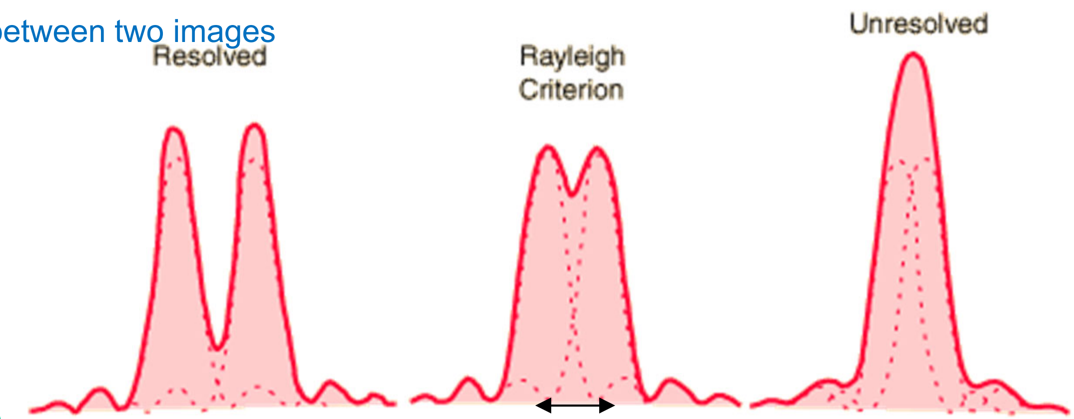
Diffraction effect

Rayleigh criterion- spatial resolution is limited by diffraction.

Assume two separate point sources can be resolved when the center of the airy disc from one overlaps the first dark ring in the diffraction pattern of second one

θ_R (angular separation between two images





ϕ_R smallest resolvable separation between two images)

$\sin\theta_R = \dfrac{\lambda}{d}$

$\boxed{\phi_R = \lambda\ f/\ d}$

Single slit

$\sin\theta_R = 1.22\,\dfrac{\lambda}{d}$

$\boxed{\phi_R = 2.44\lambda\ f/d}$

Circular aperture

$\phi_R = 1.27\lambda\ f/d$ (Gaussian beam)

ϕ_R represents the smallest spot size that can be achieved by an optical system with a circular or slit aperture of a given f-number (diffraction-limit spot size)

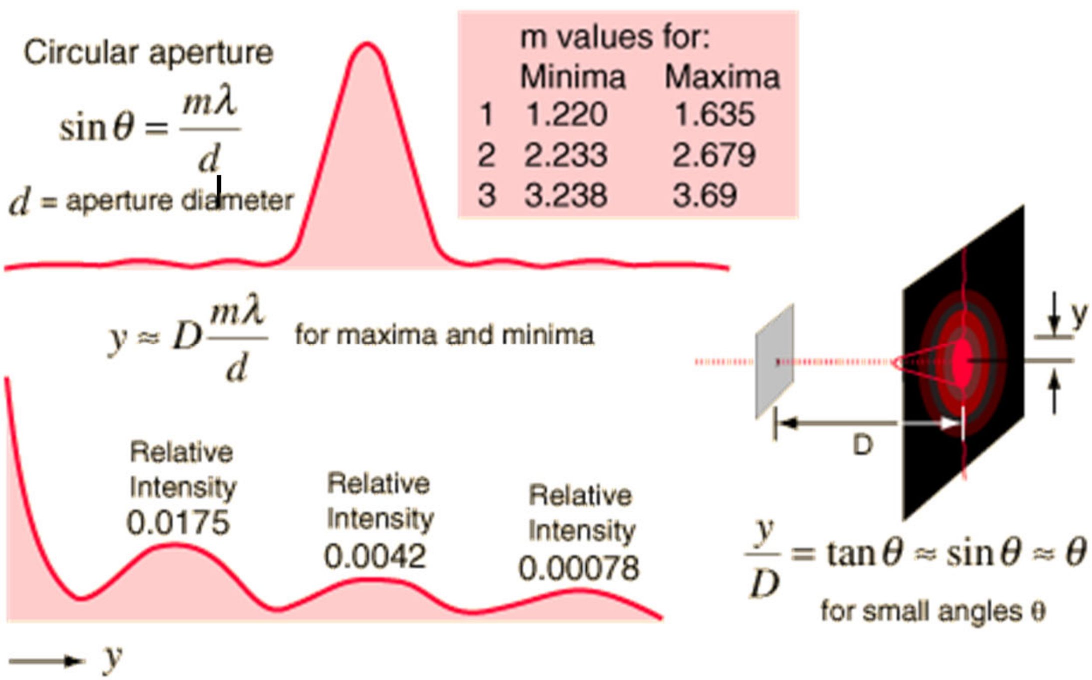
W.Wang

Diffraction–limited spot size

As we will see later when we derive irradiance distribution in the diffraction pattern of a slit is defined as

$$I = I_o\,[2J_1(\delta)/\delta]^2$$

Where $\delta = \pi d \sin\theta / m\lambda$



Since m=1.22 for 1^{st} min.

$\sin\theta = 1.22\lambda/d$

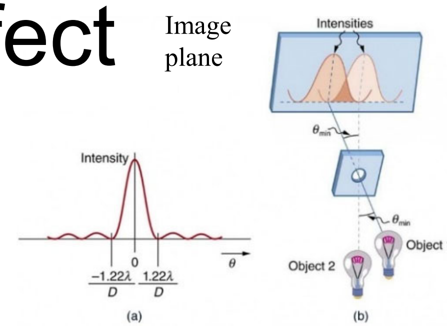
NA=Sinθ =ϕ/2f= 1/2f-number

Please read the hand written derivation for more detail: http://courses.washington.edu/me557/readings/reflection+refraction.pdf

Example of Diffraction Effect

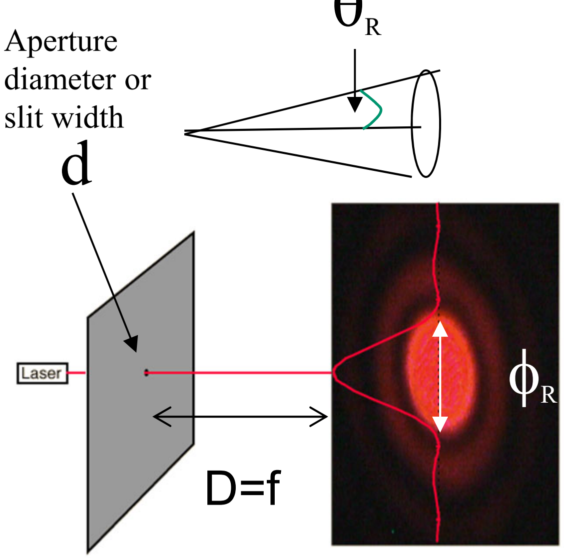
Spot size and spatial resolution

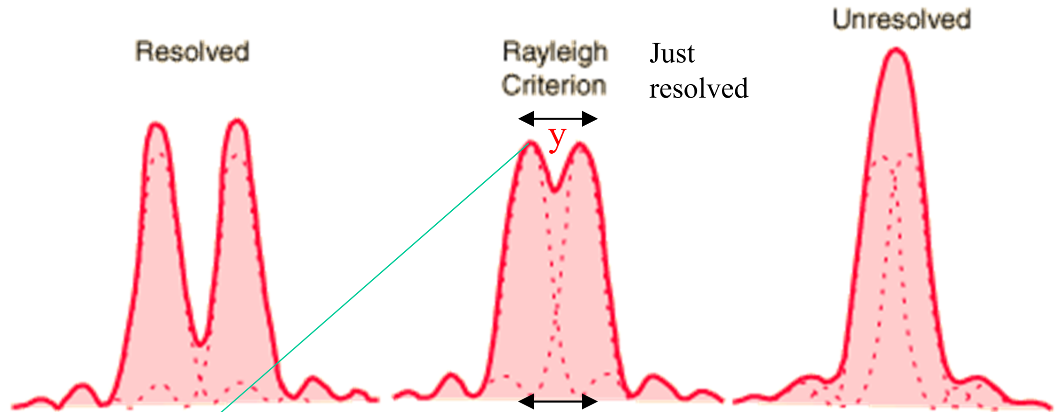
Image plane



Rayleigh criterion- spatial resolution is limited by diffraction.

Assume two separate point sources can be resolved when the center of the airy disc from one overlaps the first dark ring in the diffraction pattern of second one





Since this is the radius of the Airy disk, the resolution is better estimated by the diameter

θ_R (angular separation between two images)

$$\sin\theta_R = \frac{\lambda}{d}$$

$$\sin\theta_R = 1.22\frac{\lambda}{d}$$

Spatial resolution $\boxed{y = \lambda f / d}$ $\phi_R = 2\lambda f/d$

NA= λ/y

Single slit

$\boxed{y = 1.22\lambda f/d}$

$\phi_R = 2.44\lambda f/d$

NA= 0.61λ/y

Circular aperture

Resolvable line resolution

y= 1.27λ f/d (Gaussian beam)

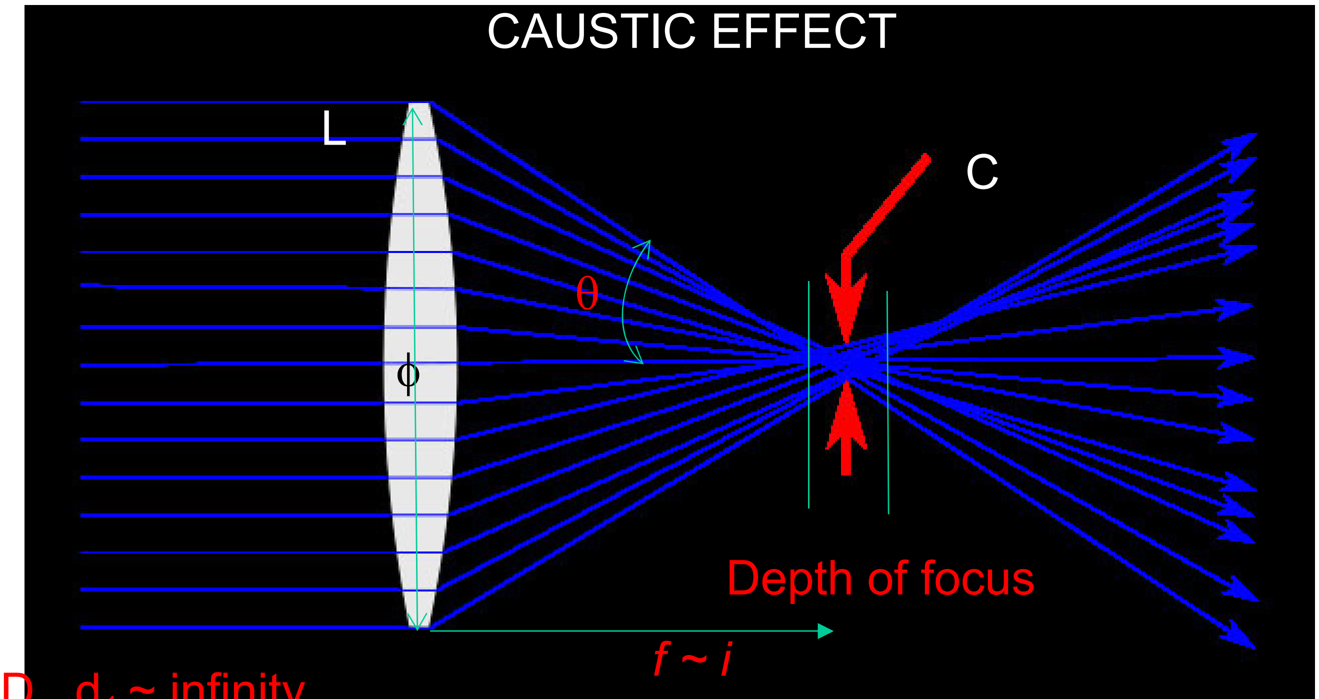
ϕ_R represents the smallest **spot size** that can be achieved by an optical system with a circular or slit aperture of a given f-number (diffraction-limit spot size)

NA=Sinθ =ϕ/2f= 1/2f-number

W.Wang

Circle of Least Confusion

Diffraction effects from wave optics and the finite aperture of a lens determine the circle of least confusion



d_2 D d_1 ~ infinity

Recall

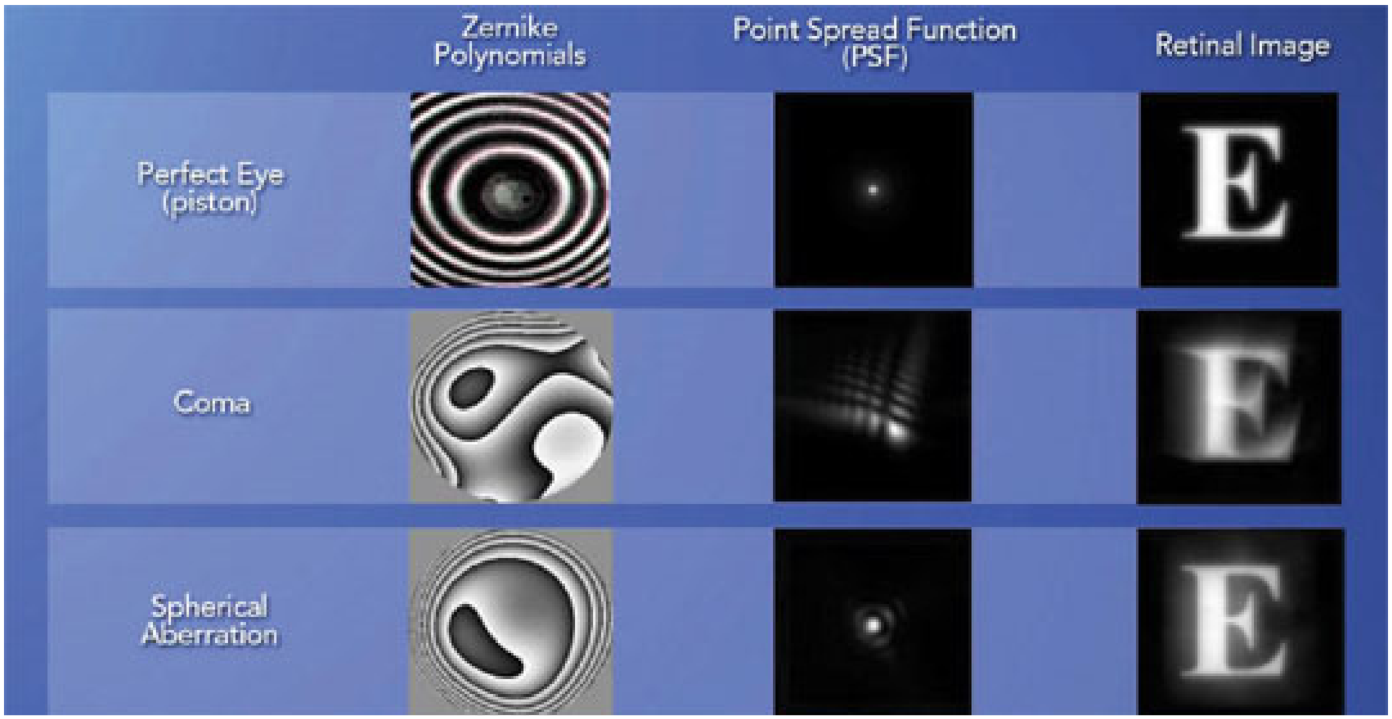
For infinite object distance

$$c = \frac{f \cdot \phi}{d_2} = \frac{f^2}{f\# d_2} = \frac{2NAf^2}{d_2}$$

$$NA = \sin\theta = \phi/(2f) = 1/2f\#$$

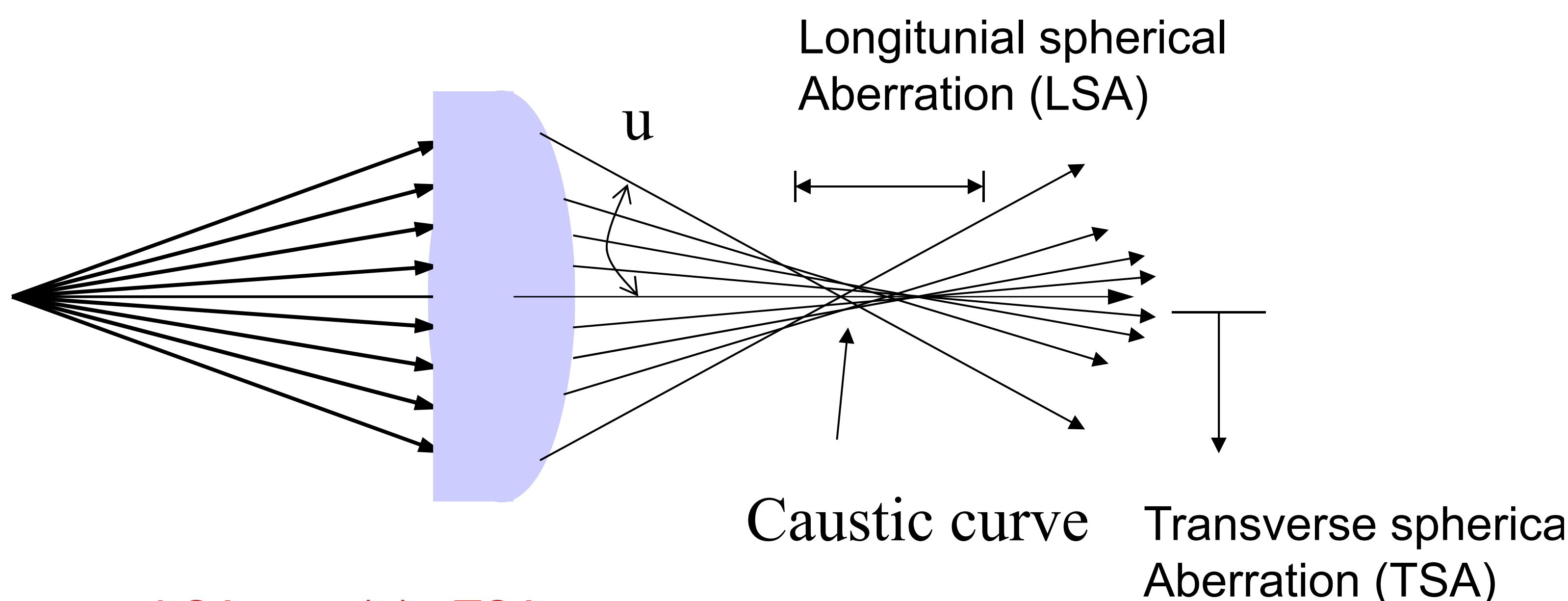
In an imperfect lens *L*, all the rays do not pass through a focal point. The smallest circle that they pass through *C* is called the circle of confusion

Vision affected by Spherical Aberration and Coma



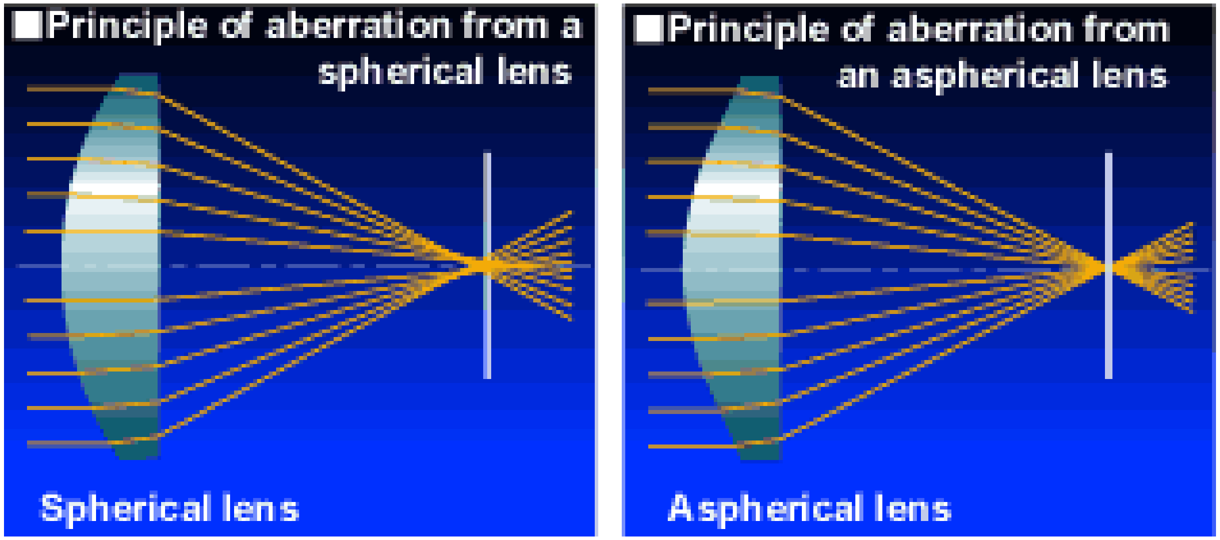
Spot Size limited by Spherical Aberration

- Spot size due to spherical aberration is 0.067f/f-number^3



LSA x tan(u)= TSA

Aspherical lens



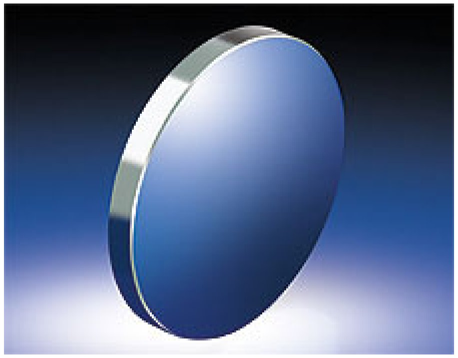
Canon)

With the spherical lens, rays coming from the lens periphery form the image before the ideal focal point. For this reason, the spherical aberration (blurred image) occurs at the center portion of the image formed. With the aspherical lens, on the contrary, even the rays coming from the lens periphery agree on the focal point, thus forming a sharp image.

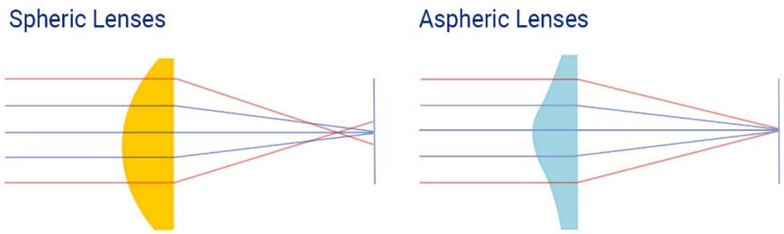
Example of Aspheric Lens

Edmund Optics 25mm Dia x 12.5mm FL Uncoated, Ge Aspheric Lens

Type of Optics	DCX Lens
Diameter (mm)	25.0
Diameter Tolerance (mm)	+0.0/-0.1
Clear Aperture (%)	90
Effective Focal Length EFL (mm)	12.5
Numerical Aperture NA	1.00
Back Focal Length BFL (mm)	11.61
Center Thickness CT (mm)	4.24
Center Thickness Tolerance (mm)	±0.10
Surface Quality	60-40
Surface Accuracy, P-V (μm)	0.3
Centering (arcmin)	3 - 5
Edges	Diamond Turned
Coating	Uncoated
Focal Length Specification Wavelength (μm)	4
Substrate	Germanium (Ge)
f/#	0.5
Type	Aspheric Lens
Wavelength Range (μm)	2 - 14
Wavelength Range (nm)	2000 - 14000
RoHS	Compliant



aspheric lenses have a more complex front surface that **gradually changes in curvature from the center of the lens out of the edge of the lens**



Thick Lens

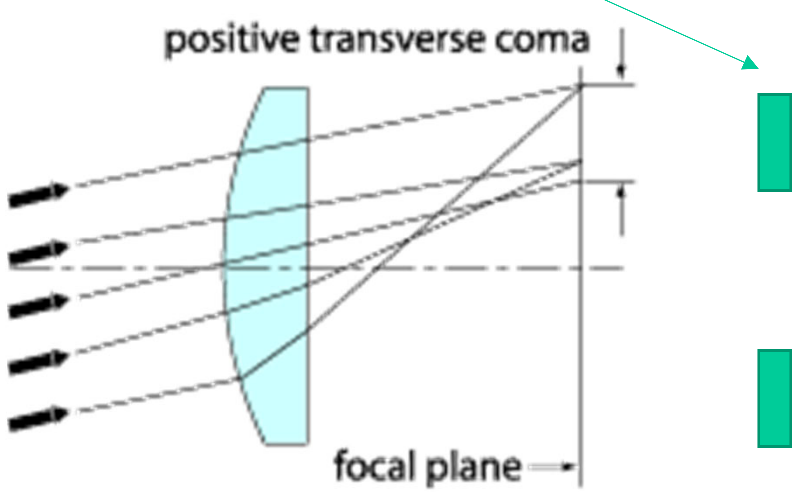
TYPE ORIENTATION		Focal Length f	BFD	FFD
GENERAL	$R_1 = R_1$, $R_2 = R_2$	$\left[(n-1)\cdot\left(\frac{1}{R_1}-\frac{1}{R_2}\right)+\frac{t_c(n-1)^2}{nR_1R_2}\right]^{-1}$	$f\cdot\left[1-\frac{t_c(n-1)}{nR_1}\right]$	$f\cdot\left[1+\frac{t_c(n-1)}{nR_2}\right]$
PLANO-CONVEX	$R_1 = R$, $R_2 = \infty$	$\frac{R}{n-1}$	$f\cdot\left[1-\frac{t_c(n-1)}{nR_1}\right]$	f=infinity
PLANO-CONCAVE	$R_1 = -R$, $R_2 = \infty$	$\frac{-R}{n-1}$	$f\cdot\left[1+\frac{t_c(n-1)}{nR_2}\right]$	f= infinity
BI-CONVEX	$R_1 = R$, $R_2 = -R$	$\left[\frac{2(n-1)}{R}-\frac{t_c(n-1)^2}{nR^2}\right]^{-1}$	$f\cdot\left[1-\frac{t_c(n-1)}{nR_1}\right]$	$f\cdot\left[1-\frac{t_c(n-1)}{nR_1}\right]$
BI-CONCAVE	$R_1 = -R$, $R_2 = R$	$-\left[\frac{2(n-1)}{R}-\frac{t_c(n-1)^2}{nR^2}\right]^{-1}$	$f\cdot\left[1+\frac{t_c(n-1)}{nR_2}\right]$	$f\cdot\left[1+\frac{t_c(n-1)}{nR_2}\right]$

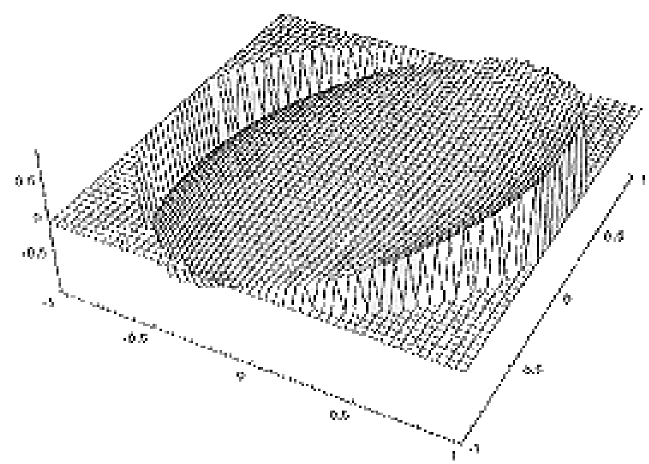
W. Wang

CVI Laser LLC

Coma

In spherical lenses, different parts of the lens surface exhibit different degrees of magnification. This gives rise to an aberration known as coma. Even if spherical aberration is corrected and the lens brings all rays to a sharp focus on axis, a lens may still exhibit coma off axis. As with spherical aberration, correction can be achieved by using multiple surfaces. Alternatively, a sharper image may be produced by judiciously placing an **aperture**, or stop, in an optical system to eliminate the more marginal rays.

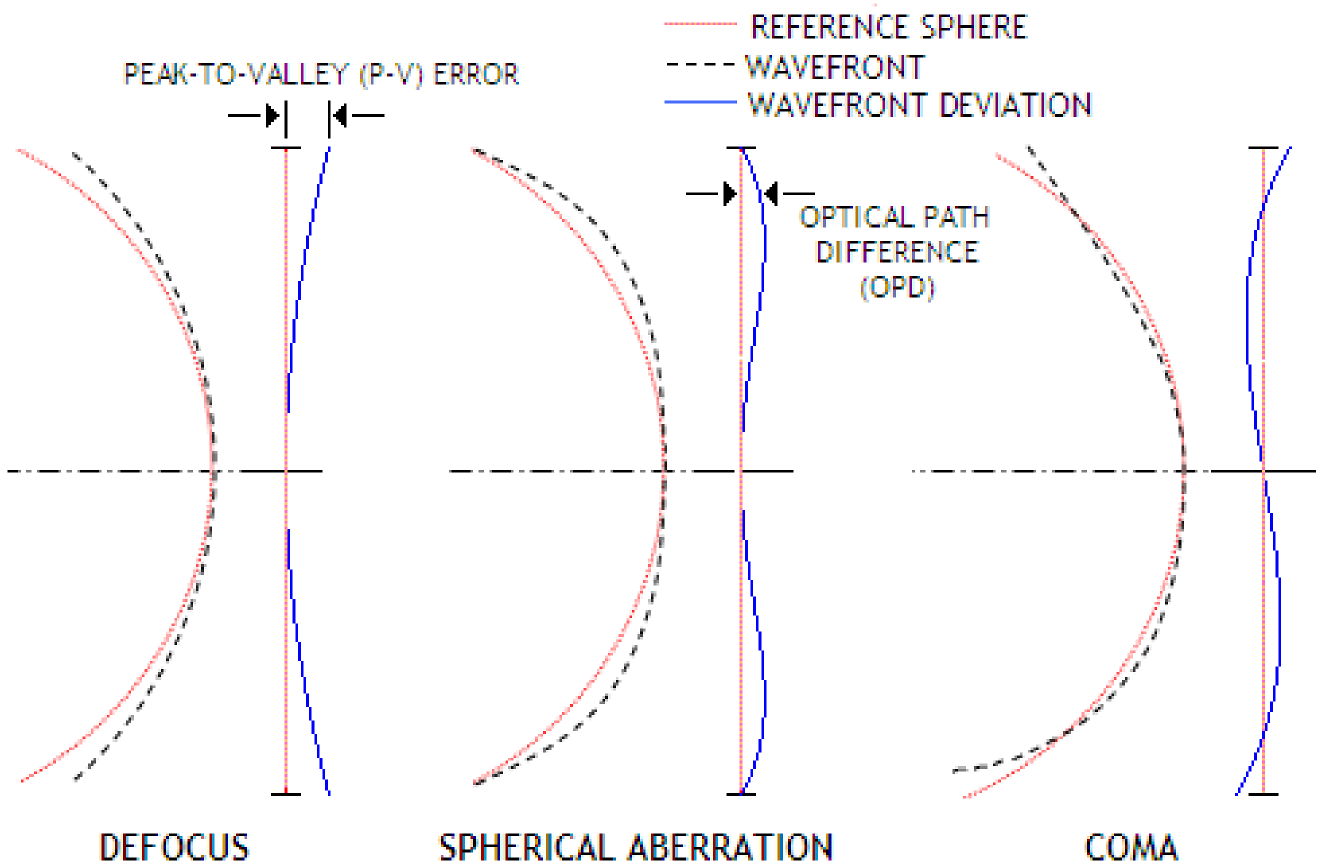




A wavefront with coma

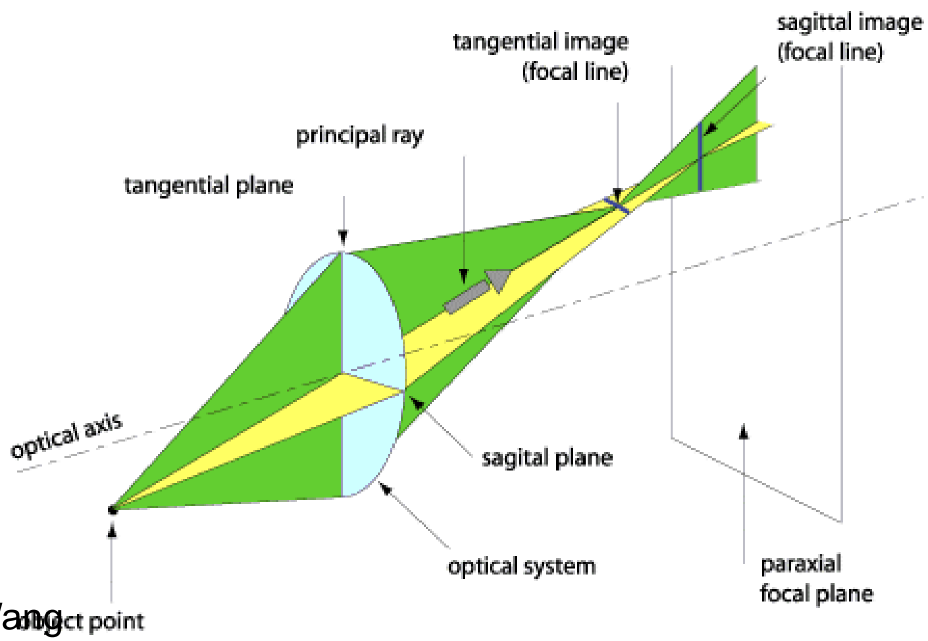
Melles Griot

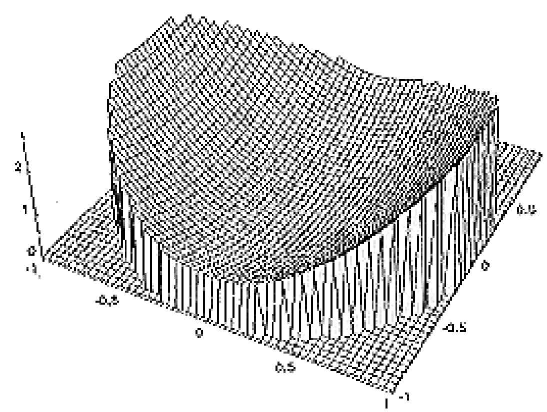
Wave front distortion by Spherical Aberration and Coma



Astigmatism

When an off-axis object is focused by spherical lens, the natural asymmetry leads to astigmatism. The amount of astigmatism in a lens depends on lens shape. The figure illustrates that tangential rays from the object come to a focus closer to the lens than do rays in the sagittal plane. When the image is evaluated at the tangential conjugate, we see a line in the sagittal direction. A line in the tangential direction is formed at the sagittal conjugate. Between these conjugates, the image is either an elliptical or a circular blur. Astigmatism is defined as the separation of these conjugates.



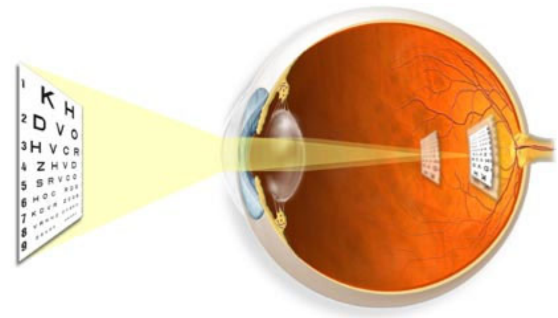


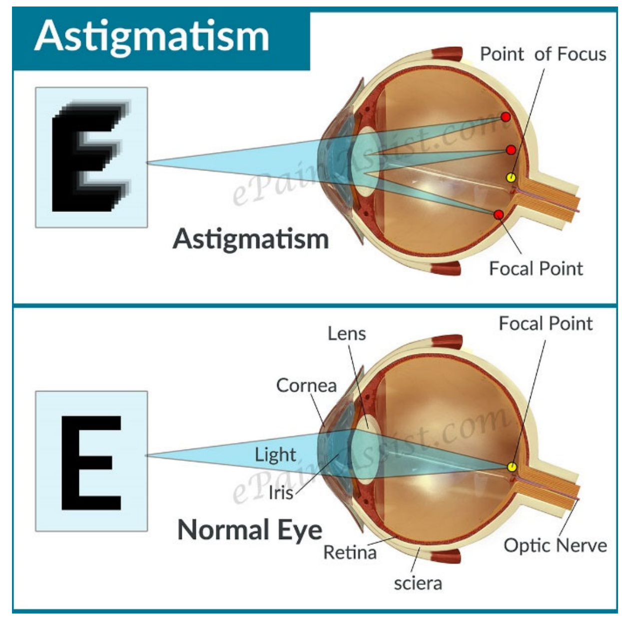
An astigmatic wavefront

Melles Griot

Astigmatism means that the cornea is oval like a football instead of spherical like a basketball. Most astigmatic corneas have two curves – a steeper curve and a flatter curve. This causes light to focus on more than one point in the eye, resulting in blurred vision at distance or near. Astigmatism often occurs along with nearsightedness or farsightedness.

Astigmatism usually occurs when the front surface of the eye, the cornea, has an **irregular curvature. Normally the cornea is smooth and equally curved in all directions** and light entering the cornea is focused equally on all planes, or in all directions. In astigmatism, the front surface of the cornea is **curved more in one direction than in the other. T**his abnormality may result in vision that is much like looking into a distorted, wavy mirror. The distortion results because of an inability of the eye to focus light rays to a point.





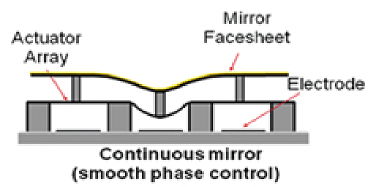
With Astigmatism (same focus clarity)
One eye: 45 degrees

W. Wang

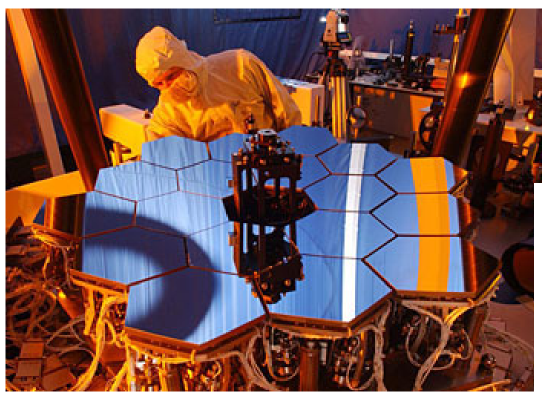
W.Wang

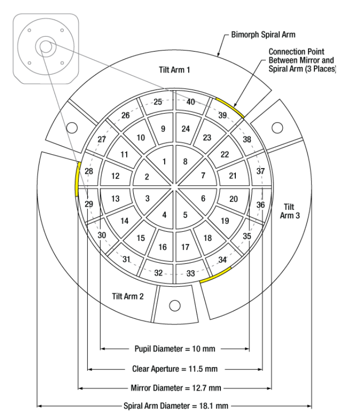
Research Topics: Wave front Correcting Lens or mirrors

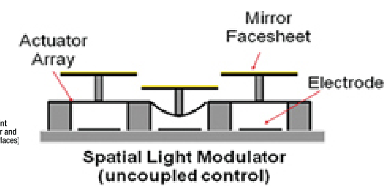
- Active Aspherical or Achromatic Lens
- Use Deformable Mirror to correct spherical aberration and prevent chromatic aberration.



How to do it with lens then if you can't have your actuator in the back?







Deformable mirrors (adaptive optics)

Spatial Light Modulator

- LC based (correcting phase shift by index change)

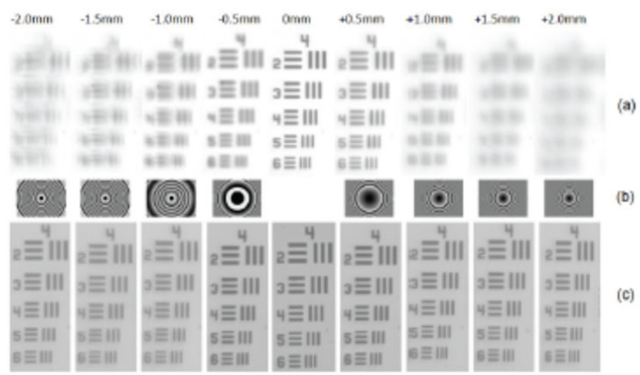


Figure 5. Experimental results, (a) The images acquired with the normal (without SLM) optical system at various object locations (b) the phase mask patterns at the SLM (c) The images acquired with our adaptive focal length systems.

Indian Journal of Science and Technology, Vol 9(48),

Lens Materials

Material	Usable Transmission Range	Index of Refraction	Features
BK7	BK7	1.52 @ 0.55 μm	Excellent all-around lens material with broad transmission and excellent mechanical properties
LaSFN9	LaSFN9	1.86 @ 0.55 μm	High-refractive-index flint glass provides more power with less curvature
SF11	SF11	1.79 @ 0.55 μm	High-refractive-index flint glass provides more power with less curvature
F2	F2	1.62 @ 0.55 μm	A good compromise between higher index and acceptable mechanical properties
BaK1	BaK1	1.57 @ 0.55 μm	Excellent all-around lens material, but has weaker chemical characteristics than BK7
Optical-Quality Synthetic Fused Silica (OQSFS)	OQSFS	1.46 @ 0.55 μm	Material provides good UV transmission and superior mechanical performance
UV-Grade Synthetic Fused Silica (UVGSFS)	UVGSFS	1.46 @ 0.55 μm	Material provides excellent UV transmission and superior mechanical performance
Optical Crown Glass	OPTICAL CROWN	1.52 @ 0.55 μm	Lower tolerance glass for mirror substrates and noncritical applications
Low-expansion borosilicate glass (LEBG)	LEBG	1.48 @ 0.55 μm	Excellent thermal stability, homogeneity, and low cost. Ideal for high-temperature windows, mirror substrates, and condenser lenses
Sapphire	SAPPHIRE	1.77 @ 0.55 μm	Excellent mechanical and thermal characteristics. Ideal material for optical windows.
Calcium Fluoride	CALCIUM FLUORIDE	1.399 @ 5 μm	This popular UV excimer laser material is used for windows, lenses, and mirror substrates

01 05 10 50 100

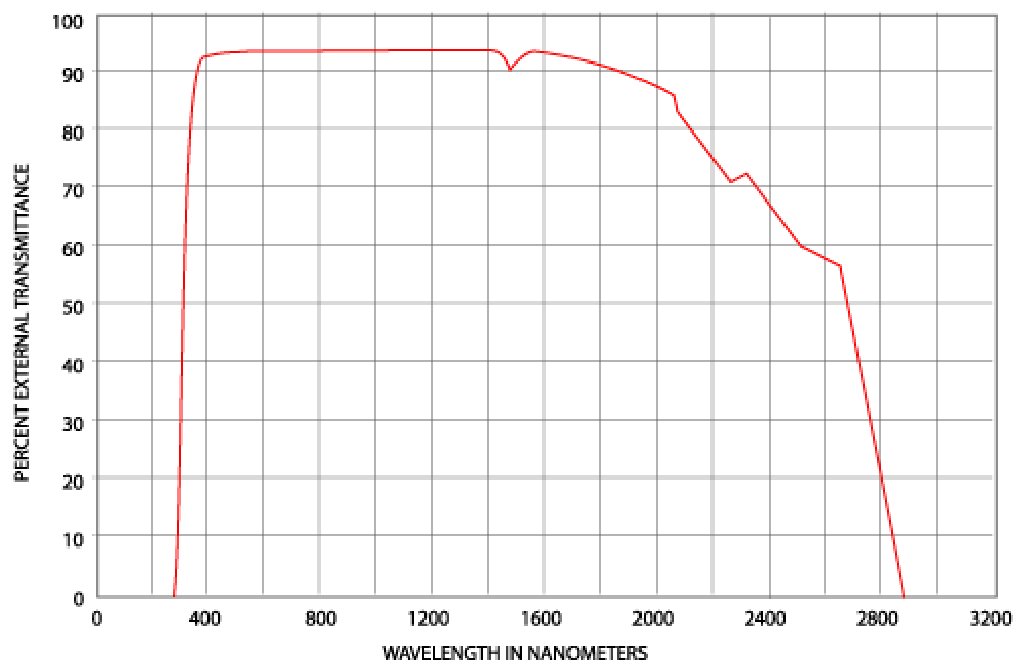
WAVELENGTH IN mm

W.Wang

BK7 Glass

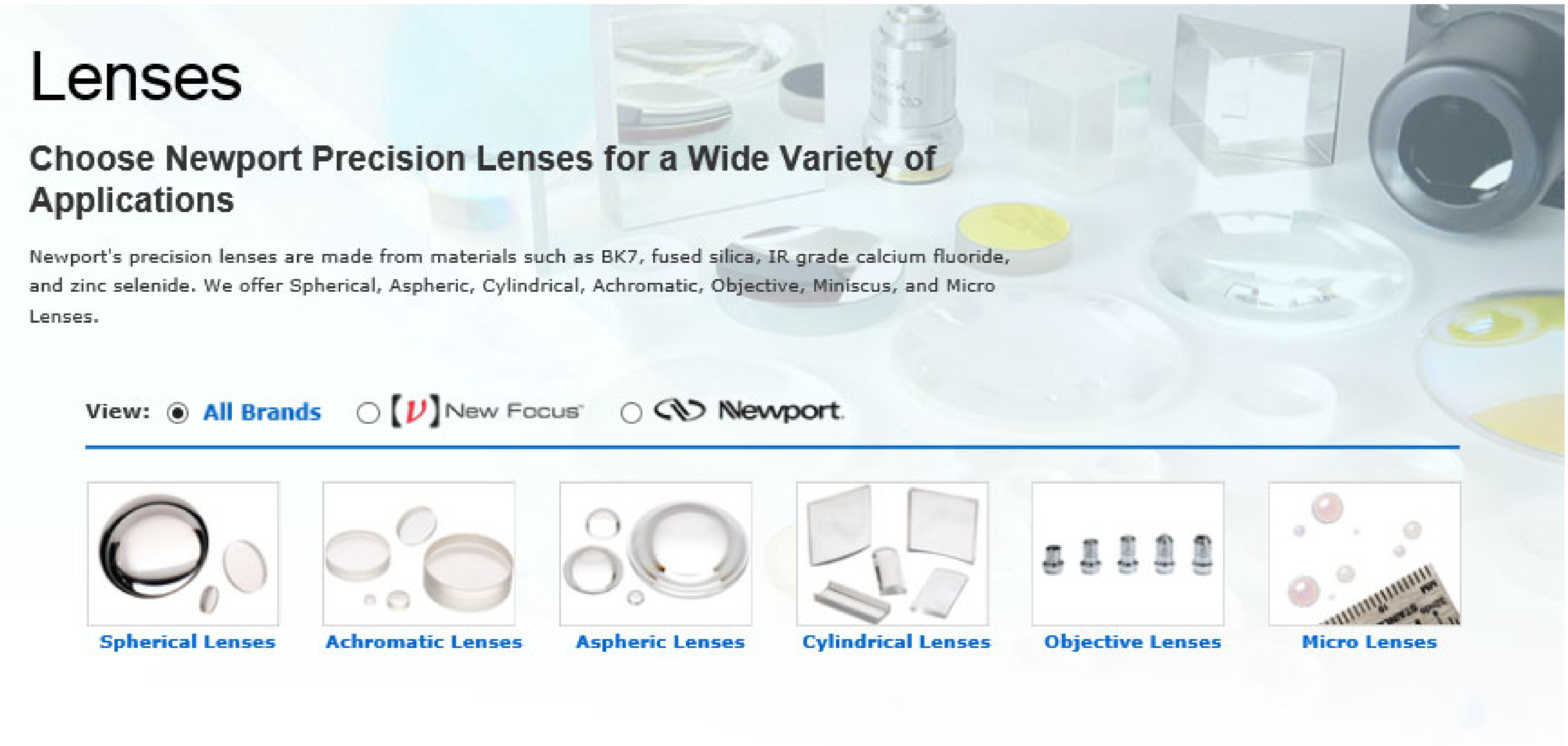
Abbé Constant: 64.17
Density:2.51 g/cm^{-3}
Young's Modulus: 8.20 x 10^9 dynes/mm^2
Poisson's Ratio: 0.206
Coefficient of Thermal Expansion (-30° to +70°C): 7.1 x 10^{-6}/°C
Coefficient of Thermal Expansion (20° to 3000°C): 8.3 x 10^{-6}/°C
Stress Birefringence, (Yellow Light): 10 nm/cm
Homogeneity within Melt: ±1x10^{-4}
Striae Grade (MIL-G-174-A): A
Transformation Temperature: 557°C
Climate Resistance: 2
Stain Resistance: 0
Acid Resistance: 1.0
Alkali Resistance: 2.0
Phosphate Resistance: 2.3
Knoop Hardness: 610
Dispersion Constants:

B_1 = 1.03961212
B_2 = 2.31792344x10^{-1}
B_3 = 1.01046945
C_1 = 6.00069867x10^{-3}
C_2 = 2.00179144x10^{-2}
C_3 = 1.03560653x10^2

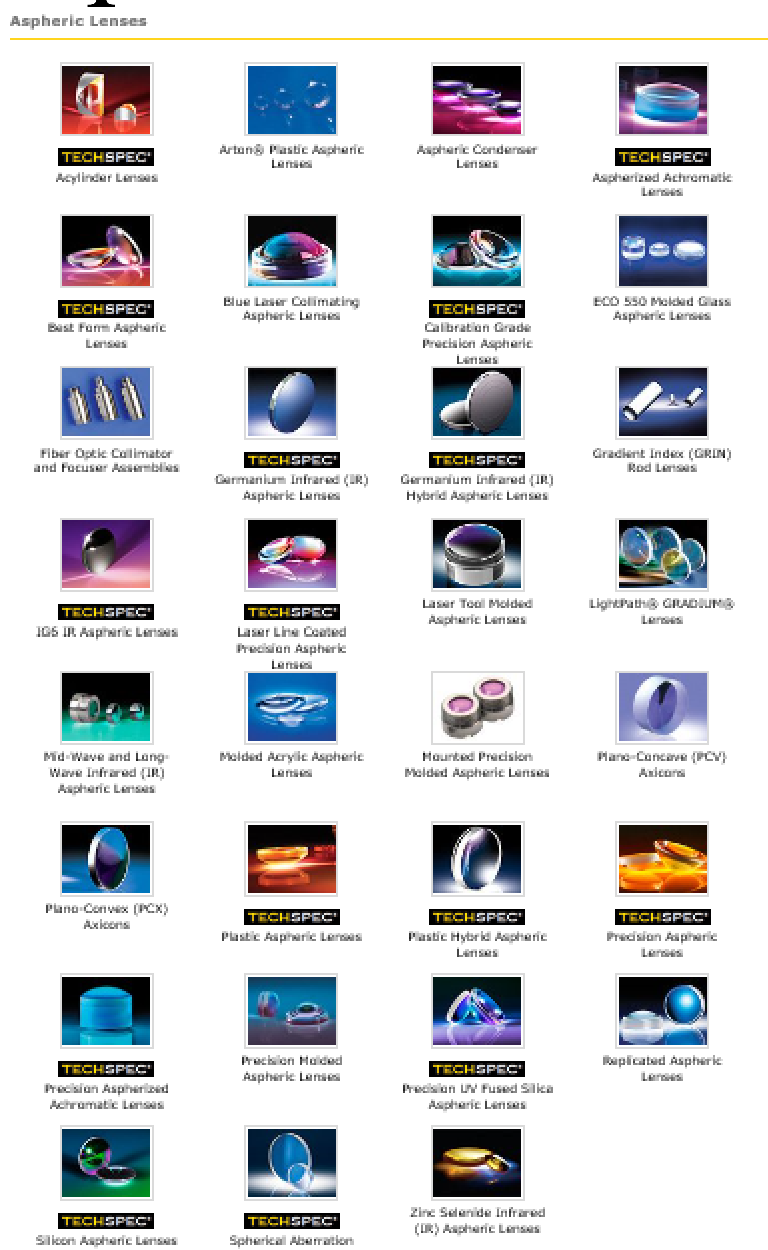


External transmission for 10-mm-thick BK7 glass

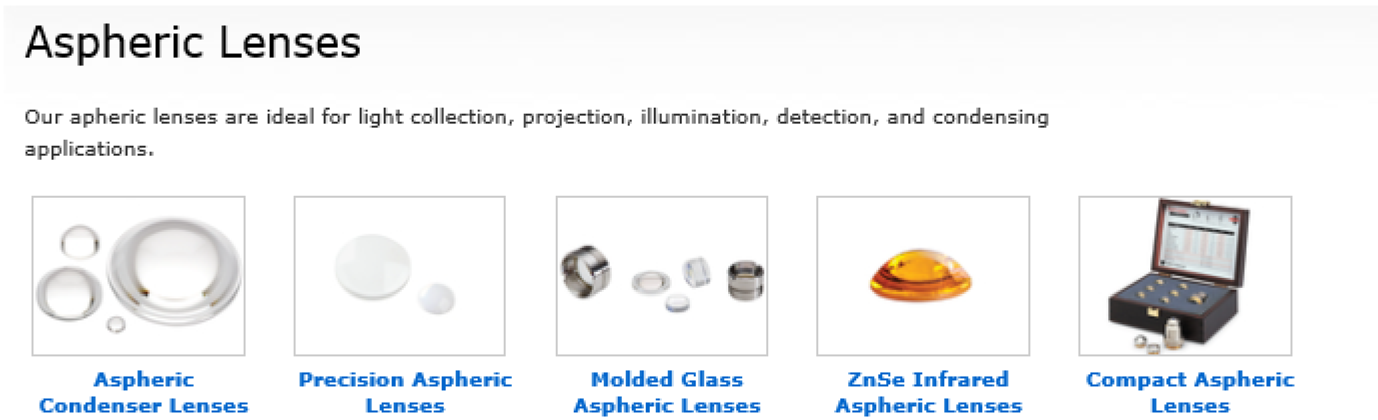
Optical Lenses



Aspherical Lenses



Aspherical Lens

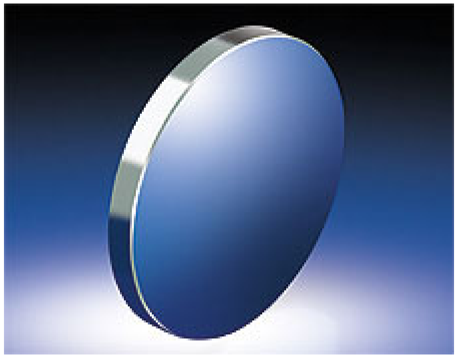


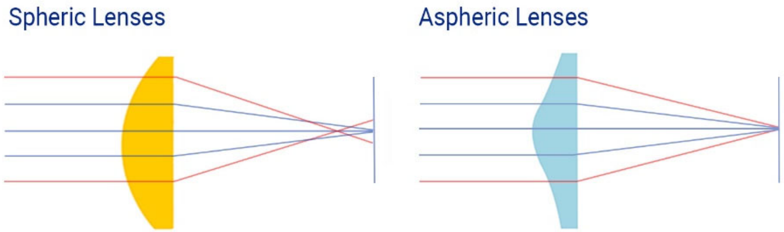
Aspheric Lenses are used to eliminate spherical aberration in a range of applications, including bar code scanners, laser diode collimation, or OEM or R&D integration. Aspheric lenses utilize a single element design which helps minimize the number of lenses found in multi-lens optical assemblies. Said another way, unlike conventional lenses with a spherical front surface, aspheric lenses have a more complex front surface that gradually changes in curvature from the center of the lens out of the edge of the lens. This reduction in total element count not only helps decrease system size or weight, but also simplifies the assembly process. Integrating aspheres into an application such as focusing the output of a laser diode may not only decrease total cost, but may also outperform assemblies designed with traditional spherical optical lenses.

Example of Aspheric Lens

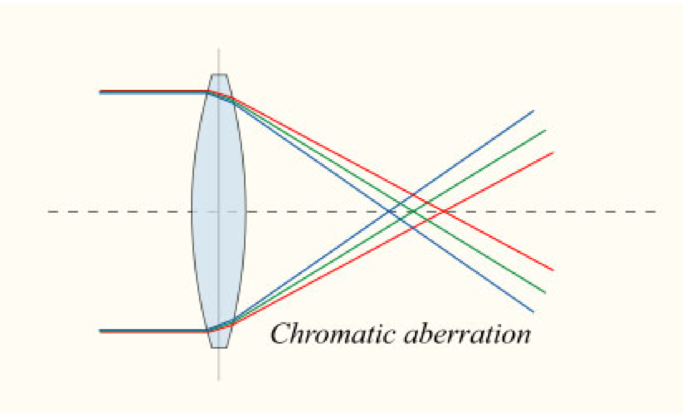
Edmund Optics 25mm Dia x 12.5mm FL
Uncoated, Ge Aspheric Lens

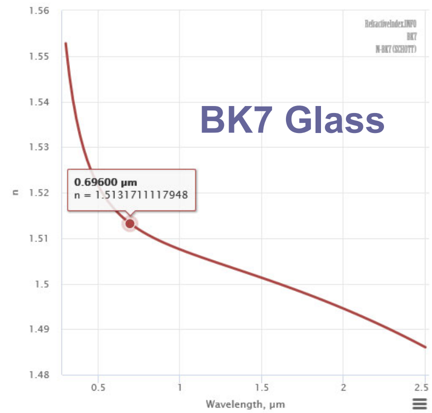
Type of Optics	DCX Lens
Diameter (mm)	25.0
Diameter Tolerance (mm)	+0.0/-0.1
Clear Aperture (%)	90
Effective Focal Length EFL (mm)	12.5
Numerical Aperture NA	1.00
Back Focal Length BFL (mm)	11.61
Center Thickness CT (mm)	4.24
Center Thickness Tolerance (mm)	±0.10
Surface Quality	60-40
Surface Accuracy, P-V (μm)	0.3
Centering (arcmin)	3 - 5
Edges	Diamond Turned
Coating	Uncoated
Focal Length Specification Wavelength (μm)	4
Substrate	Germanium (Ge)
f/#	0.5
Type	Aspheric Lens
Wavelength Range (μm)	2 - 14
Wavelength Range (nm)	2000 - 14000
RoHS	Compliant





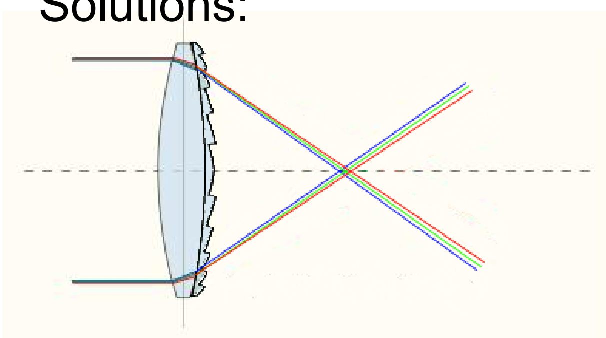
Chromatic Aberration

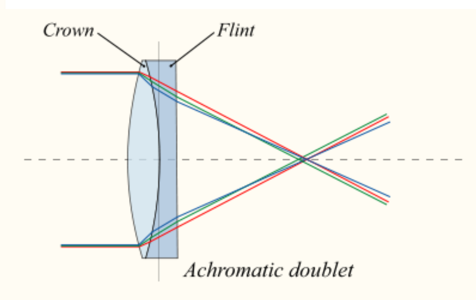




Chromatic aberration of a single lens causes different wavelengths of light to have differing focal lengths

Solutions:





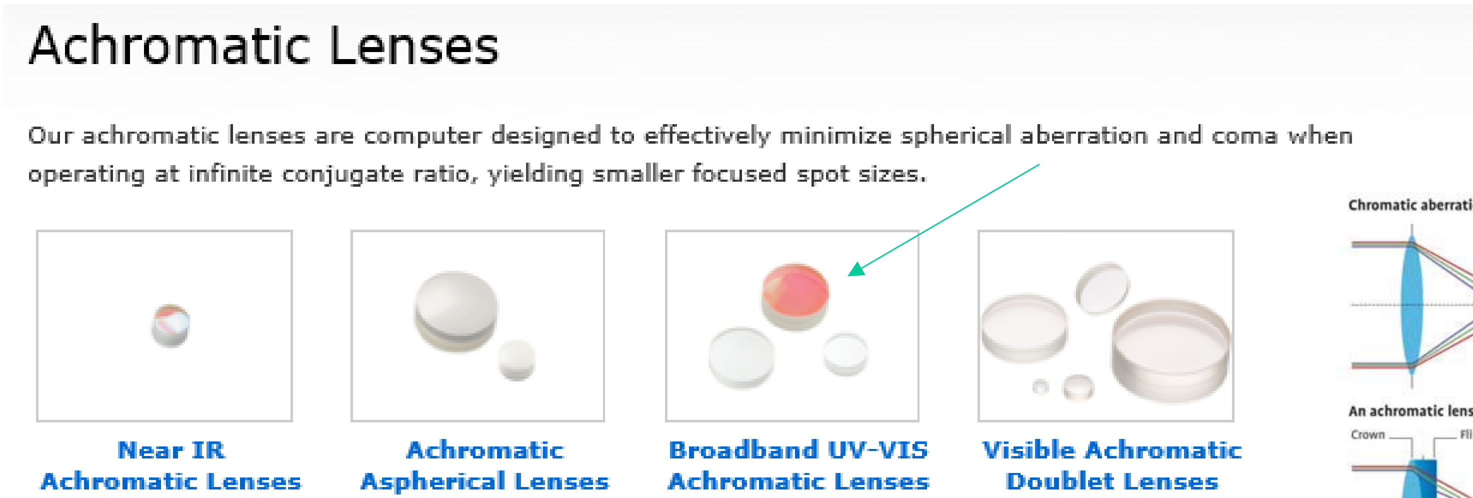
Diffractive optical element with complementary dispersion properties to that of glass can be used to correct for color aberration

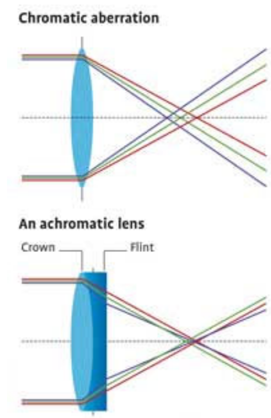
For an achromatic doublet, visible wavelengths have approximately the same focal length

Above with correction
Below without correction

wikipedia

Achromatic Lens



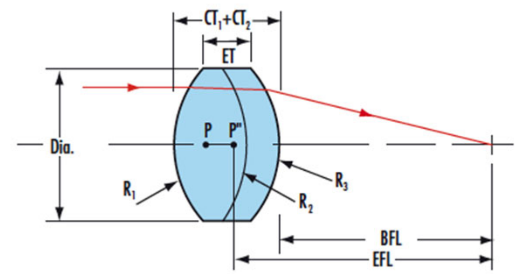


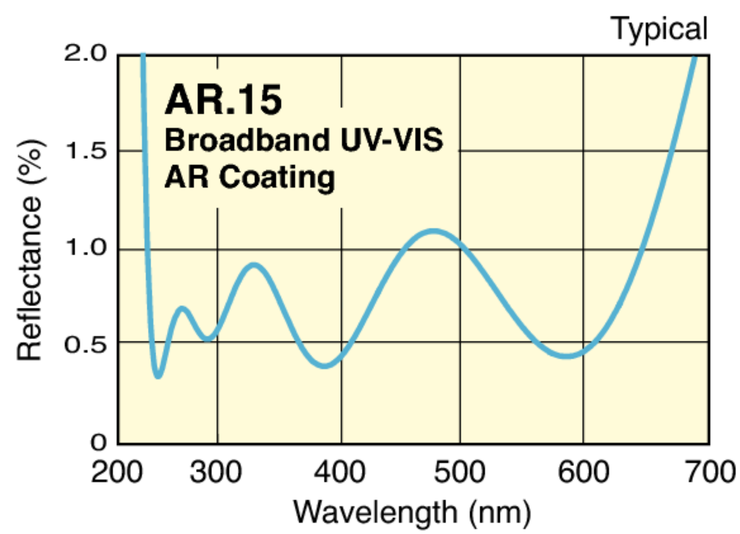
Achromatic Lenses are used to minimize or eliminate chromatic aberration. The achromatic design also helps minimize spherical aberrations. Achromatic Lenses are ideal for a range of applications, including fluorescence microscopy, image relay, inspection, or spectroscopy. An Achromatic Lens, which is often designed by either cementing two elements together or mounting the two elements in a housing, creates smaller spot sizes than comparable singlet lenses.

Example of Achromatic Lens

Newport Broadband Achromatic L
12.50 mm, 40.00 mm EFL, 360 nn
nm

Model	PAC12AR.15
Lens Shape	Plano-Convex
Antireflection Coating	345-700 nm
Lens Material	N-FK5 & F2
Diameter	0.49 in. (12.5 mm)
Effective Focal Length	40.0 mm
Surface Quality	40-20 scratch-dig
Center Thickness	5.5 mm
T_e	4.30 mm
Lens Type	Achromatic Doublet
BFL	37.61 mm
f/#	3.2
R	24.47 mm
R_2	-19.17 mm
R_3	-50.19 mm
Clear Aperture	11.50 mm
Chamfers	0.25 mm
Chamfers Angle/Tolerance	45°
Center Thickness (T_c) Tolerance	± 0.1 mm
Edge Thickness (T_e) Tolerance	Reference
Focal length tolerance	±2%
Coating Code	AR.15
T_{c1}	3.50 mm
T_{c2}	2.00 mm





Achromatic Lenses

Anti-Reflection Coatings

Without anti-reflection With anti-reflection

Geometric Optics

We define a **ray** as the path along which light energy is transmitted from one point to another in an optical system. It represents location and direction of energy transfer ad direction of light propagate. The basic laws of geometrical optics are the law of reflection and the law of refraction.

Law of reflection: $|\theta_r| = |\theta_i|$
Snell's law, or **the law of refraction**: $n_i \sin\theta_i = n_t \sin\theta_t$.

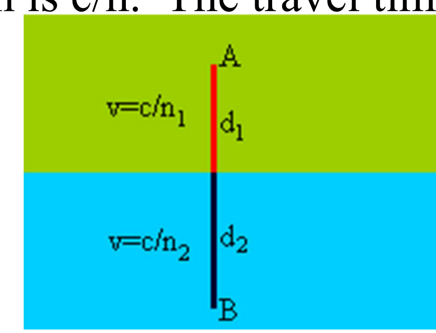
If not being reflected or refracted, a light ray travels in a straight line.

The **optical path length** of a ray traveling from point A to point B is defined as c times the time it takes the ray to travel from A to B.

Assume a ray travels a distance d_1 in a medium with index of refraction n_1 and a distance d_2 in a medium with index of refraction n_2.

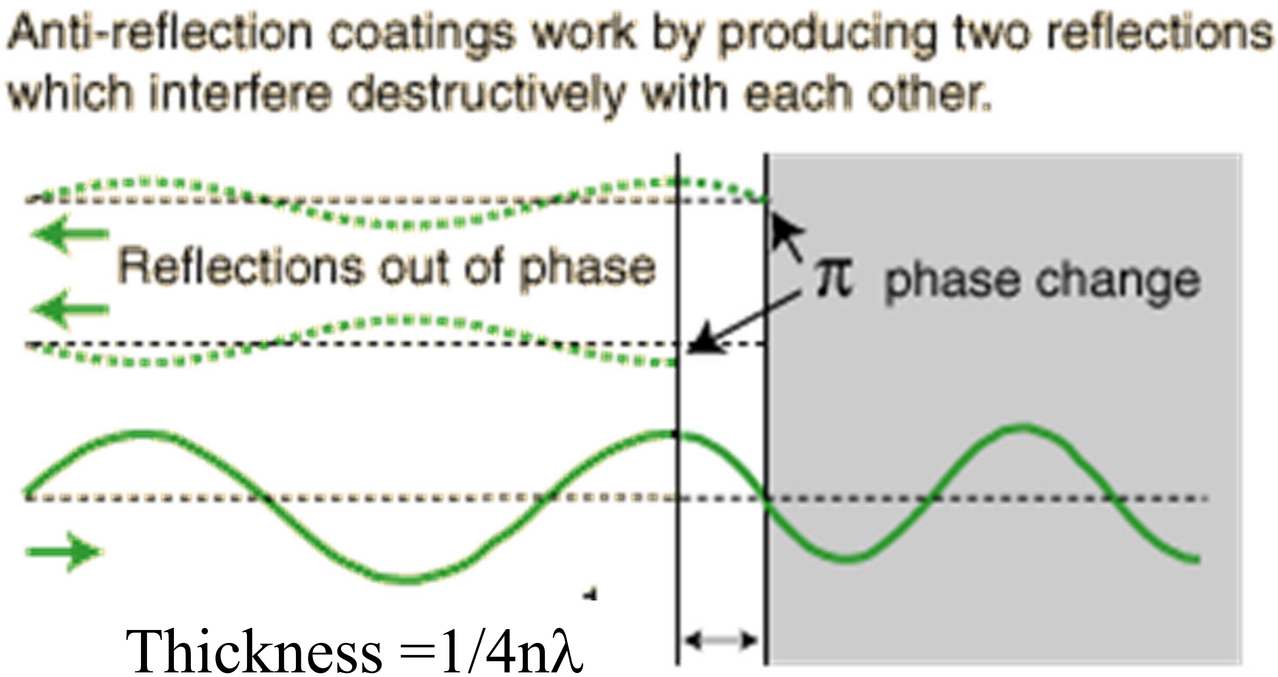
The speed of light in a medium with index of refraction n is c/n. The travel time from A to B therefore is

$$t = n_1 d_1/c + n_2 d_2/c.$$



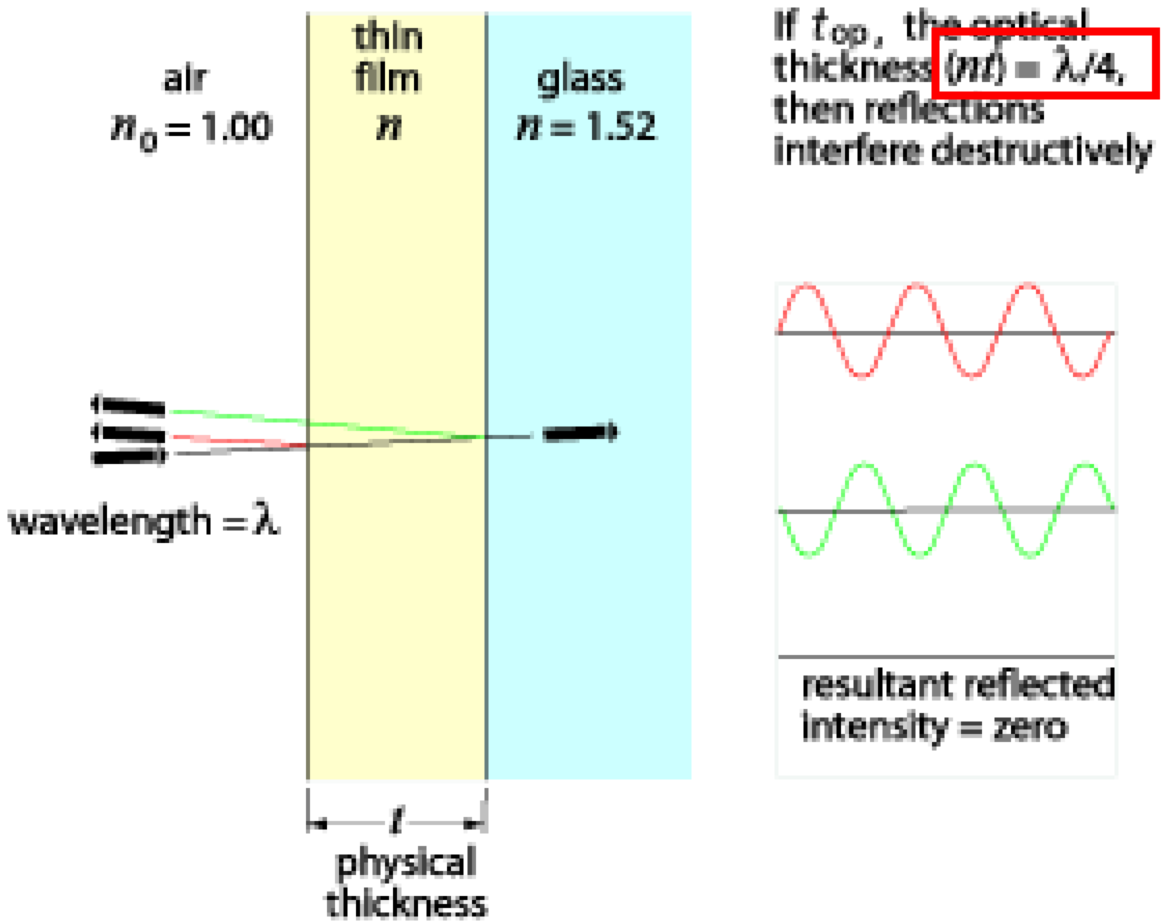
The optical path length is OPL = $n_1 d_1 + n_2 d_2$.

Anti-Reflection Coatings



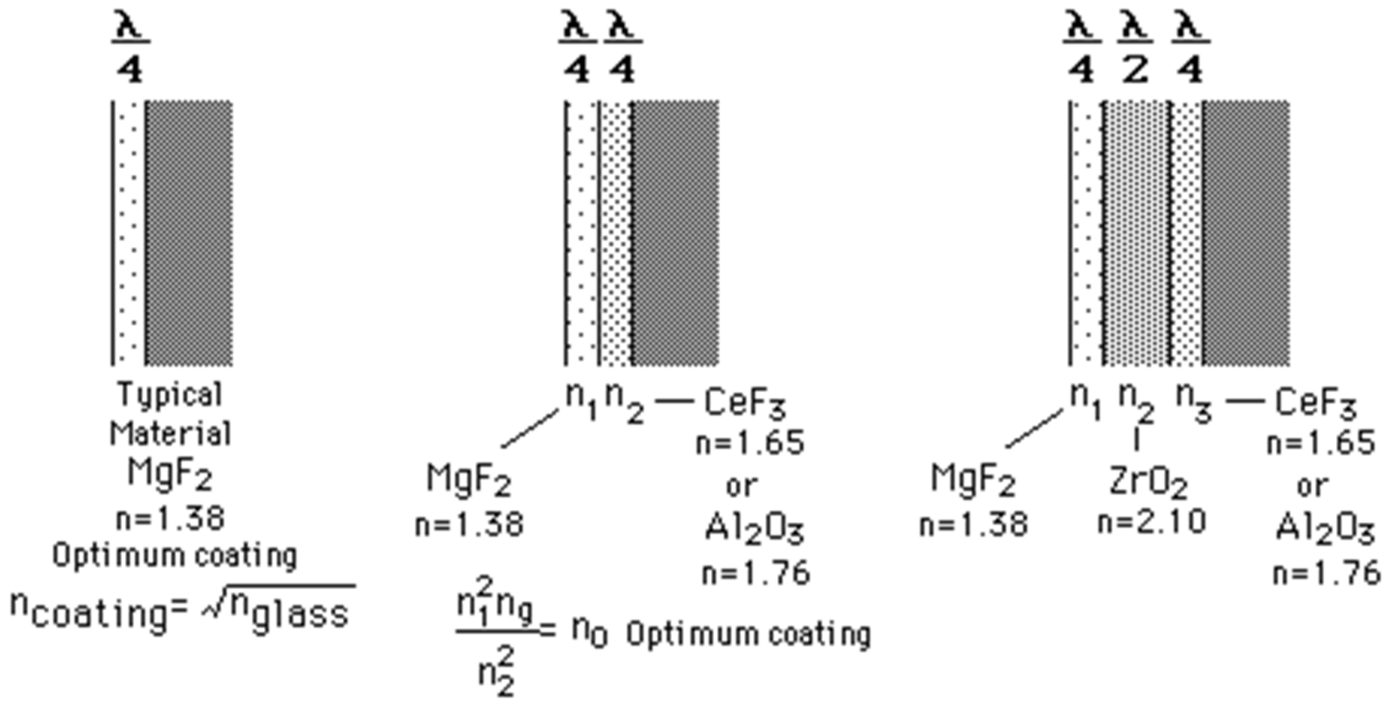
OPL = nL= 1/4λ

Antirefelction coating



OPL

Multi-Layer Anti-Reflection Coatings

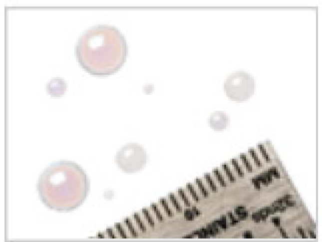


Micro Lenses

Micro Lenses

mico lenses are commonly used for laser collimating and focusing, laser-to-fiber coupling, fiber-to-fiber coupling, and fiber-to-detector coupling.

Gradient Index Micro Lenses



Spherical Ball Micro Lenses

Microlens Arrays

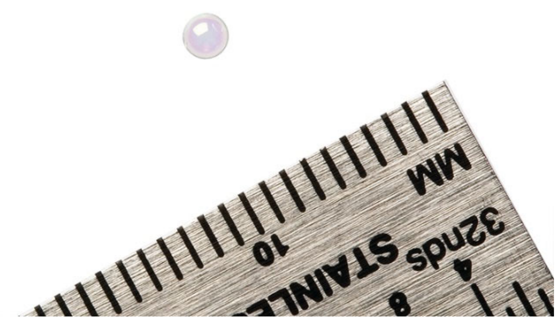
$$2W_o' = \frac{4}{\pi}\lambda(f/\phi); f/\phi = F\#$$

$$1/f = (n-1)(1/R_1 - 1/R_2)$$

$$1/p + 1/q = 1/f$$

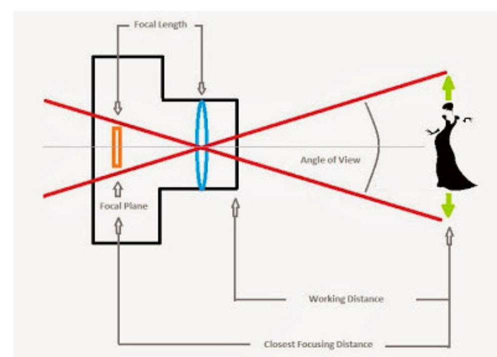
Example of Micro Lens

Newport Spherical Ball Micro Lens, 1.0 mm 0.55 mm FL, 0.05 mm WD, Uncoated

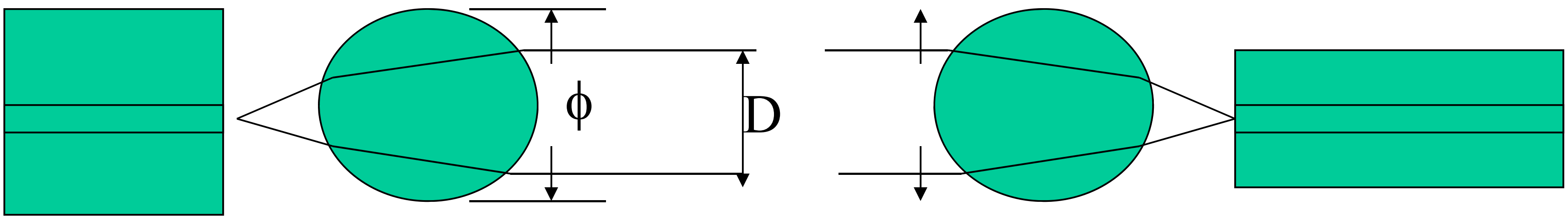


Model	LB1
Lens Shape	Ball
Diameter	0.04 in. (1.0 mm) ~ ϕ
Lens Material	Grade A LaSF N9
Antireflection Coating	Uncoated
Effective Focal Length	0.55 mm
Working Distance	0.05 mm
Clear Aperture	0.8 mm = ϕ
Diameter Tolerance	± 1 µm

$$\left[\frac{2(n-1)}{R} - \frac{t_c(n-1)^2}{nR^2}\right]^{-1}$$



Micro Lens: Spherical ball lenses for fiber coupling



Spheres are arranged so that fiber end is located at the focal points. The output from the first sphere is then collimated. If two are Aligned axially to each other, the beam will be transferred from one focal point to another.

F-number and Numerical Aperture of Lens

The f-number (focal ratio) is the ratio of the focal length f of the lens to its clear aperture ϕ (effective diameter). The f-number defines the angle of the cone of light leaving the lens which ultimately forms the image. This is important concept when the throughput or light-gathering power of an optical system is critical, such as when focusing light into a monochromator or projecting a high power image.):

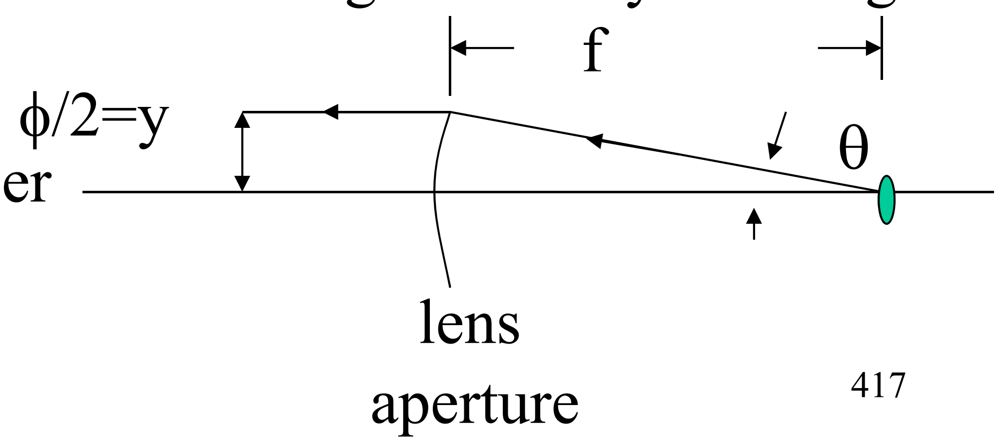
$$\text{f-number} = f/\phi$$

Numeric aperture is defined as sine of the angle made by the marginal ray with the optical axis:

$$NA = \sin\theta = \phi/(2f) = 1/2\text{f-number}$$

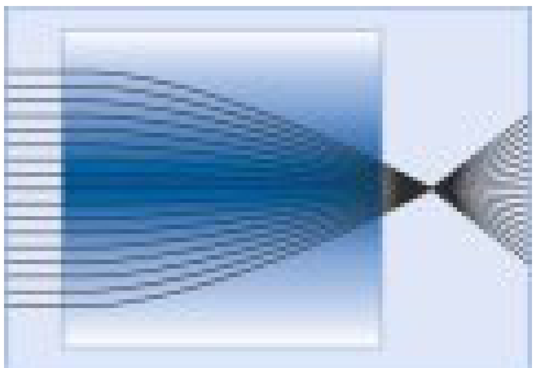
↑

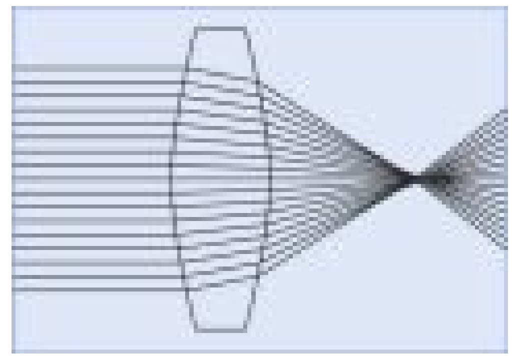
Acceptance angle



GRIN Lens

Gradient index micro lenses represent an innovative alternative to conventional spherical lenses since the lens performance depends on a continuous change of the refractive index within the lens material.





1. GRIN objective lenses with **an angle of view of 60° are produced in standard diameters of 0.5, 1.0 und 1.8 mm**. Typical **object distances are between 5 mm and infinity.**

2. Instead of curved shaped surfaces only plane optical surfaces are used which facilitate assembly. The light rays are continuously bent within the lens until finally they are focused on a spot.

W.Wang

Ray in Inhomogeneous Medium

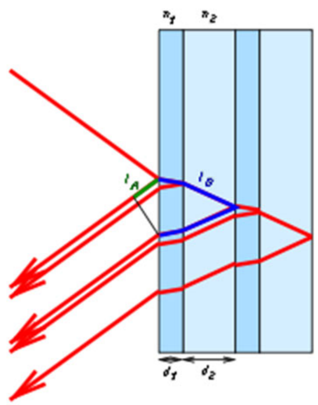
In an inhomogeneous medium, the refractive index varies in a continuous manner and, in general, the ray paths are curved!

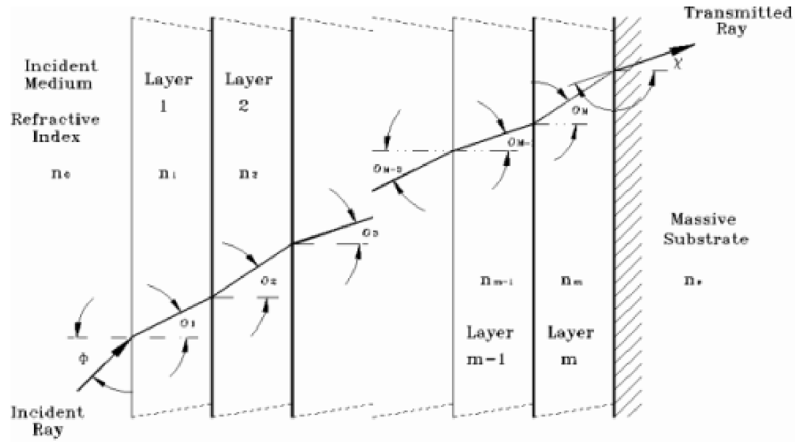
At each interface, the light ray satisfies Snell's law and one obtains:

$n_1 \sin\theta_1 = n_2 \sin\theta_2 = n_3 \sin\theta_3 = ...$

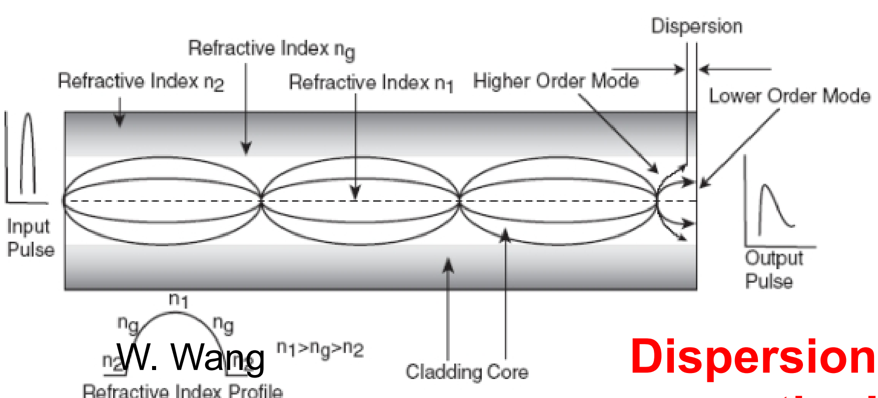
Thus, we may state that the product

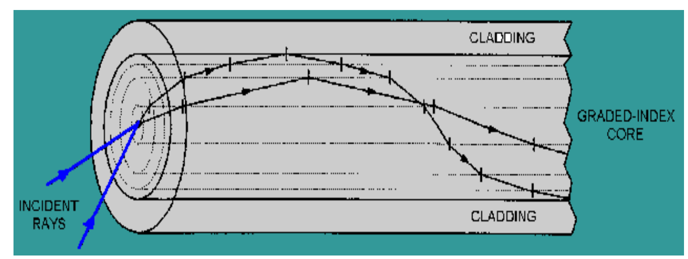
$n(x)\sin\theta(x) = n(x)\sin\phi(\phi)$





Ray bend along layered structure gradually changing index





Example of rays travel inside a graded index multimode fiber

Dispersion correction!!!!!

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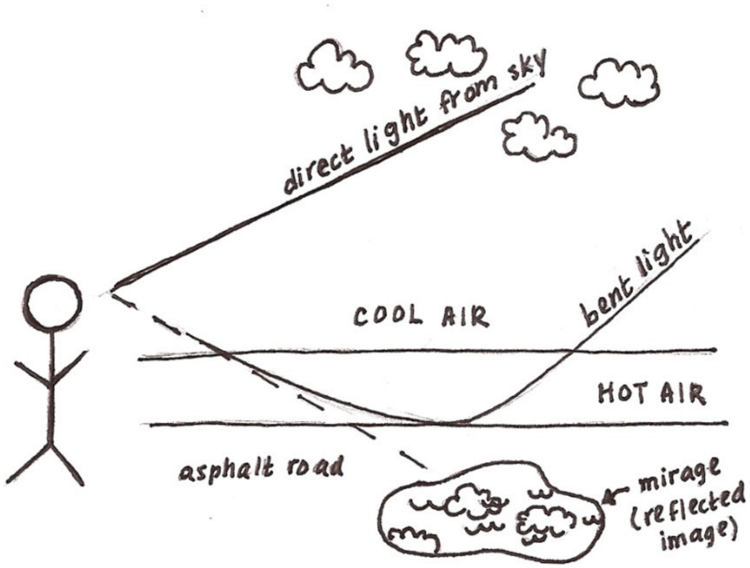
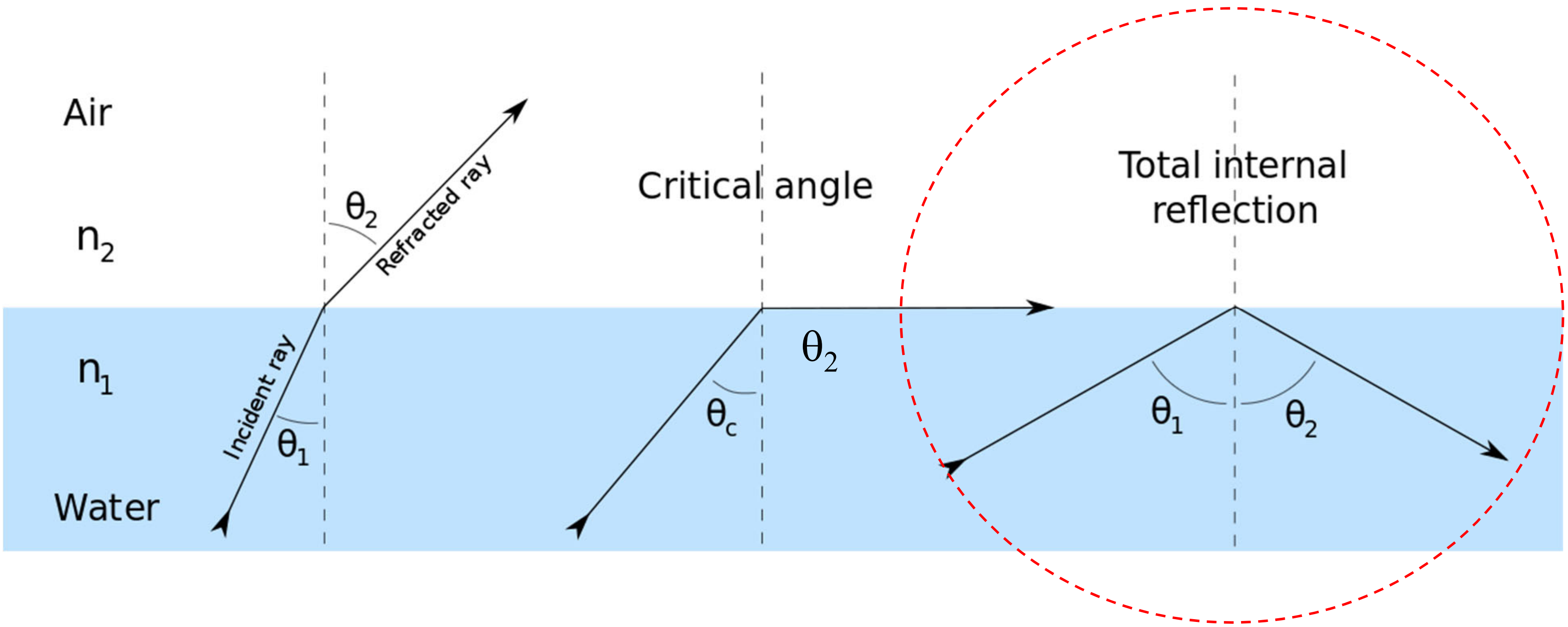
Inferior Mirage



Due to total internal reflection and gradient indices
Able to see the mirage and with correct colors

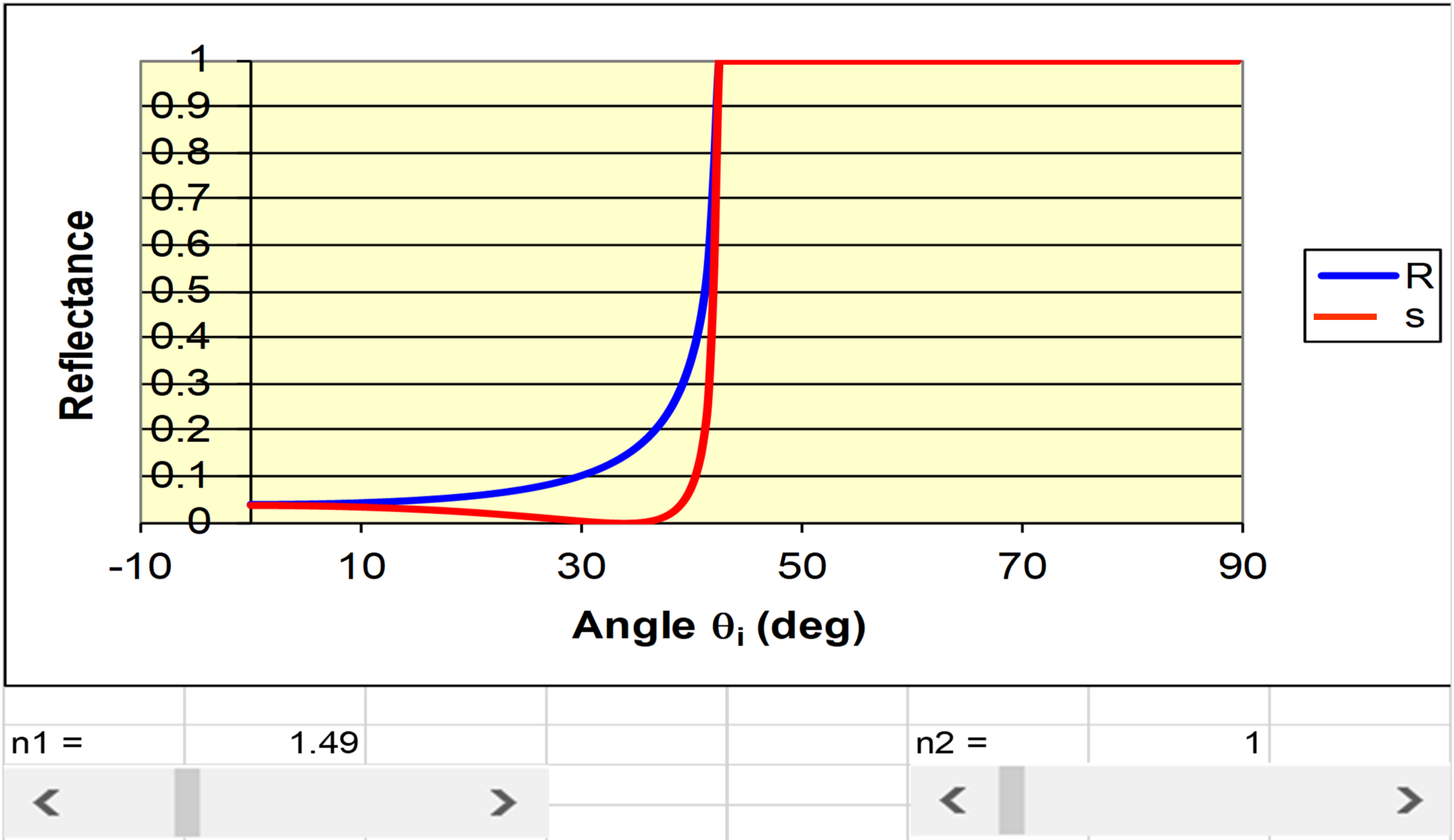
The photograph was taken by Professor Piotr Pieranski of Poznan University of Technology in Poland; used with permission from Professor Pieranski.

Total Reflection



$n_1 \sin\theta_1 = n_2 \sin\theta_2$ $n_1 > n_2$

$\theta_2 = 90^o$ ➡ $$\theta_c = \theta_1 = \sin^{-1}\frac{n_2}{n_1}$$



graph of the reflectance R for s- and p-polarized light as a function of n_1, n_2, and θ_1 $n_{acrylate} = 1.488 \Rightarrow \theta_{critical} = 42.4^o$

The usual form of the inferior mirage is shown in the figure. The air is normal except for a thin heated layer, so the rays are mostly straight lines, but strongly curved upwards in the heated layer. An object AB appears to the observer at O reflected at A'B', inverted and below the horizon, in front of the shining pool that is the reflection of the sky. Except for inversion, the object is not distorted. Note that the rays to the top and bottom of the object are *crossed*. This crossing is necessary to produce the reflection. The object AB is also visible by direct rays in the normal way. It is not very unusual to see such inverted images, but in many cases there is nothing to be imaged, so just the shining pool of the sky is seen. The inferior mirage with this geometry is frequently seen while driving in the summer, and cars ahead may be seen reflected in it. It can also be seen on walls exposed to the sun by placing your eye close to the wall. Dark rocks on a sandy plain can appear to be vegetation around open water, the classic desert mirage of fable.

$$n\,(P,T)=1+0.000293 \times (P/P_o)\,(T_o/T)$$

(e.g. T_1=47°C and T_2=67°C, $\theta_{1critical}$ =89.68°)

Where Po =atmospherically pressure, To is the standard temperature at 300oK

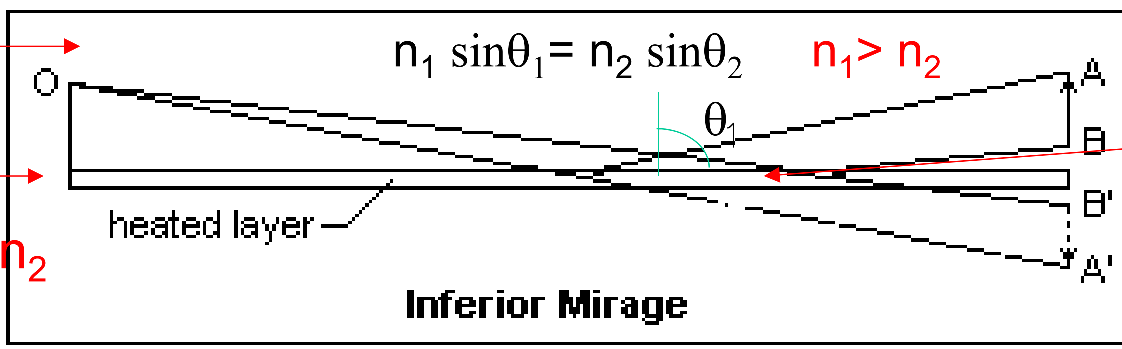
Eye at that angle will see mirage

n_1

n_2

$T_2 > T_1$ so $n_1 > n_2$

w.wang



Total reflection

Earth Atmosphere Model

Metric Units

Glenn Research Center

For h > 25000 (Upper Stratosphere)

$$T = -131.21 + .00299\,h$$

$$p = 2.488 * \left[\frac{T + 273.1}{216.6}\right]^{-11.388}$$

For 11000 < h < 25000 (Lower Stratosphere)

$$T = -56.46$$

$$p = 22.65 * e^{(1.73 - .000157\,h)}$$

For h < 11000 (Troposphere)

$$T = 15.04 - .00649\,h$$

$$p = 101.29 * \left[\frac{T + 273.1}{288.08}\right]^{5.256}$$

Not PV = nRT

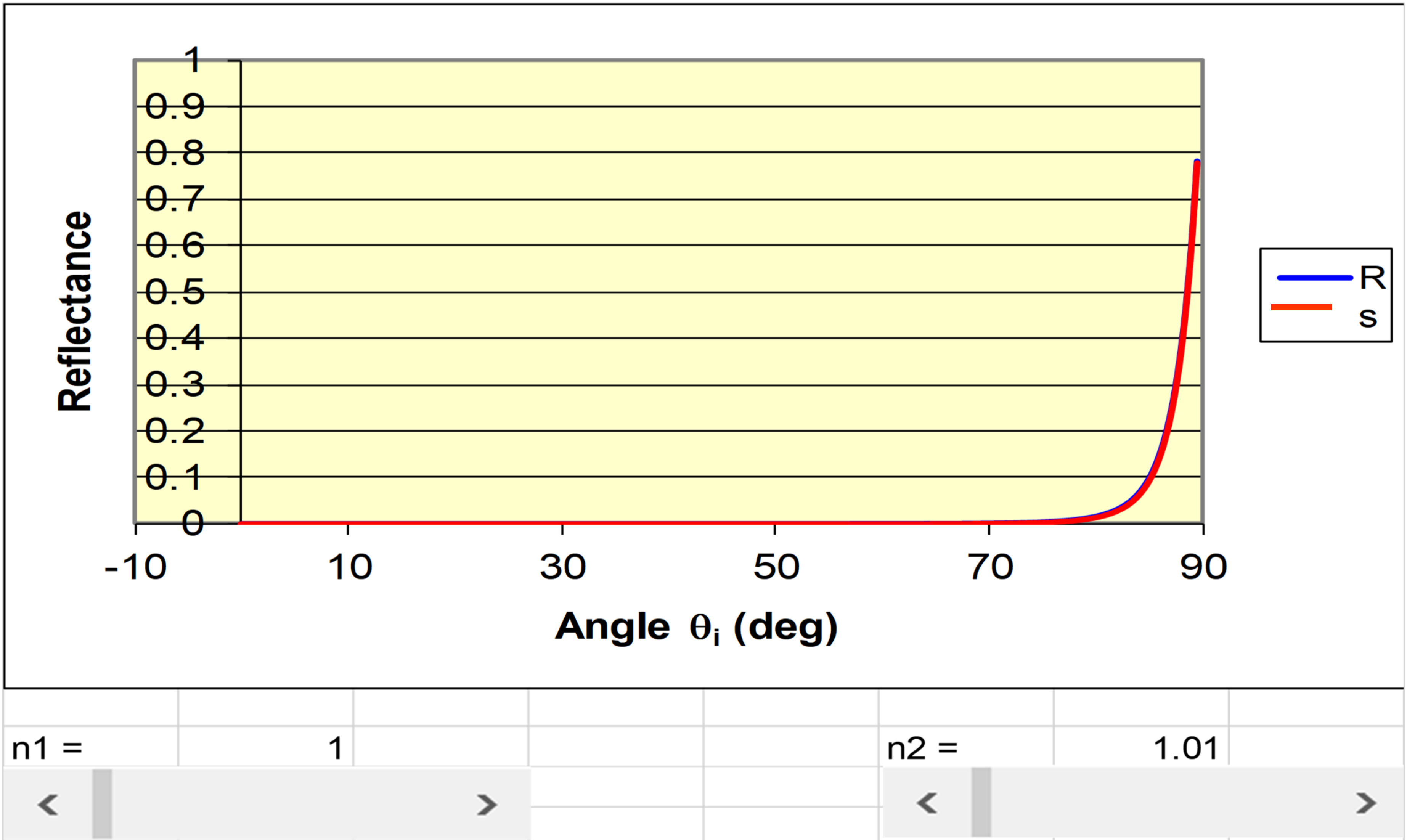
ρ = density (kg/cu m)

p = pressure (K–Pa)

$$\rho = p / (.2869 * (T + 273.1))$$

T = temperature (^{0}C)

h = altitude (m)



graph of the reflectance R for s- and p-polarized light as a function of n_1, n_2, and θ_1 $n_{desert\ air}$ = 1.0039 => $\theta_{critical} = 88^o$

Stooping

$n_1 \sin\theta_1 = n_2 \sin\theta_2$ $n_1 > n_2$

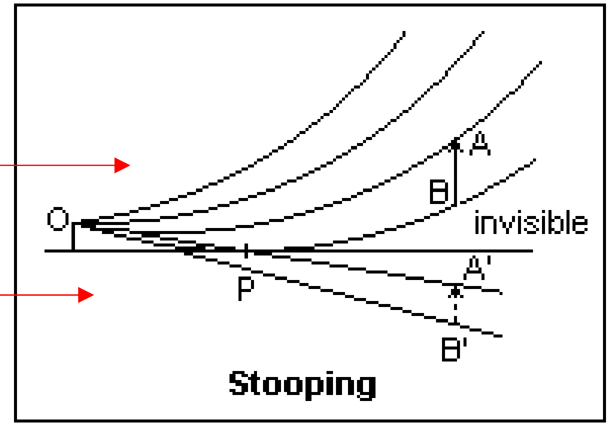
$n(P,T) = 1 + 0.000293 \times (P/Po)\,(To/T)$

Where Po =atmospherically pressure, To is the standard temperature at 300oK

n_1

n_2

$T_2 > T_1$



A less common variant of the inferior mirage is shown at the right, which is known as *stooping*. The observer at O sees a reduced image A'B' of an object AB against the sky. The curvature of the rays is greatly enhanced in this diagram to show what is happening. Note that there are no direct rays in this case, and no reflection, since there are no crossed rays. The erect image A'B' is all that is observed. The point P is observed on the horizon, and objects further away are invisible, as below the horizon. The curvature of the rays for this phenomenon is induced by an abnormal density aloft, as by a cold layer on top of a warm surface layer. Since the change in density is not large, this is a subtle effect, but one that should exist rather commonly, but in amounts too small to be perceived without careful observation and comparison of far and near objects. A distant object will seem to be smaller with respect to a nearby object than it normally is, and the horizon will draw in, making the earth seem smaller.

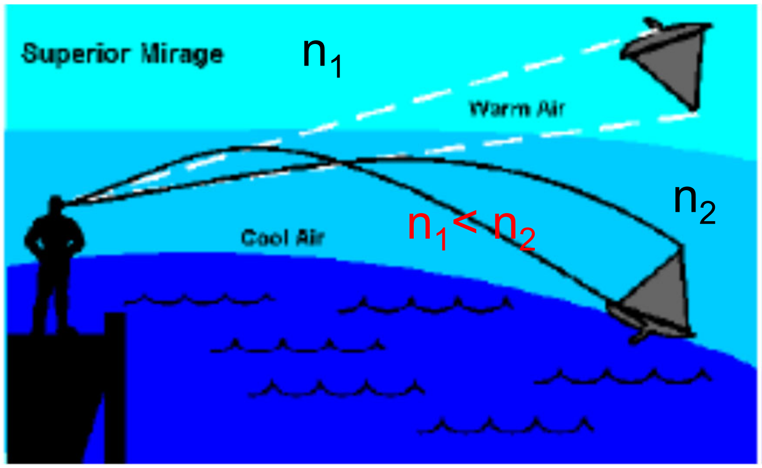
Super Mirage

$n(P,T) = 1 + 0.000293 \times (P/Po)(To/T)$

Where Po =atmospherically pressure, To is the standard temperature at 300oK

$T_1 > T_2$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$ $n_1 < n_2$



The superior mirage occurs under reverse atmospheric conditions from the inferior mirage. For it to be seen, **the air close to the surface must be much colder than the air above it**. This condition is common over snow, ice and cold water surfaces. When very cold air lies below warm air, light rays are bent downward toward the surface, thus tricking our eyes into thinking an object is located higher or is taller in appearance than it actually is. Figure on the above is adapted from http://www.islandnet.com/~see/weather/elements/mirage1.htm

Looming

$n_1 \sin\theta_1 = n_2 \sin\theta_2$ $n_2 > n_1$

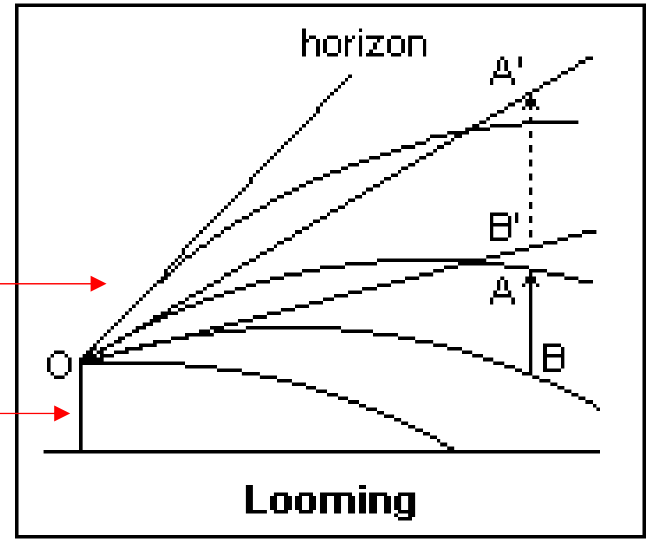
$n(P,T) = 1 + 0.000293 \times (P/Po)(To/T)$

Where Po =atmospherically pressure, To is the standard temperature at 300oK

n_1

n_2

$T_{1} > T_2$

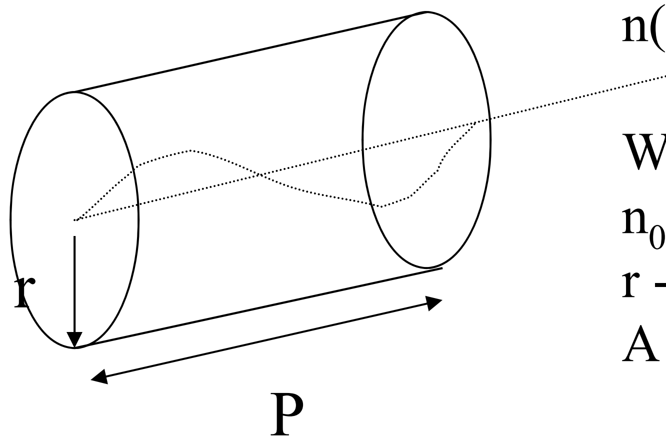


The diagram shows ray conditions in a superior mirage, in the phenomenon known as looming. Ray curvature is again greatly enhanced to make clear what is observed. The horizon is seen apparently above the horizontal, so the earth seems to be saucer-shaped, and the horizon is distant. An object AB appears as an enlarged erect image A'B'. As in stooping, there is no direct view with straight rays. This figure assumes that the density gradient is uniform from the surface up, giving approximately circular rays. A very strong general inversion, as above snow-covered ground in cloudy weather, may produce looming, which lends a very mysterious appearance to the landscape. It may be possible to see objects that are normally hidden behind obstacles appear above them and looking closer.

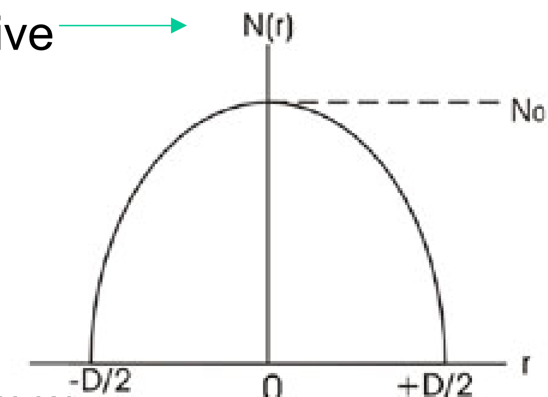
Why a GRIN ?

- Conventional designs
  - Conical elements
  - Aberrated lenses
  - Diffractive
- GRIN design
  - Well suited for small-diameter designs
  - Allows back-focal offset, unlike conical element
  - Simpler alignment than multiple lens solutions
  - Lower dispersion than diffractive axicon

GRIN Lenses



Refractive index



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$n(r) = n_o(1\text{-}Ar^2/2)$

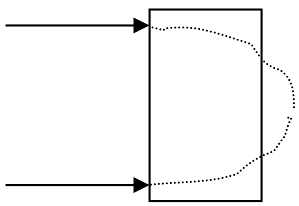
Where
n_0 -- Index of Refraction at the Center
r -- Diameter of Grin Lens
A -- Gradient Constant

The quadratic n(r) results in a sinusoidal ray path

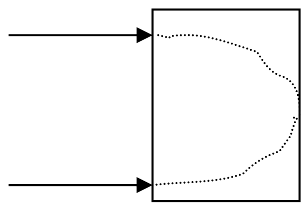
$P = 2\pi/A^{0.5}$

For length L = P/4 => quarter pitch lens
= P/2 => ½ pitch lens

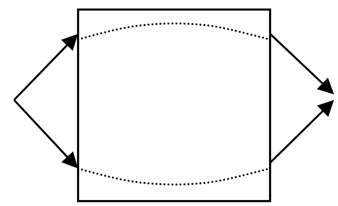
GRIN Lenses



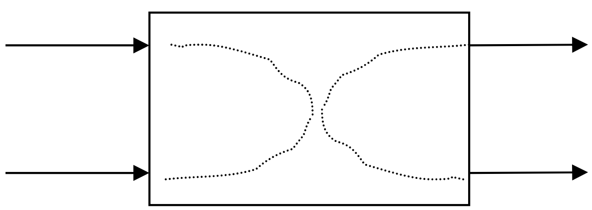
0.23P



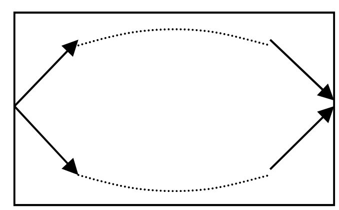
0.25P



0.29P

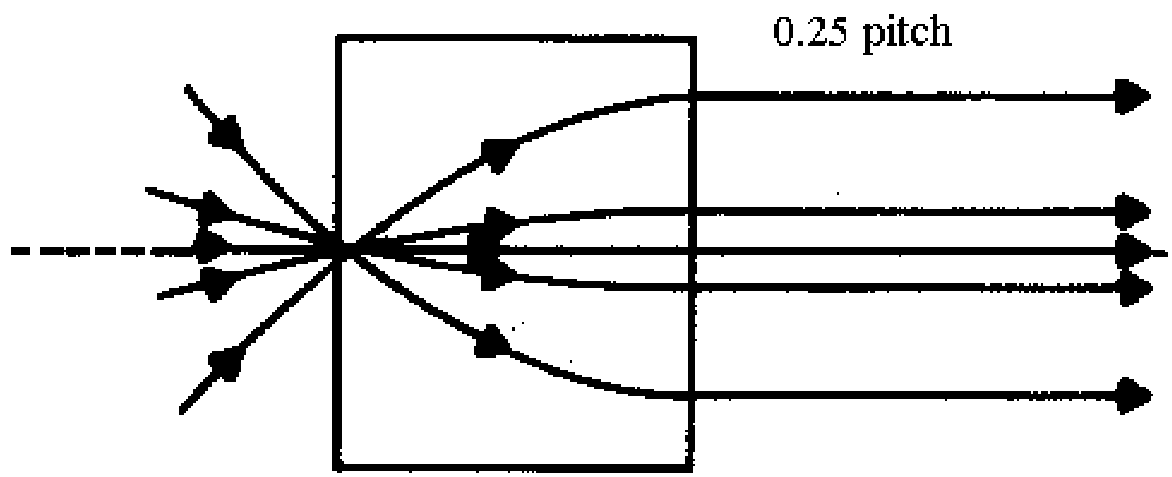


0.5P



0.5P

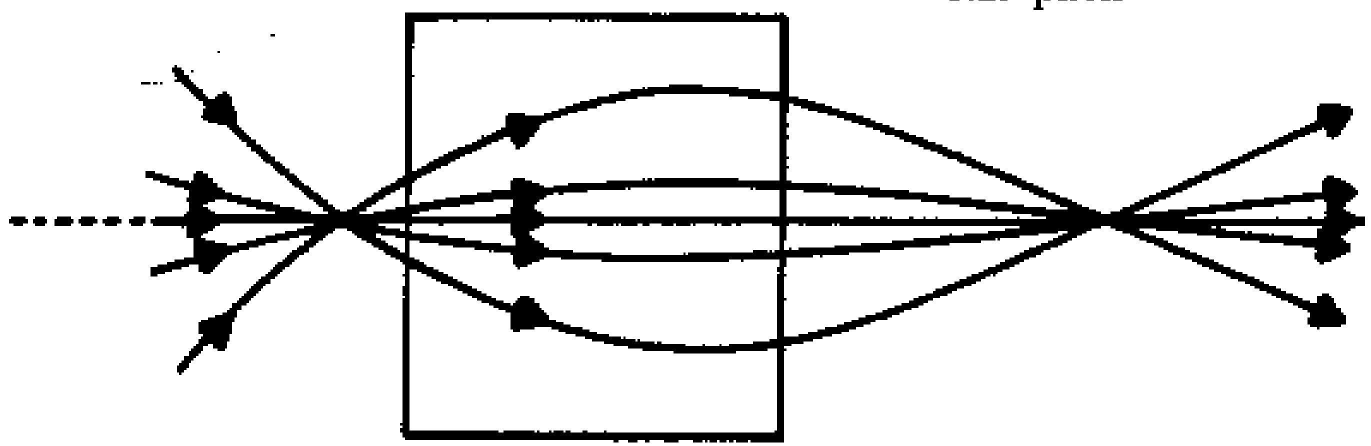
GRIN Lens



Light exiting a fiber can be collimated into a parallel beam when the output end of the fiber is connected to the GRIN lens. (0.25P)

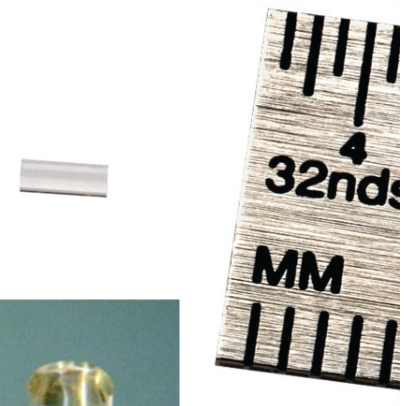
GRIN Lens

Focusing of the fiber output onto a small detector or focusing of the output of a source onto the core of a fiber can be accomplishing by increasing the length of the GRIN lens to 0.29 pitch. Then the source can be moved back from the lens and the transmitted light can be refocused at some point beyond the lens. Such an arrangement is useful for coupling sources to fibers and fibers to detectors.

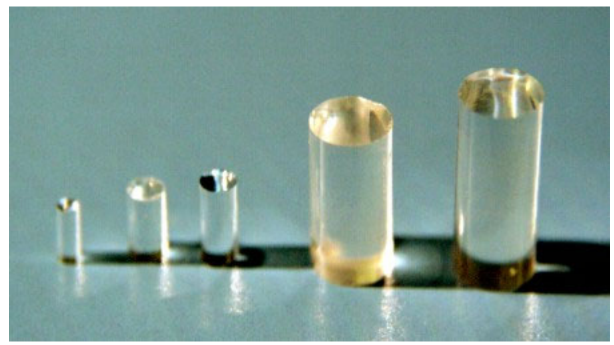


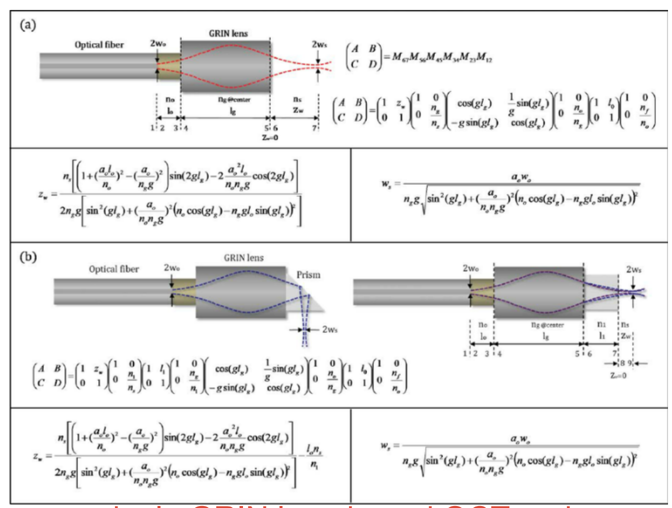
Example of Grin Lens

NewPort Plano-Angled Gradient Index Micro Lens,1.8mm,.46 NA,1300nm,0.23 Pitch



Model	LGI1300-1A
Lens Shape	Plano-Angled
Diameter	0.07 in. (1.8 mm)
Lens Material	SELFOC® radial gradient index oxide glass
Antireflection Coating	1300 nm
Working Distance	0.26 mm
Clear Aperture	Central 70% of diameter
Diameter Tolerance	+0.005/-0.010 mm



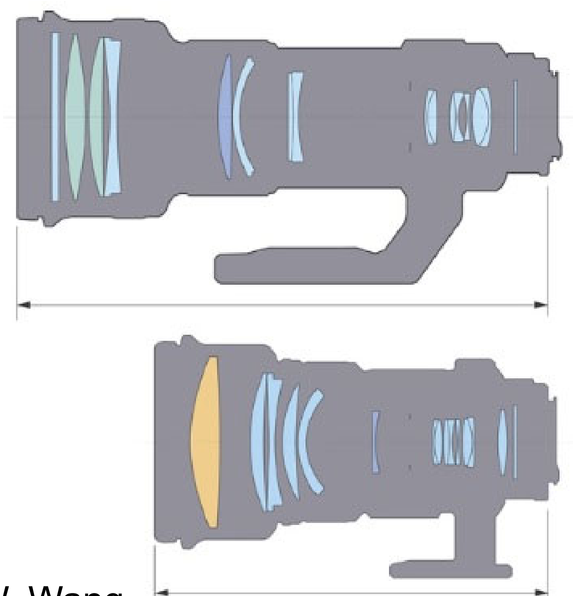


single GRIN lens-based OCT probe

Research Project: How to make an active tunable lens

Diffractive Lens

- Using Diffractive grating lens instead of refractive lens
- Size reduction
- Diffraction grating can be used to introduce corrections, rather than create aberrations



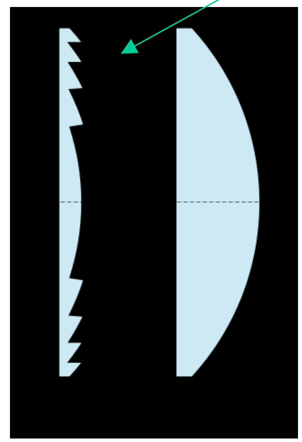
Connon DO Diffractive Optics Lens
(shown here 70-300mm DO lens)
Photokina 2000 exhibition in Cologne

Fresnel Lens

- Using grating to simulate the refractive lens effect (e.g. flash light, slde projector lens)
- The Fresnel lens reduces the amount of material required compared to a conventional lens by dividing the lens into a set of concentric annular sections. An ideal Fresnel lens would have an infinite number of sections. In each section, the overall thickness is decreased compared to an equivalent simple lens. This effectively divides the continuous surface of a standard lens into a set of surfaces of the same curvature, with stepwise discontinuities between them.

- Fresnel lens design allows a substantial reduction in thickness (and thus mass and volume of the materials)

Notice prims is used again here for refracting light into different directions

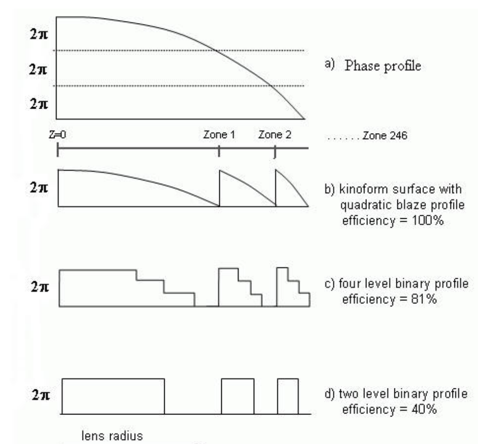


Cross section of a conventional spherical plano-convex lens of equivalent power

Close-up view of a flat Fresnel lens shows concentric circles on the surface

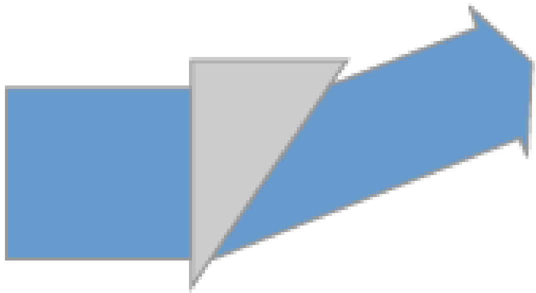
wikipedia

Close-up view of a flat Fresnel lens shows concentric circles on the surface



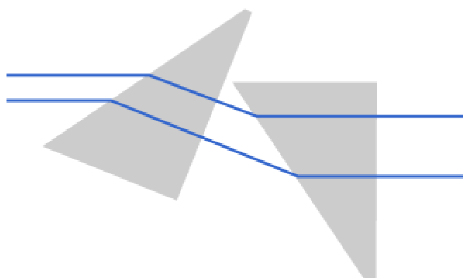
gratings

Beam Reduction and Expander

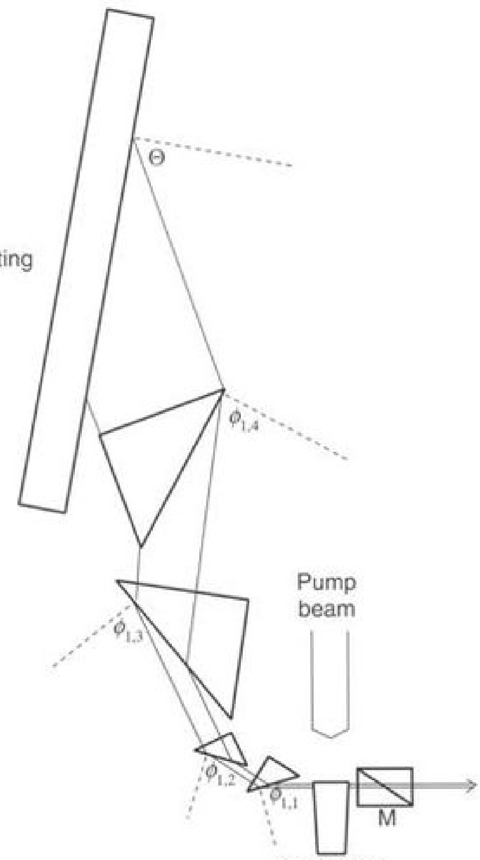


Use Smell's Law

An anamorphic prism. The output beam is substantially narrower than the input beam

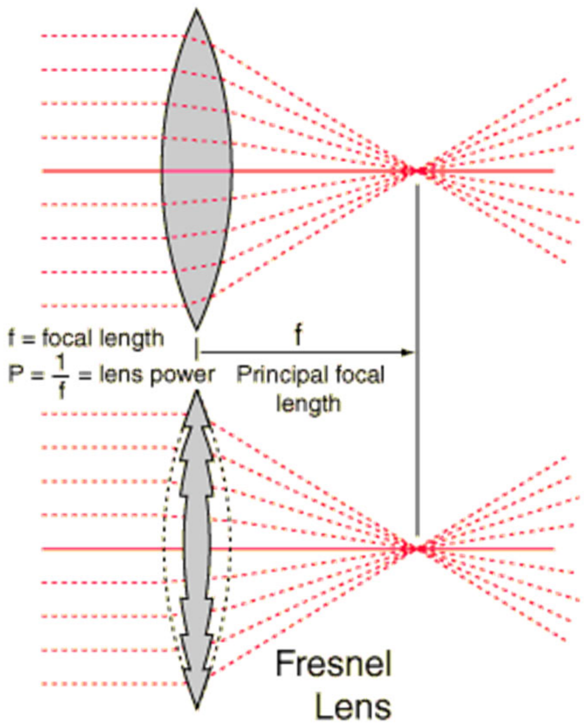


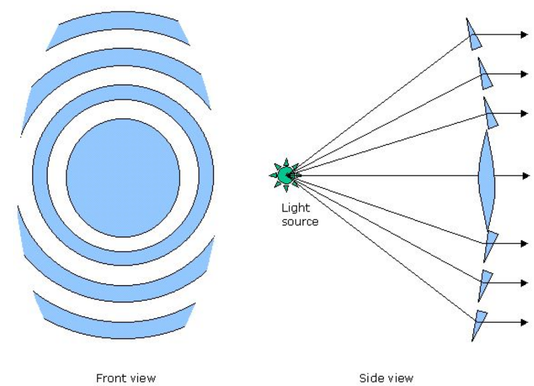
An anamorphic prism pair with refractive index of 1.5, where Brewster's angle is used on one side of each prism, and normal incidence on the other one. Two parallel beams passing through the prisms are shown. Their distance changes, and likewise their beam radii in the direction of the plane are changed. The prism pair thus works as a beam expander if the input beam comes from the left side. Of course, the beam radius in the direction perpendicular to the drawing plane is not changed.



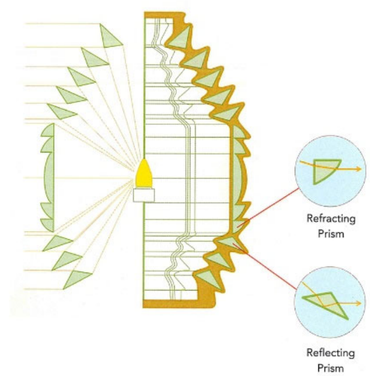
Long-pulse tunable laser oscillator utilizing a multiple-prism beam expander

Diffractive Lens





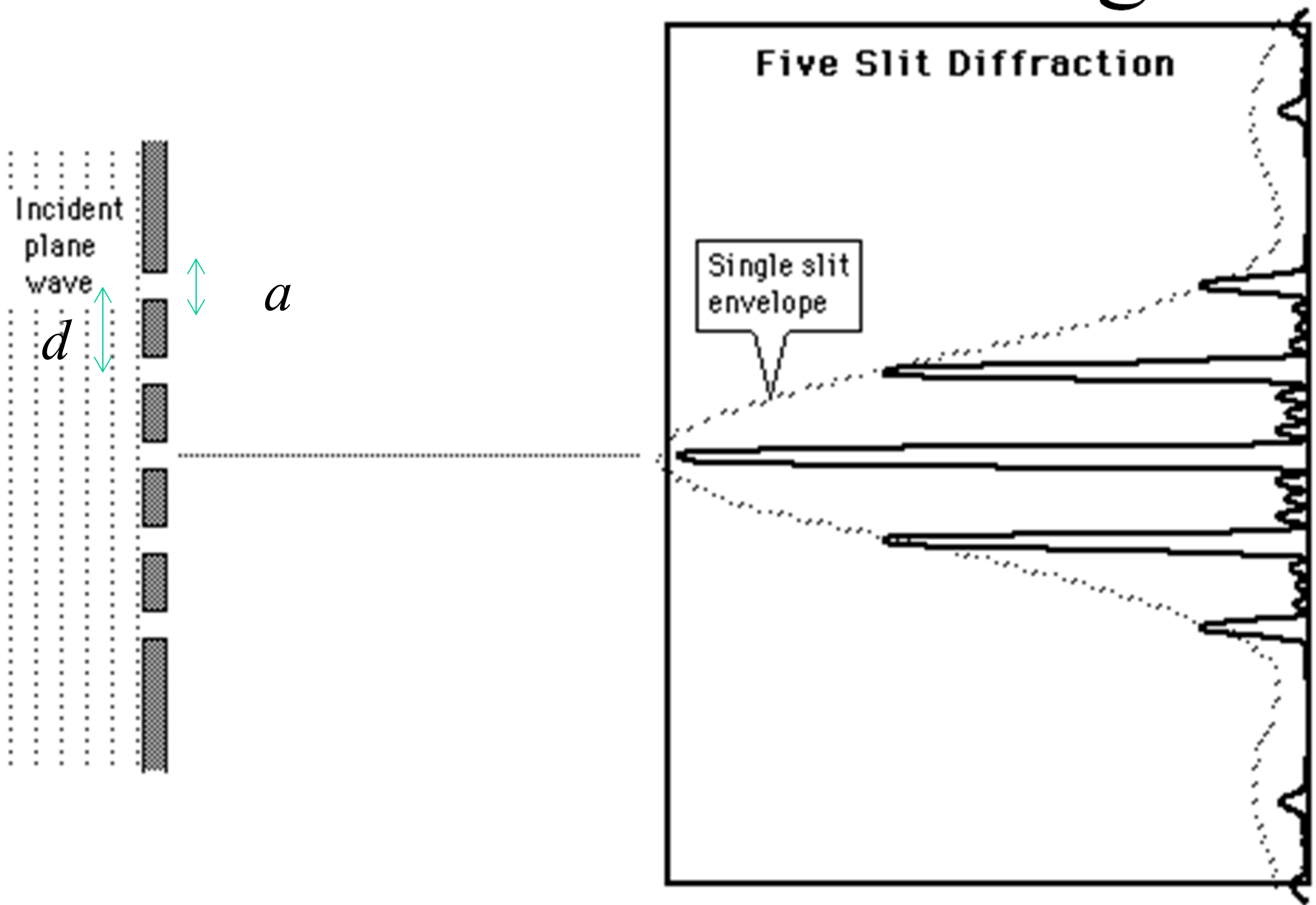
Multiples lenses



Cutaway illustration of Fresnel's lighthouse-lens system.
Some of the prisms in a Fresnel lens refract the light, allowing it to pass through in the desired direction. Some of the prisms reflect the light internally off one face, completely changing its course before it exits.

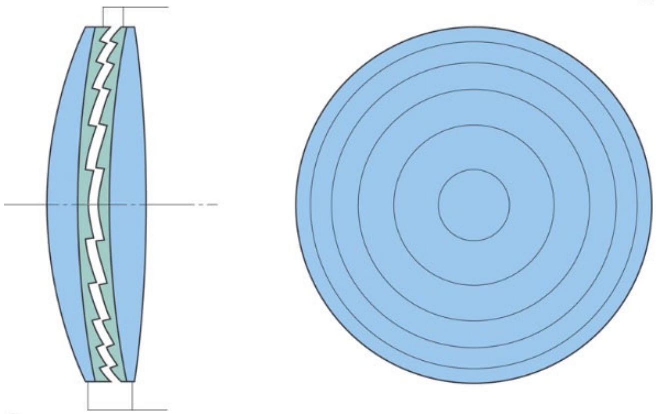
Multiples prisms

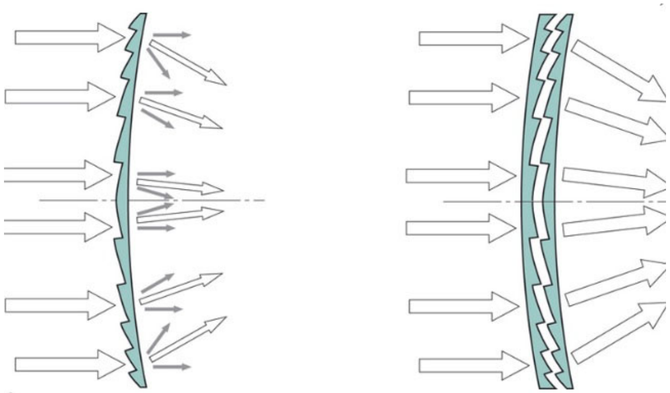
1 D Diffractive Grating

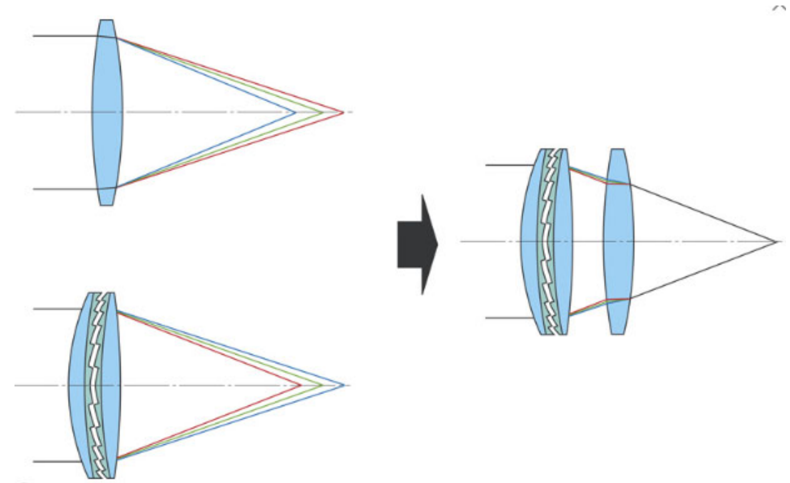


Slit separation d ~ slit width a

Cannon Diffractive Lens







Chromatic aberration, where light of different wavelengths comes to a focus at different positions on the optical axis, is a characteristic of both conventional glass elements (left top) and the Multi-layer Diffractive Optical (DO) Element (left bottom). However, the DO element focuses the wavelengths in a reverse order to conventional optical elements. By combining a DO element with a conventional element (right), chromatic aberration can be eliminated.

A single diffraction grating (left) creates a lot of superfluous light which degrades the final image. By combining two gratings (right), Canon has overcome this problem.

W.Wang

Things needed for Planar Lens or Diffraction Lens Design

- Application
- Lens system
- Ray transfer matrix analysis (e.g. ZMAX)
- Fabrication and testing (optical components)
- Optical performance test: line resolution, distortion, MTF, etc.

Week 9

- Course Website: http://courses.washington.edu/me557/optics
- Reading Materials:

  - Week 9 reading materials are from:

http://courses.washington.edu/me557/readings/

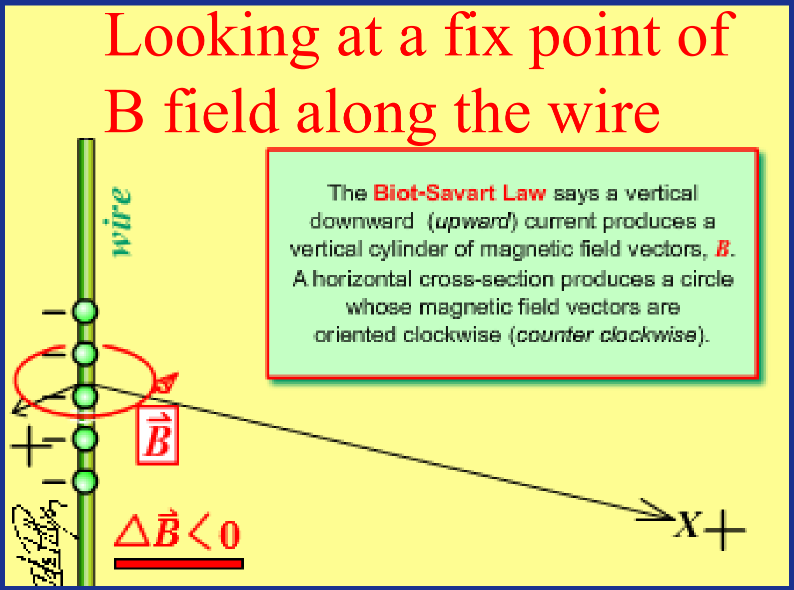
- HW 2 due week 11
- Design Project 2 assigned: Prism Design due Monday Week 13
- Final project meeting (week 14)
- Discussing proposal ideas Week 12

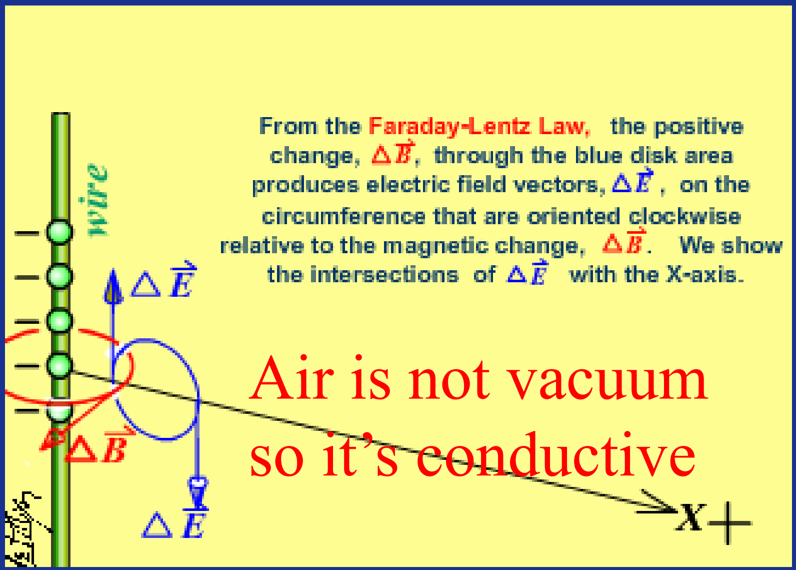
This week

- Review light as a wave, Maxwell equation, wave equation, Boundary condition, phase matching condition (law of reflection and refraction), reflection refraction coefficient
- Diffraction and interference and grating
- Prims

Electromagnetic Wave

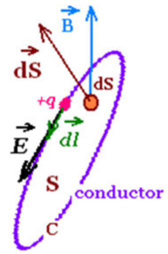
- Electromagnetic waves are created by the vibration of an electric charge. This vibration creates a wave which has both an electric and a magnetic component



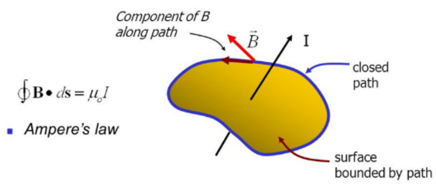


Maxwell Equations

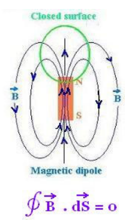
Differential form



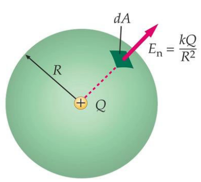
Faraday's Law $\nabla \times E = \frac{-\partial B}{\partial t}$



Ampere's Law $\nabla \times H = J + \frac{\partial D}{\partial t}$



Gauss's Law for Magnetism $\nabla \bullet B = 0$



Gauss's Law for Electricity $\nabla \bullet D = \rho$

E = *Electric Field (V/m)*	ρ = *charge density* (c/m^3)	i = *electric current (A)*
B = *Magnetic flux density*$(Web/m^2, T)$	ε_0 = *permittivity*	J = *current density*(A/m^2)
D = *Electric flux density* (c/m^2)	μ_0 = *permeability*	c = *speed of light*
H = *Magnetic Field (A/m)*	Φ_B = *Magnetic flux (Web)*	P = *Polarization*

From Ampere's and Faraday's law and a vector calculus identity,

$$\nabla \times (\nabla \times E) = \nabla(\nabla \bullet E) - \nabla^2 E$$, Wave equation

becomes $$\boxed{\nabla^2 E + \omega^2 \mu_o \varepsilon_o E = 0}$$

We consider the simple solution where E field is parallel to the x axis and its function of z coordinate only, the wave equation then becomes,

$$\frac{\partial^2 E_x}{\partial z^2} + \omega^2 \mu_o \varepsilon_o E_x = 0$$

A solution to the above differential equation is

Wave equation in free space based on above condition

$$\boxed{E = \hat{x} E_o e^{-jkz}}$$

Substitute above equation into wave equation yields,

$$(-k^2 + \omega^2 \mu\varepsilon) E = 0 \quad \Rightarrow \quad \boxed{k^2 = \omega^2 \mu\varepsilon}$$ (dispersion relation)

Trigonometric Functions in Terms of Exponential Functions

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i} \qquad \csc x = \frac{2i}{e^{ix} - e^{-ix}}$$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2} \qquad \sec x = \frac{2}{e^{ix} + e^{-ix}}$$

$$\tan x = \frac{e^{ix} - e^{-ix}}{i\left(e^{ix} + e^{-ix}\right)} \qquad \cot x = \frac{i\left(e^{ix} + e^{-ix}\right)}{e^{ix} - e^{-ix}}$$

Exponential Function vs. Trigonometric and Hyperbolic Functions

$$\boxed{e^{ix} = \cos x + i \sin x} \qquad e^{x} = \cosh x + \sinh x$$

Remember exponential term can be put in terms o trigonometric function

Hyperbolic Functions in Terms of Exponential Functions

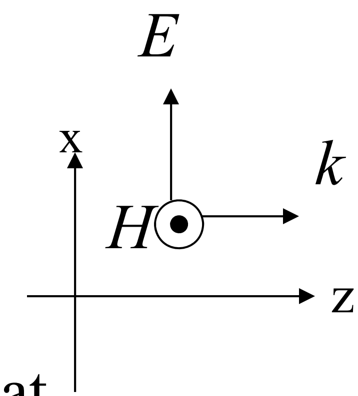
$$\sinh x = \frac{e^{x} - e^{-x}}{2} \qquad \operatorname{csch} x = \frac{2}{e^{x} - e^{-x}}$$

$$\cosh x = \frac{e^{x} + e^{-x}}{2} \qquad \operatorname{sech} x = \frac{2}{e^{x} + e^{-x}}$$

$$\tanh x = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}} \qquad \coth x = \frac{e^{x} + e^{-x}}{e^{x} - e^{-x}}$$

Let's transform the solution for the wave equation into real space and time, (assume time harmonic field)

$$E(z,t) = \mathrm{Re}\{Ee^{j\omega t}\} = \boxed{\hat{x}E_o \cos(\omega t - kz)}$$



$k = 2\pi/\lambda$, where k = wave number

Image we riding along with the wave, we asked what Velocity shall we move in order to keep up with the wave, The answer is phase of the wave to be constant

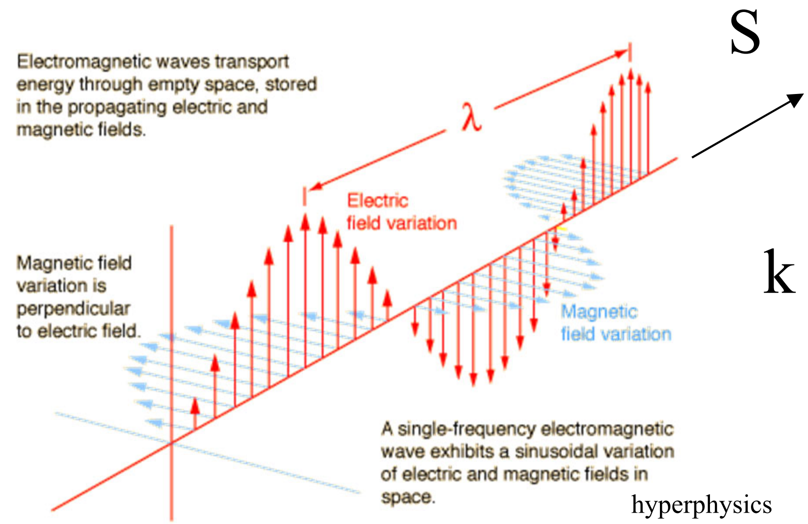
$$\omega t - kz = \text{a constant}$$

The velocity of propagation is therefore given by,

$$\boxed{\frac{dz}{dt} = v = \frac{\omega}{k} = \frac{1}{\sqrt{\mu_o \varepsilon_o}}} \quad \text{(phase velocity)}$$

Poynting's Theorem

For a time –harmonic electromagnetic wave, the power density per unit area associate with the wave is defined in complex Representation by vector S,



hyperphysics

$$S = E \, x \, H^* \ (\mathrm{W/m^2})$$

(or)

$$S = \frac{ExB^*}{2\mu_o} = \frac{|E|^2}{2Z_o}\hat{z} \Longrightarrow (\text{Intensity}) \ \boxed{I(P) = \frac{|E|^2}{2Z_o}} \text{ where } z_o = \text{air impedance} = \sqrt{\frac{\mu_o}{\varepsilon_o}}$$

Use Ampere's law and electric elasticity equation

Time average pontying vector <s> is defined as average of the Time domain Poynting vector S over a period T=2π/ω.

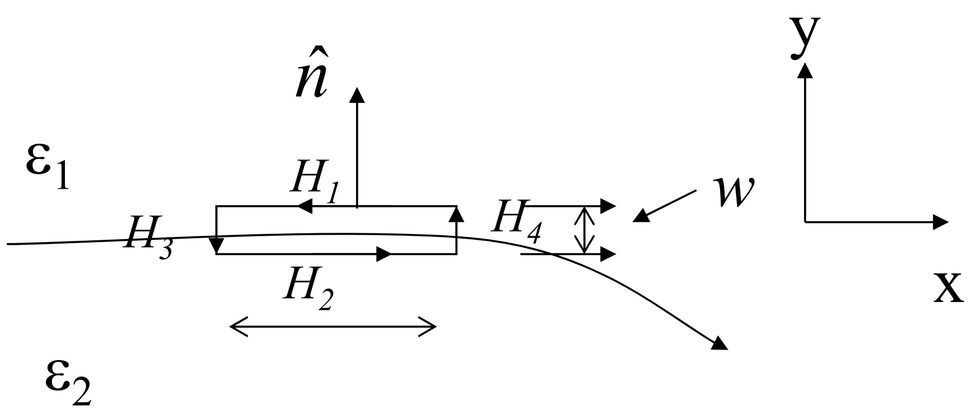
$$< S >= \frac{1}{2\pi}\int_0^{2\pi} d(\omega t) E(x,y,z,t)\times H(x,y,z,t)$$

$$\text{(or)} \qquad < S >= \frac{1}{2}\mathrm{Re}\{E\times H\}$$

Boundary Conditions

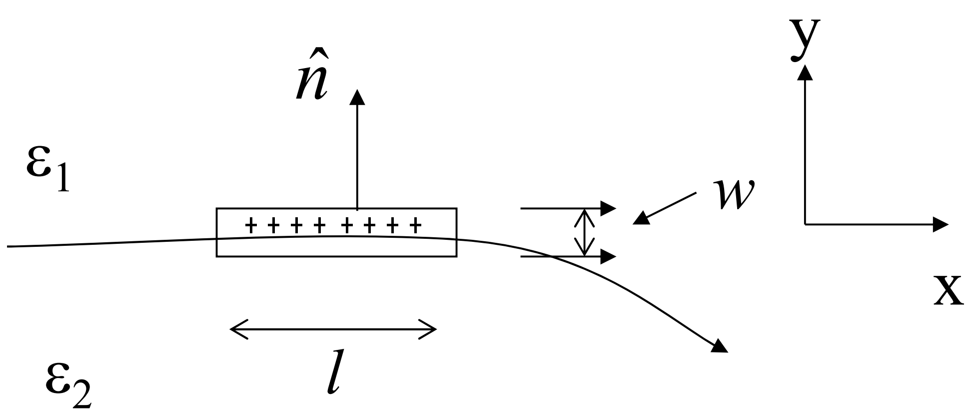
Tangential Components:

- Faraday's Law, $\boxed{\hat{n}\times(E_1 - E_2) = 0}$
- Ampere's Law $\boxed{\hat{n}\times(H_2 - H_1) = J_s}$



Normal components:

- Gauss Law of Magnetism $\boxed{(B_1 - B_2)\bullet\hat{n} = 0}$
- Gauss Law of Electricity $\boxed{(D_1 - D_2)\bullet\hat{n} = \rho}$



On the surface of a perfect conductor, $E_2 = 0$ and $H_2 = 0$

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Polarization

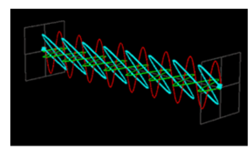
Let's assume the real time-space *E* vector has x and y components:

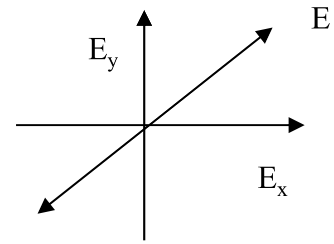
$$E(z,t) = a\cos(\omega t - kz + \phi_a)\hat{x} + b\cos(\omega t - kz + \phi_b)\hat{y}$$

$E_y/E_x = Ae^{j\phi}$

linearly polarized: $\phi_b - \phi_a = 0..or\,\pi$

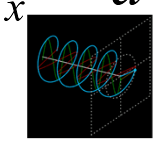
$$E_y = \pm(\frac{b}{a})E_x$$

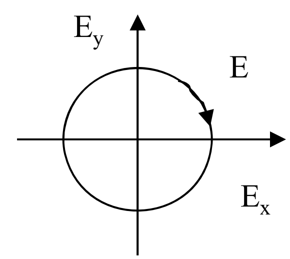




circularly polarized: $\phi_b - \phi_a = \pm\frac{\pi}{2}$

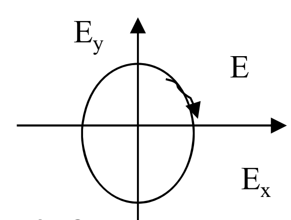
$$\frac{E_y}{E_x} = \frac{b}{a} = 1$$





Elliptically polarized: $\phi_b - \phi_a = anything..except..0, \pi, \pm\frac{\pi}{2}$

$$\frac{E_y}{E_x} = \frac{b}{a} = anything$$



Reflection and Transmission (TE, S wave, I)

Fresnel Equation

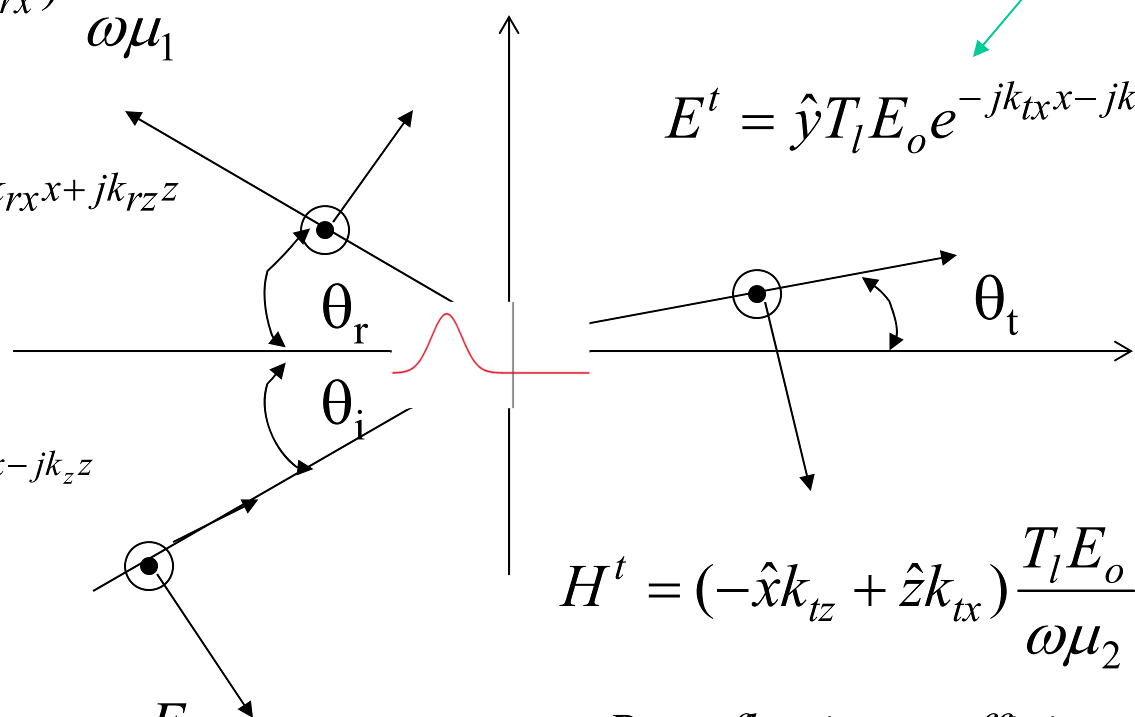
$\mu_1, \varepsilon_1, n_1$ $\mu_2, \varepsilon_2, n_2$

Negative sign means positive propagating direction

$$H^r = (\hat{x}k_{rz} + \hat{z}k_{rx})\frac{R_l E_o}{\omega\mu_1} e^{-jk_{rx}x + jk_{rz}z}$$

$$E^t = \hat{y}T_l E_o e^{-jk_{tx}x - jk_{tz}z}$$

$$E^r = \hat{y}R_l E_o e^{-jk_{rx}x + jk_{rz}z}$$



$$E^i = \hat{y}E_o e^{-jk_x x - jk_z z}$$

$$H^t = (-\hat{x}k_{tz} + \hat{z}k_{tx})\frac{T_l E_o}{\omega\mu_2} e^{-jk_{tx}x - jk_{tz}z}$$

$$H^i = (-\hat{x}k_z + \hat{z}k_x)\frac{E_o}{\omega\mu_1} e^{-jk_x x - jk_z z}$$

R_l=reflection coefficient
T_l=transmission coefficient

TE = transverse electric, perpendicularly polarized (E perpendicular to plan of incident)

If neither two are perfect conductors, J_s=0, then boundary conditions requires both the tangential electric-filed and magnetic-field components be continuous at z =0 thus,

$$e^{-jk_x x} + R_l e^{-jk_{rx} x} = T_l e^{-jk_{tx} x} \qquad \text{(E component)}$$

$$\frac{-k_z}{\omega\mu_1} e^{-jk_x x} + \frac{k_{rz}}{\omega\mu_1} R_l e^{-jk_{rx} x} = \frac{-k_{tz}}{\omega\mu_2} T_l e^{-jk_{tx} x} \qquad \text{(B component)}$$

For the above equations to hold at all x, all components must be the same, thus we get the phase matching condition:

$$k_x = k\sin\theta_i = k_{rx} = k_r \sin\theta_r = k_{tx} = k_t \sin\theta_t$$

From this we obtain law of reflection:

$$\boxed{\theta_i = \theta_r}$$

Since $k = k_r$ *because* $k^2 = k_r^2 = \omega^2 \mu_1 \varepsilon_1 = k_1$

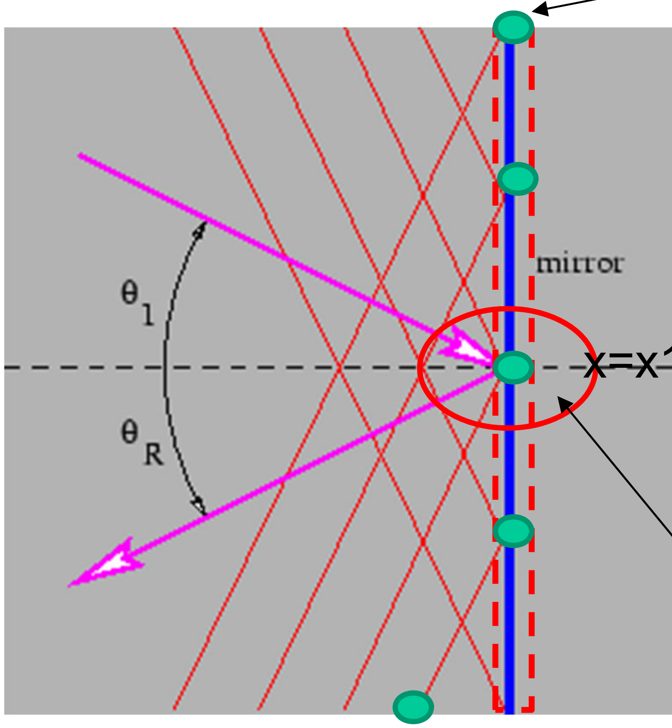
And Snell's Law:

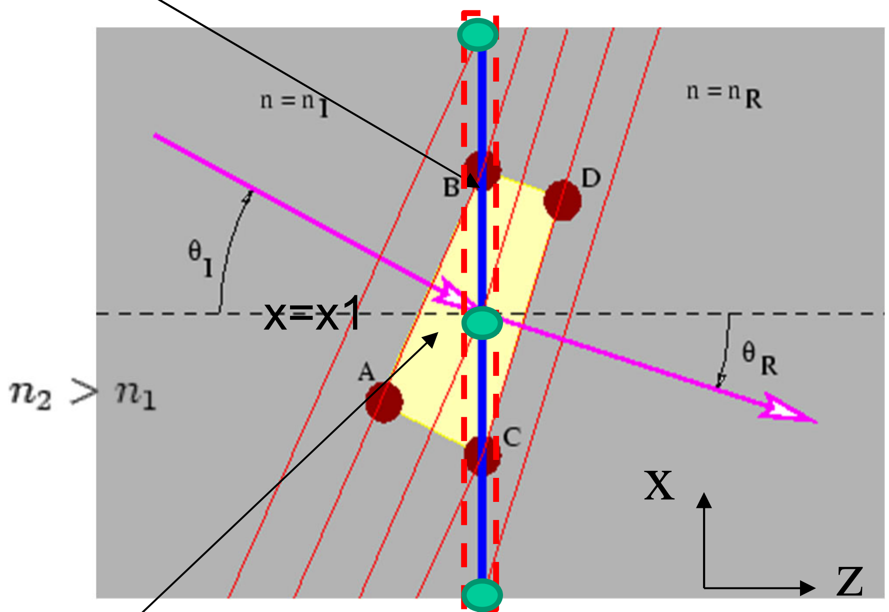
$$\boxed{n_1 \sin\theta_1 = n_2 \sin\theta_2}$$

$$\begin{cases} n_1 = c\sqrt{\mu_1 \varepsilon_1} = \dfrac{c}{\omega} k_1 \\ n_2 = c\sqrt{\mu_2 \varepsilon_2} = \dfrac{c}{\omega} k_2 \end{cases}$$

Pictorial explanation

Recall the phase map in the ray optics at $z = 0$, all phase in x direction must be the same:





Because $\hat{n}\times(E_1 - E_2) = 0$ ➡ $E_o e^{-jk_x x} + R_l E_o e^{-jk_{rx} x} = T_l E_o e^{-jk_{tx} x}$

We know $\phi_{ix} = \phi_{rx} = \phi_{tx}$ therefore $k_x x = k_{rx} x = k_{tx} x$ for any x at z=0.

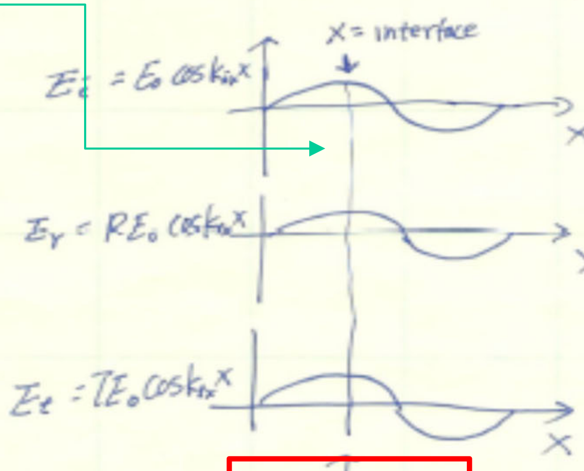
Then $k_x = k\sin\theta_i = k_{rx} = k_r \sin\theta_r = k_{tx} = k_t \sin\theta_t$ and $1 + R_l = T_l$

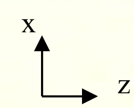
456

Mathematic explanation

Has to be the same

The reason is because we need to have same phase front ! because plane wave incident





from $\boxed{\text{phase is same at all } x}$ and $\hat{n} \times (E_1 - E_2) = 0$

$$\Rightarrow \boxed{1 + R_e = T_e} \quad \text{— (1)}$$

(ii) From phase matching condition

$$k_{ix} = k_{rx} = k_{tx}$$

$k_i = \omega\sqrt{\mu_1 \epsilon_1}$

$k_r = \omega\sqrt{\mu_1 \epsilon_1}$

$k_t = \omega\sqrt{\mu_2 \epsilon_2}$

$$k_i \sin\theta_i = k_i \sin\theta_r = k_t \sin\theta_t$$

(same as Law of reflection) $\Rightarrow \boxed{\theta_i = \theta_r}$ — (2)

$$\omega\sqrt{\mu_1 \epsilon_1} \sin\theta_i = \omega\sqrt{\mu_2 \epsilon_2} \sin\theta_t$$

[same as Law of refraction] $\sqrt{\mu_1 \epsilon_1} \sin\theta_i = \sqrt{\mu_2 \epsilon_2} \sin\theta_t \Rightarrow \boxed{n_1 \sin\theta_i = n_2 \sin\theta_t}$ — (3)

(iii) using B.C. for magnetic field

If neither two are perfect conductors, J_s=0,

$\hat{n} \times (H_1 - H_2) = 0$ tangential B field is continuous

z = impedance

$= \eta = \sqrt{\frac{\mu}{\varepsilon}}$

$\eta = \sqrt{\frac{\mu_0}{\varepsilon_0}}$

$\vec{H}_0 = \frac{\hat{k} \times \vec{E}}{\eta}$ ← (based on Faraday's law)

solved using Faraday's or Ampere's Law

$\vec{H}_i = \frac{-k_{iz}}{\omega\mu_1} E_0 e^{-jk_{ix}x} e^{-jk_{iz}z}$

$\vec{H}_r = \frac{k_{rz}}{\omega\mu_1} E_0 R_e e^{-jk_{rx}x} e^{jk_{rz}z}$

$\vec{H}_t = \frac{-k_{tz}}{\omega\mu_2} E_0 T_e e^{-jk_{tx}x} e^{-jk_{tz}z}$

at $z=0$ & using phase matching condition

$$\boxed{\frac{-k_{iz}}{\mu_1} + \frac{k_{rz}}{\mu_1} R_e = \frac{-k_{tz}}{\mu_2} T_e} \quad (4)$$

$k_{ix} = k_{rx} = k_{tx}$

(iv) To get R_ℓ & T_ℓ :

multiply $\mu_1 k_{tz}$ to $1 + R_\ell = T_\ell$ (eq 1)

$$\boxed{\mu_1 k_{tz} + \mu_1 k_{tz} R_\ell = \mu_1 k_{tz} T_\ell}$$

& multiply $\mu_2 \mu_1$ to equation (4)

$$\boxed{-\frac{k_{iz}}{\mu_1} + \frac{k_{rz}}{\mu_1} R_\ell = -\frac{k_{tz}}{\mu_2} T_\ell} \quad (4)$$

$$\boxed{-\mu_2 k_{iz} + \mu_2 k_{rz} R_\ell = -\mu_1 k_{tz} T_\ell}$$

Combine

we get

$$(\mu_2 k_{rz} + \mu_1 k_{tz}) R_\ell = (-\mu_1 k_{tz} + \mu_2 k_{iz})$$

$$\Rightarrow \boxed{R_\ell = \frac{-\mu_1 k_{tz} + \mu_2 k_{iz}}{\mu_1 k_{tz} + \mu_2 k_{rz}}}$$

$$\boxed{T_\ell = 1 + R_\ell = \frac{2 \mu_2 k_{iz}}{\mu_1 k_{tz} + \mu_2 k_{iz}}}$$

$$\Rightarrow R_\ell = \frac{-\mu_1 \omega \sqrt{\mu_2 \epsilon_2} \cos\theta_t + \mu_2 \omega \sqrt{\mu_1 \epsilon_1} \cos\theta_i}{\mu_1 \omega \sqrt{\mu_2 \epsilon_2} \cos\theta_t + \mu_2 \omega \sqrt{\mu_1 \epsilon_1} \cos\theta_r} = \frac{-\sqrt{\epsilon_2}\cos\theta_t + \sqrt{\epsilon_1}\cos\theta_i}{\sqrt{\epsilon_2}\cos\theta_t + \sqrt{\epsilon_1}\cos\theta_r}$$

multiply top & bottom by C_0

$$= \frac{-n_2 \cos\theta_t + n_1 \cos\theta_i}{n_2 \cos\theta_t + n_1 \cos\theta_r}$$

Recall

$$\begin{cases} n_1 = c\sqrt{\mu_1 \varepsilon_1} = \dfrac{c}{\omega} k_1 \\ n_2 = c\sqrt{\mu_2 \varepsilon_2} = \dfrac{c}{\omega} k_2 \end{cases}$$

For TE wave (perpendicular polarization)

$$|R_e|^2 = \left| \frac{n_1 \cos\theta_i - n_2 \cos\theta_t}{n_1 \cos\theta_i + n_2 \cos\theta_t} \right|^2$$

$$I(P) = \frac{|E|^2}{2Z_o}$$

Recall Intensity is square of E field

Do the same derivation like TE for TM to get the TM polarization R an T

For TM (parallel polarization)

$$|R_{ee}|^2 = \left| \frac{n_1 \cos\theta_t - n_2 \cos\theta_i}{n_1 \cos\theta_t + n_2 \cos\theta_i} \right|^2$$

For TE wave

$$1 + |R_e|^2 = |T_e|^2$$

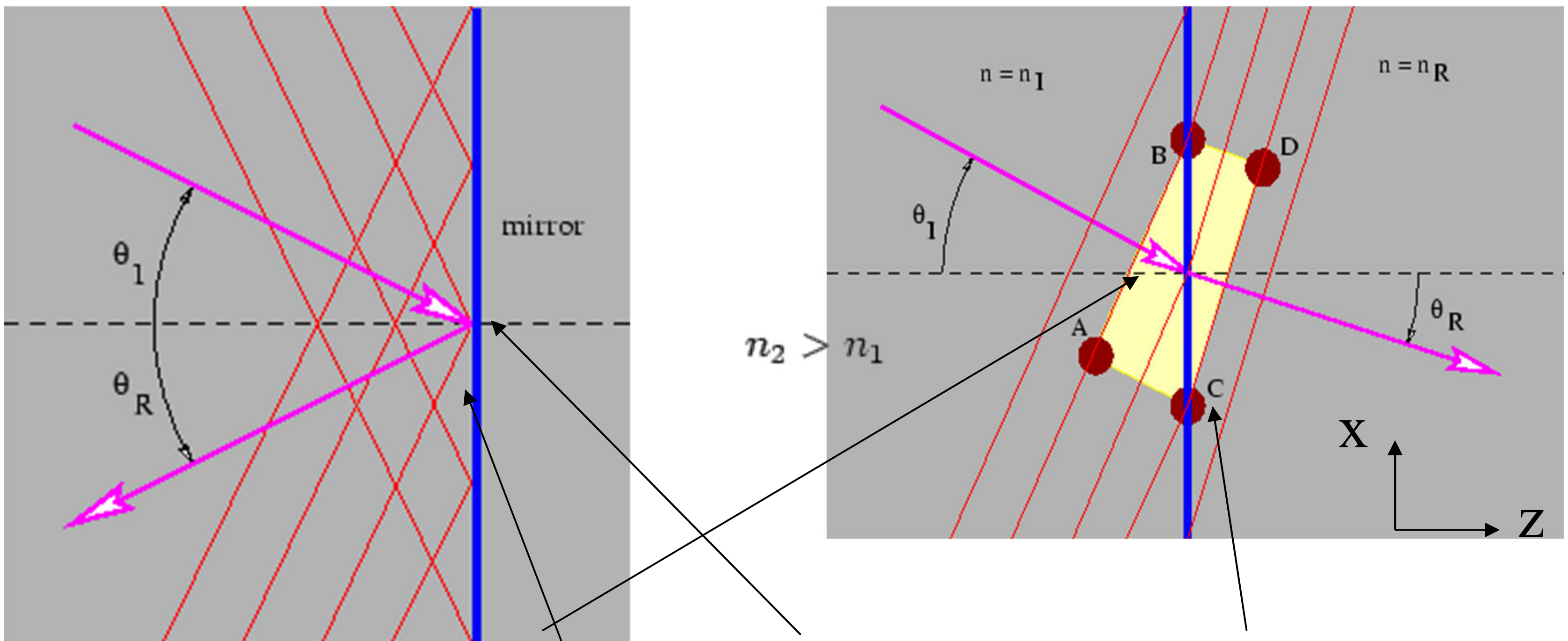
For TM wave

$$1 + |R_{ee}|^2 = |T_{ee}|^2$$

Here we verfy the two laws in the ray theory:

1) the laws of reflection
2) The law of refraction.

$$\theta_i = \theta_r \qquad\qquad n_1 \sin\theta_1 = n_2 \sin\theta_2$$



$$k_x = k \sin\theta_i = k_{rx} = k_r \sin\theta_r = k_{tx} = k_t \sin\theta_t$$

To Find Reflection and Transmission Coefficient, substitute solution for E^i, E^r, E^t, into wave equation

$$\nabla^2 E^i + \omega^2 \mu_1 \varepsilon_1 E^i = 0$$

$$\nabla^2 E_r + \omega^2 \mu_1 \varepsilon_1 E_r = 0$$

$$\nabla^2 E^t + \omega^2 \mu_2 \varepsilon_2 E^t = 0$$

We find,

$$k_x^2 + k_z^2 = k_1^2 = k_{rx}^2 + k_{rz}^2$$

$$k_{tx}^2 + k_{tz}^2 = k_2^2$$

Using phase matching condition, we get,

$$1 + R_l = T_l$$

$$1 - R_l = \frac{\mu_1 k_{tz}}{\mu_2 k_z} T_l$$

$$\Rightarrow$$

$$R_l = \frac{\mu_2 k_z - \mu_1 k_{tz}}{\mu_2 k_z + \mu_1 k_{tz}}$$

$$T_l = \frac{2\mu_2 k_z}{\mu_2 k_z + \mu_1 k_{tz}}$$

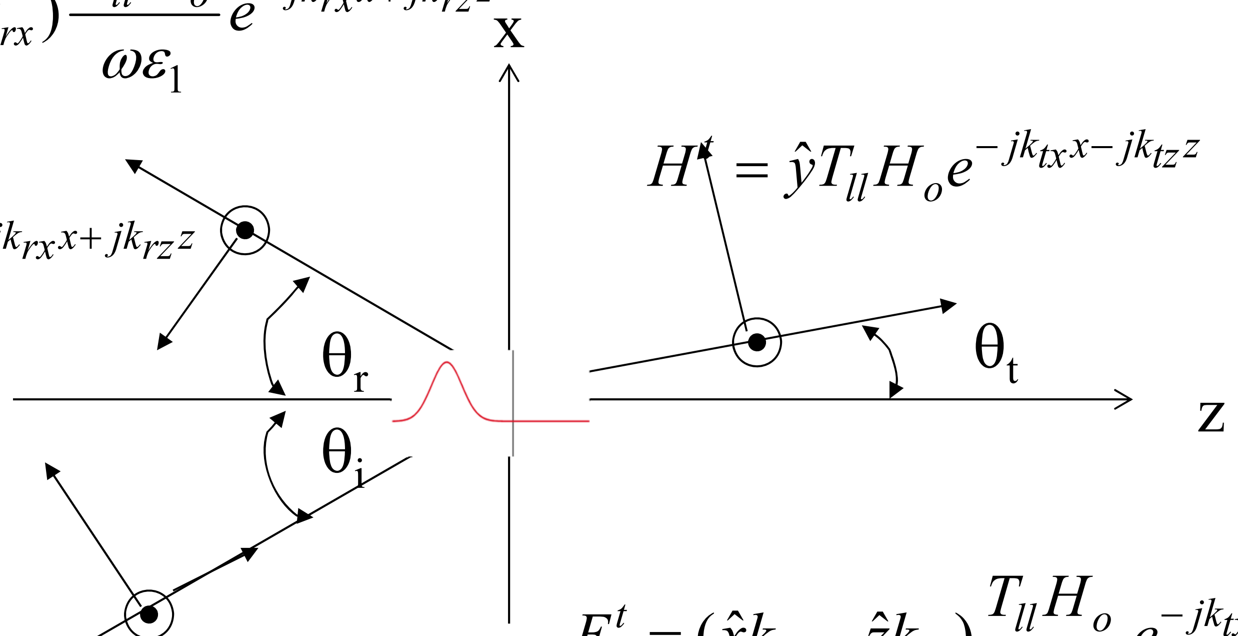
Reflection and Transmission (TM, P wave)

$\mu_1, \varepsilon_1, n_1$ $\qquad$ $\mu_2, \varepsilon_2, n_2$

$$E^r = (-\hat{x}k_{rz} - \hat{z}k_{rx})\frac{R_{ll}H_o}{\omega\varepsilon_1}e^{-jk_{rx}x + jk_{rz}z}$$

$$H^r = \hat{y}R_{ll}H_o e^{-jk_{rx}x + jk_{rz}z}$$

$$H^t = \hat{y}T_{ll}H_o e^{-jk_{tx}x - jk_{tz}z}$$



$$E^t = (\hat{x}k_{tz} - \hat{z}k_{tx})\frac{T_{ll}H_o}{\omega\varepsilon_2}e^{-jk_{tx}x - jk_{tz}z}$$

$$E^i = (\hat{x}k_z - \hat{z}k_x)\frac{H_o}{\omega\varepsilon_1}e^{-jk_x x - jk_z z}$$

$$H^i = \hat{y}H_o e^{-jk_x x - jk_z z}$$

R_{ll}= reflection coefficient

T_{ll}= transmission coefficient

TM = transverse magnetic, parallel polarized (E parallel to plan of incident)

Substitute solution for E^i, E^r, E^t, into wave equation

We get,

$$1 + R_{ll} = T_{ll}$$

$$1 - R_{ll} = \frac{\varepsilon_1 k_{tz}}{\varepsilon_2 k_z} T_{ll}$$

$$\Rightarrow$$

$$R_{ll} = \frac{\varepsilon_2 k_z - \varepsilon_1 k_{tz}}{\varepsilon_2 k_z + \varepsilon_1 k_{tz}}$$

$$T_{ll} = \frac{2\varepsilon_2 k_z}{\varepsilon_2 k_z + \varepsilon_1 k_{tz}}$$


No one is born hating another person because of the colour of his skin, or his background, or his religion.
People must learn to hate, and if they can learn to hate, they can be taught to love,
for love comes more naturally to the human heart than its opposite.
Nelson Mandela

Week 12

- Course Website: http://courses.washington.edu/me557/optics
- Reading Materials:

  - Week 12 reading materials are from:

http://courses.washington.edu/me557/readings/

- Problem solving HW 2 Due Week 14
- Please finish your Lab 1
- Prism Design project due week 14
- Discuss Final Project Proposal (this Thurdsday)
- Proposal due next Monday (Week 13)

Last week

- Reflection and refraction using wave theory
- Critical and Brewster's angles
- Diffraction, Interference and grating
-

This week

- Interference and Grating
- Prism
- Birefringent materials
- Phase plate
- Light Coupling system

Evading Radar Coverage

Best stealthy vehicle of all!!!!

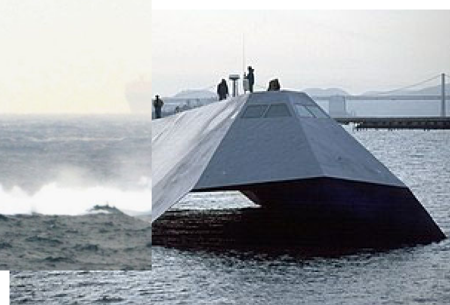
Making a plane non-detectable by Conventional RADAR

In different

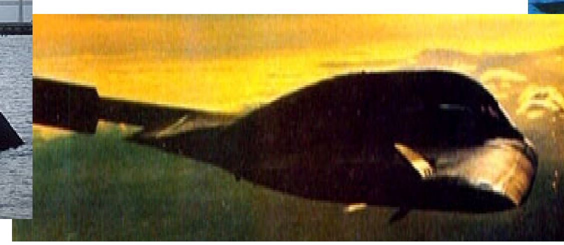
- -terrain
- -Weather (temperature, rain, and snow…)
- -Altitude (lower or higher)

corvette

W. Wang



Sea Shadow



B2

F117

Radar Jamming Planes (electronic warfare)

Northrop Grumman EA-6B Prowler

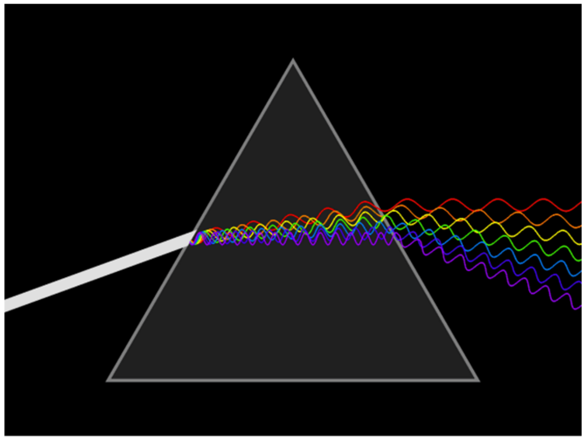
E-2C-AEWA Hawkeye

EA-18G Growler

replacement

Prism

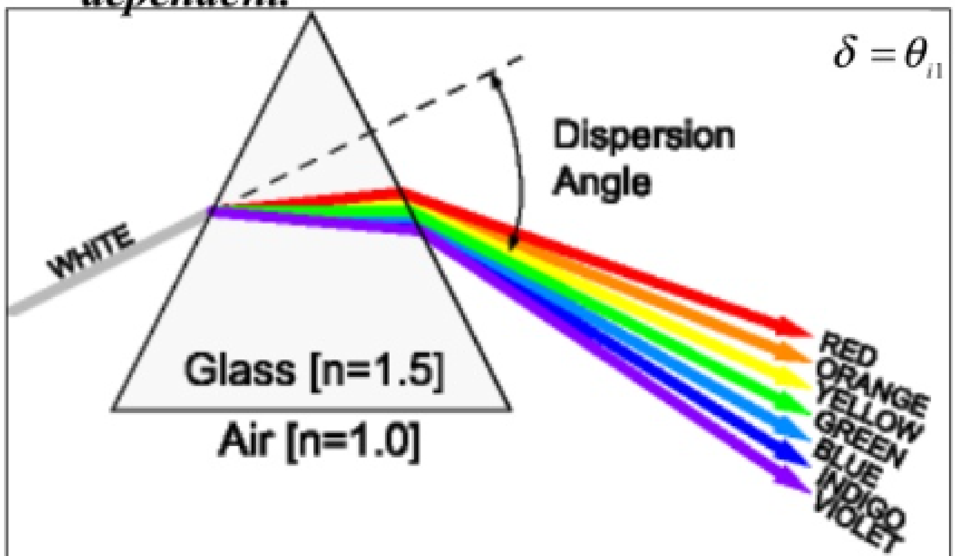
- Dispersing white light
- Deflect and steering light
- Mirrors
- Beam expander and reducer
- Used in coupling light into integrated optical system
- Beam splitter and combiner
- Polarizer and phase shifter



Dispersion Prisms

A light beam striking a face of a prism at an angle is partly reflected and partly refracted. The amount of light reflected is given by Fresnel's equations, and the direction of the reflected beam is given by the law of reflection (angle of incidence = angle of reflection). The refracted light changes speed as it moves from one medium to another. This speed change causes light striking the boundary between two media at an angle to proceed into the new medium at a different angle, depending on the angle of incidence, and on the ratio between the refractive indices of the two media (Snell's law). Since the refractive index varies with wavelength, light of different colors is refracted diffferently. Blue light is slowed down more than red light and will therefore be bent more than red light.

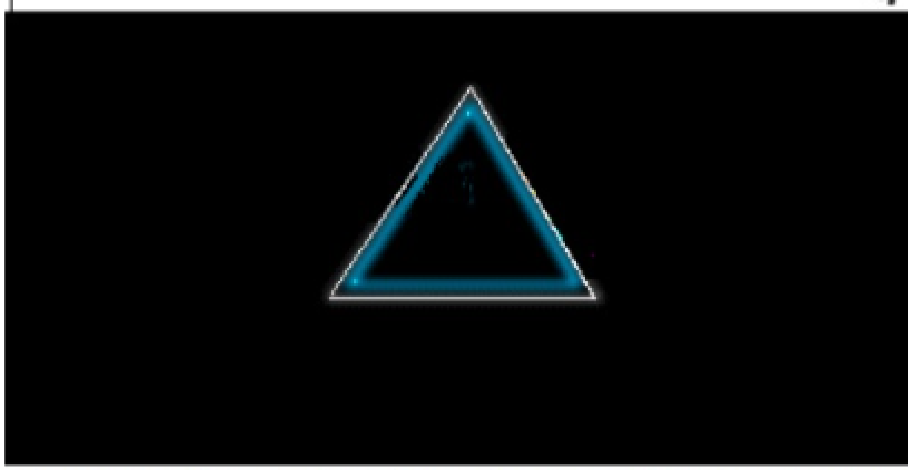
In 1672 Newton wrote "A New Theory about Light and Colors" in which he said that the white light consisted of a mixture of various colors and the diffraction was color dependent.

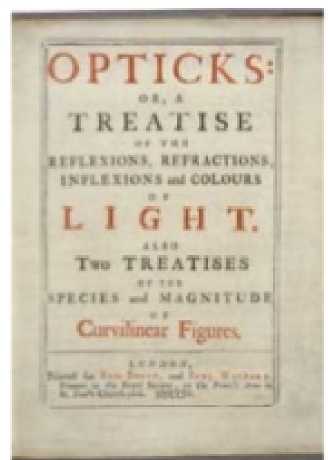


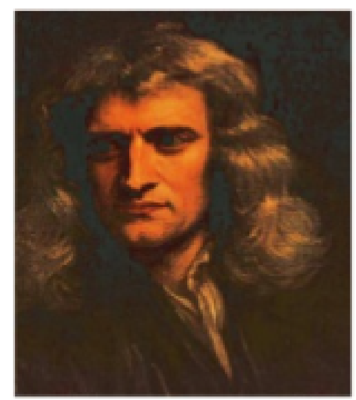
$$\delta = \theta_{i1} + \sin^{-1}\left\{\sin\alpha\left[n(\lambda)^2 - \sin^2\theta_{i1}\right]^{1/2} - \cos\alpha\sin\theta_{i1}\right\} - \alpha$$

Color	*λ_0 (nm)*	*υ [THz]*
Red	***780 - 622***	***384 – 482***
Orange	***622 - 597***	***482 – 503***
Yellow	***597 - 577***	***503 – 520***
Green	***577 - 492***	***520 – 610***
Blue	***492 - 455***	***610 – 659***
Violet	***455 - 390***	***659 - 769***

1 nm = 10^{-9}m, 1 THz = 10^{12} Hz




OPTICKS:
TREATISE
REFLEXIONS, REFRACTIONS, INFLEXIONS and COLOURS
LIGHT.
Two TREATISES
SPECIES and MAGNITUDE
Curvilinear Figures.



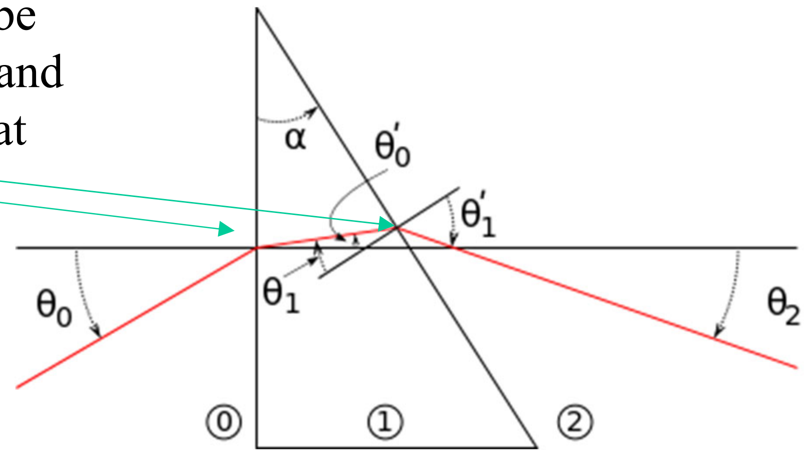
Isaac Newton
1542 - 1727

11

http://physics.nad.ru/Physics/English/index.htm

Return to Table of Content

Ray angle deviation and dispersion through a prism can be determined by tracing a sample ray through the element and using Snell's law at each interface. For the prism shown at right, the indicated angles are given by



$$\theta_0' = \arcsin\left(\frac{n_0}{n_1}\sin\theta_0\right)$$

$$\theta_1 = \alpha - \theta_0'$$

$$\theta_1' = \arcsin\left(\frac{n_1}{n_2}\sin\theta_1\right)$$

$$\theta_2 = \theta_1' - \alpha$$

All angles are positive in the direction shown in the image. For a prism in air $n_o=n_2\sim 1$. Defining $n=n_1$, the deviation angle δ is given by

$$\delta = \theta_0 + \theta_2 = \theta_0 + \arcsin\left(n \sin\left[\alpha - \arcsin\left(\frac{1}{n}\sin\theta_0\right)\right]\right) - \alpha$$

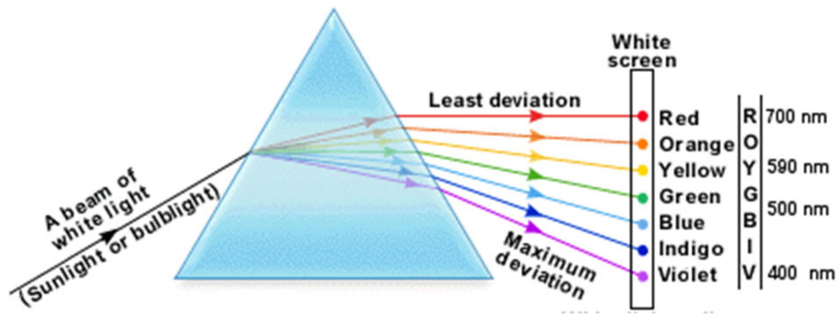
If the angle of incidence θ_o and prism apex angle α are both small, $\sin\theta \approx \theta$ and $\arcsin x \approx x$ if the angles are expressed in radians. This allows the nonlinear equation in the deviation angle δ to be approximated by

$$\delta \approx \theta_0 - \alpha + \left(n\left[\left(\alpha - \frac{1}{n}\theta_0\right)\right]\right) = \theta_0 - \alpha + n\alpha - \theta_0 = (n-1)\alpha\ .$$

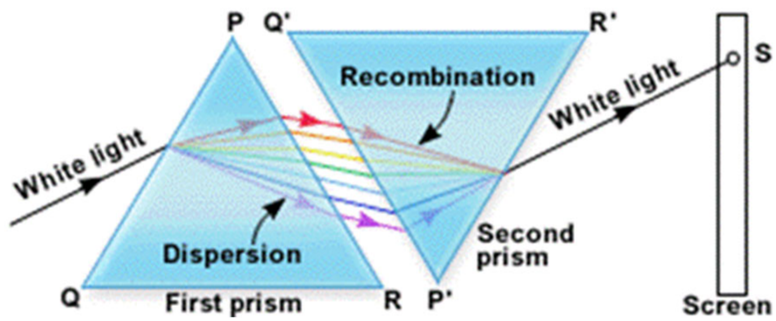
The deviation angle depends on wavelength through *n*, so for a thin prism the deviation angle varies with wavelength according to

$$\delta(\lambda) \approx [n(\lambda) - 1]\alpha$$

Dispersing White Light

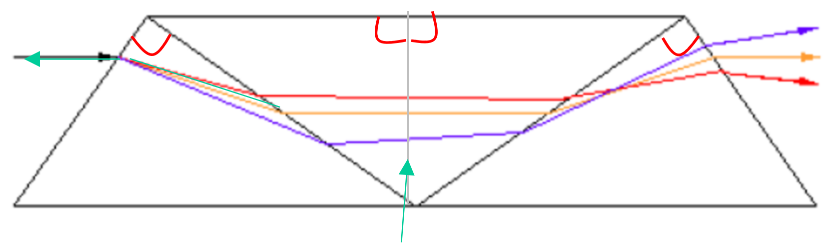


Reversable



Direct vision spectroscope

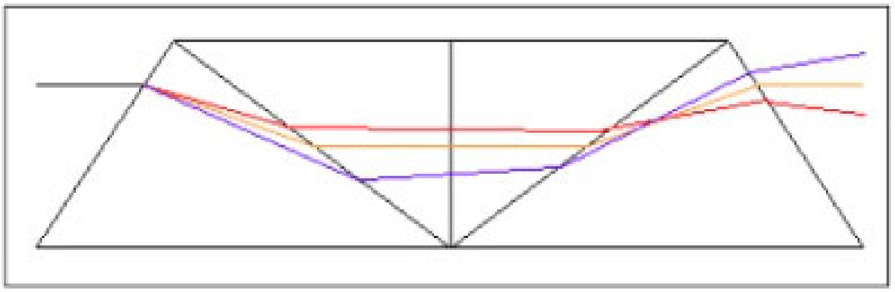
Three prism arrangement, known as a **double Amici prism**, consists of a symmetric pair of **right angled prisms** of **a given refractive index**, and **two right angled prisms of a different refractive index in the middle**. It has the useful property that the **center wavelength refracted back into the direct line of the entrance beam (ex credit)**. The prism assembly is thus a direct-vision prism, and is commonly used as such in hand-held spectroscopes.



isosceles triangle

<u>***Dispersing Prisms (continue – 1)***</u>

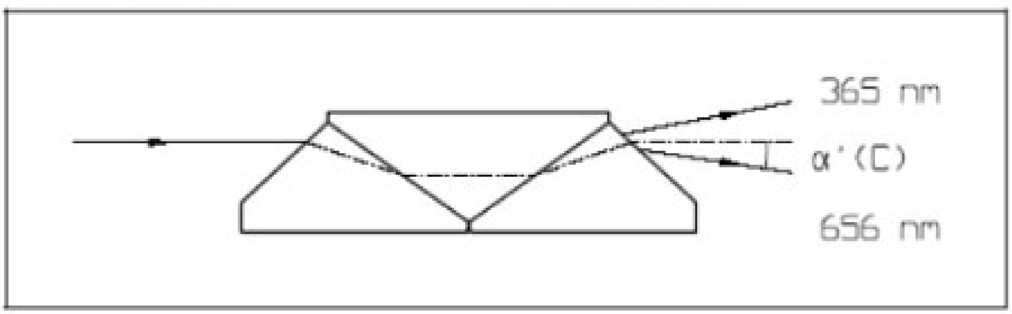
Amici Prism



Giovanni Amici
(1786-1863)

Amici-Prismen

Amici Prisms



λ		α'
365 nm	(i)	10.2 °
405 nm	(h)	3.2 °
436 nm	(g)	0.0 °
486 nm	(F)	-3.3 °
546 nm	(e)	-5.7 °
589 nm	(D)	-6.9 °
656 nm	(C)	-8.2 °

Disperse at different angle. Based on angles we know what wavelength

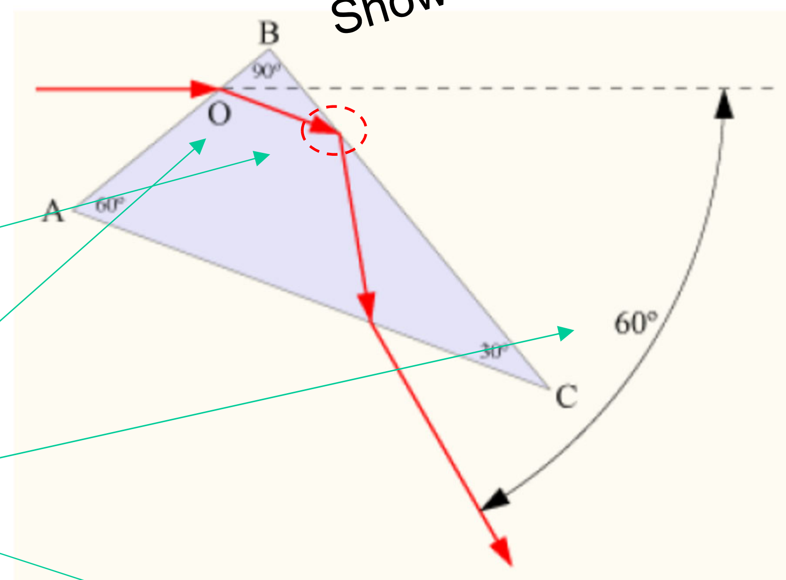
Dispersion Angles

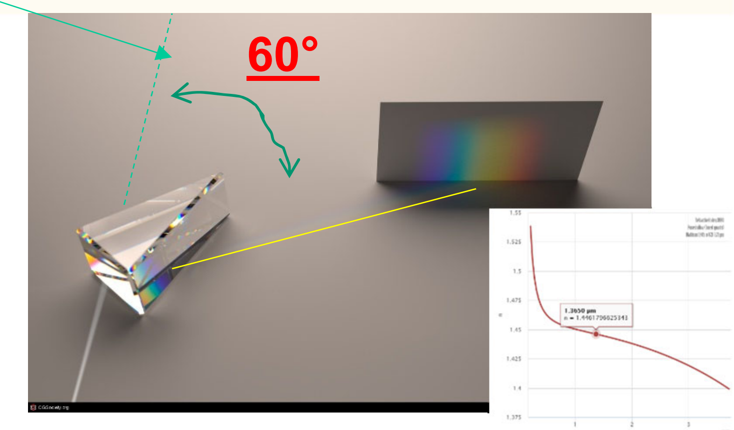
- Dispersion angle can be optimized by either base angle of prism, incident angle or refractive index of the prism.

Abbe Prism

Show refractometer

This is a right-angled prism with **30°-60°-90° triangular faces**. A beam of light is refracted as it enters face AB, undergoes **total internal reflection** from face BC, and is refracted again on exiting face AC. **One particular wavelength of the light exits the prism at a deviation angle of exactly 60°. This is the minimum possible deviation of the prism**, all other wavelengths being deviated by greater angles. By rotating the prism around any point O on the face AB, the wavelength which is deviated by 60° can be selected. Thus, the Abbe prism is a type of constant deviation dispersive prism.





$n_1\sin\theta_1=n_2\sin\theta_2$

Abbe Number

Dispersion can be fine-tuned by selecting glasses with the appropriate refractive index characteristics for a particular application. In general, the dispersion properties of various glass formulations are compared through **Abbe numbers, which are determined by measuring the refractive indices of specific reference wavelengths passed through the glass**. Abbe numbers for popular glasses utilized in prism construction are listed in Table 1. As is evident from examining the table, **lower Abbe numbers refer to higher dispersive power**, translating into a **greater angular spread of colors in the emerging light spectrum**.

Glass Formula	Refractive Index	Abbe Number
Fused Quartz	1.4585	67.8
BK 7	1.5168	64.17
Light Barium Crown	1.5411	59.9
Light Flint	1.5725	42.5
Dense Flint Glass	1.620	36.37
Extra Dense Flint Glass	1.6725	32.20
Very Dense Flint Glass	1.728	28.41

The Abbe number, V_D (variation of refractive index versus wavelength), , of a material is defined as

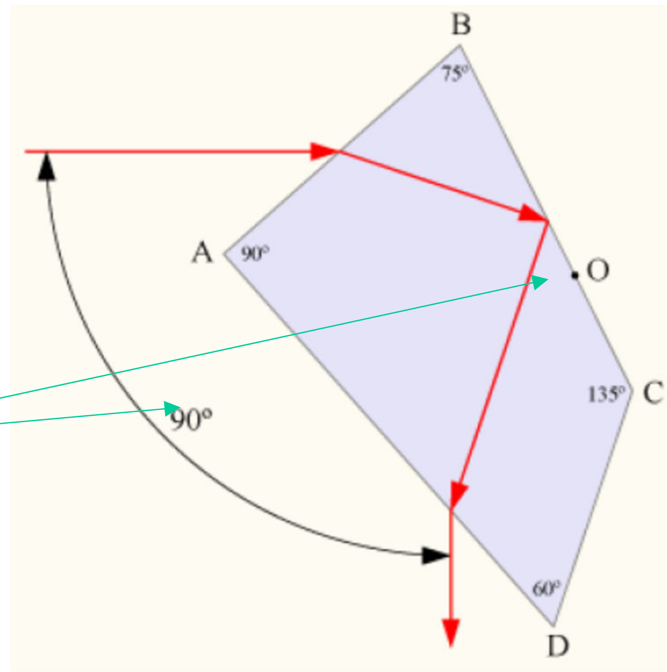
$$V_D = \frac{n_D - 1}{n_F - n_C},$$

where n_C, n_D and n_F are the refractive indices of the material at the wavelengths of the Fraunhofer C, D_1, and F spectral lines (656.3 nm, 589.3 nm, and 486.1 nm respectively). (see how dispersive the material is)

W.Wang

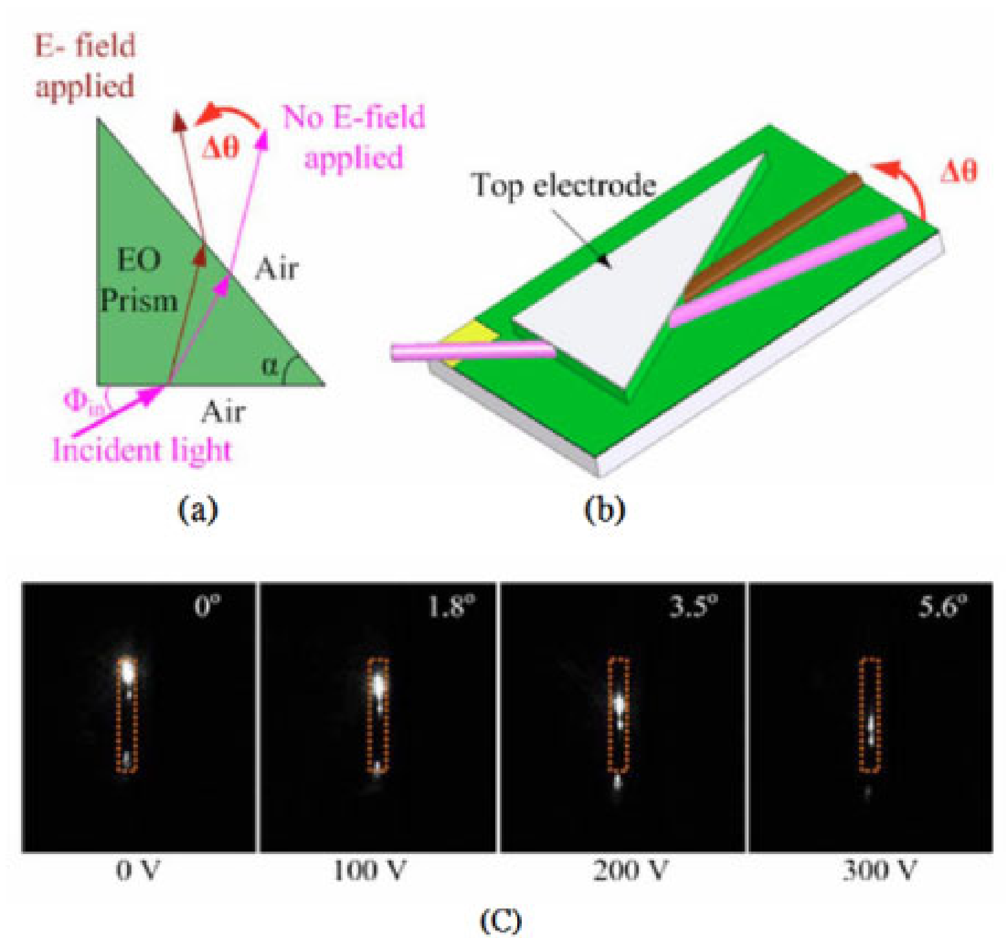
Pellin-Broca prism

This is similar to the Abbe prism but consist of a four-sided block of glass with **90°, 75°, 135°, and 60° angles on the end faces**. Light enters the prism through face AB, undergoes **total internal reflection from face BC**, **and exits through face AD**. The refraction of the light as it enters and exits the prism is such that one particular wavelength of the light **is deviated by exactly 90°.** As the prism is rotated around a point O, one-third of the distance along face BC, the selected wavelength which is deviated by 90° is changed without changing the geometry or relative positions of the input and output beams. It is commonly used to **separate a single wavelength from a light beam containing multiple wavelengths**. Thus, this is a type of constant deviation dispersive prism.



Dispersion in space with selective angle

Beam Deflector



Electro-optical (EO) scanner

Monochromatic
input light

What other ways can you come up with Beam deflector designs? (extra credit)

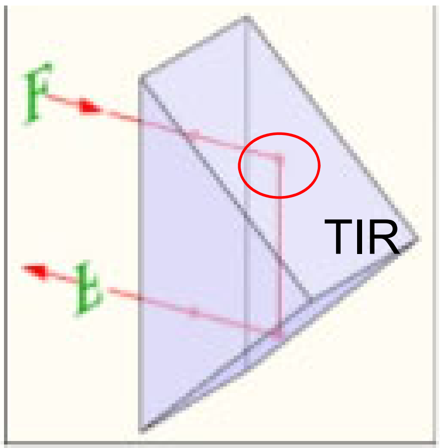
- Mechanical
- Electromagnetic means?
- Multiprisms?

Reflective Prisms (Mirror)

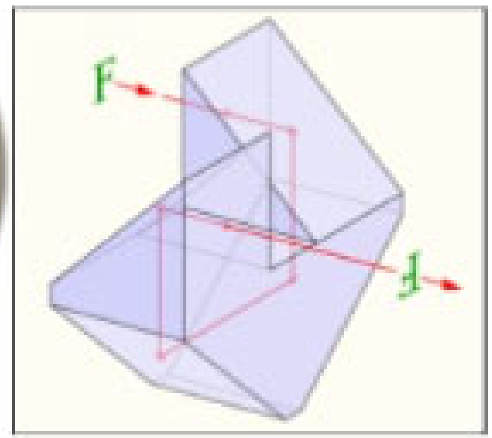
Reflective prisms utilize the "**internal reflection**" at the surfaces. **In order to avoid dispersion, light must enter and exit the prism orthogonal to a prism surface.** If light inside the prism hits one of the surfaces at a sufficiently steep angle, there is total internal reflection, and all of the light is reflected in accordance with the law of reflection (angle of incidence = angle of reflection). This makes a prism a very useful substitute for **a planar mirror** in some situations because it also has **no dipersion but still absorption loss (ε is complex)**

Thus, reflective prisms may be used to **alter** the direction of light beams, to **offset** the beam, and to **rotate or flip** images.

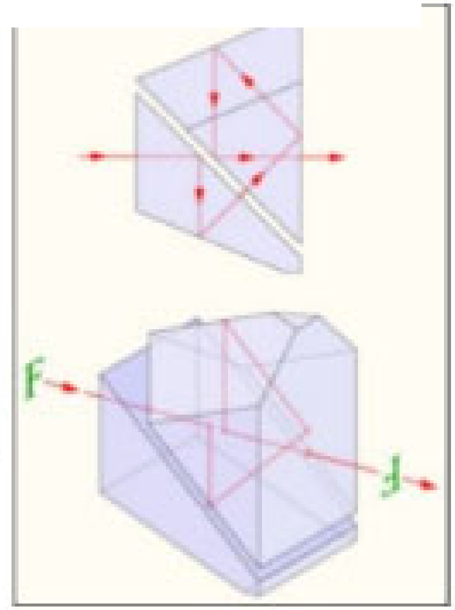
Reflecting Prisms



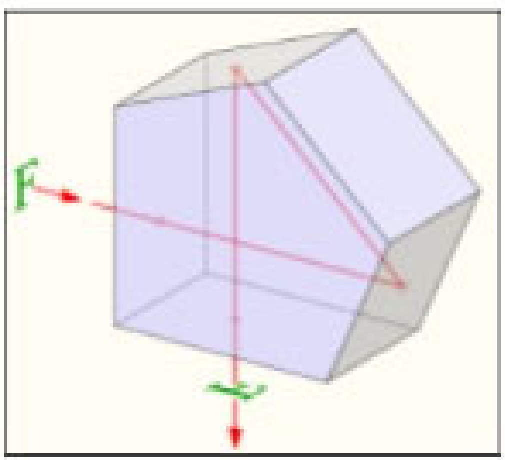
Porro Prism



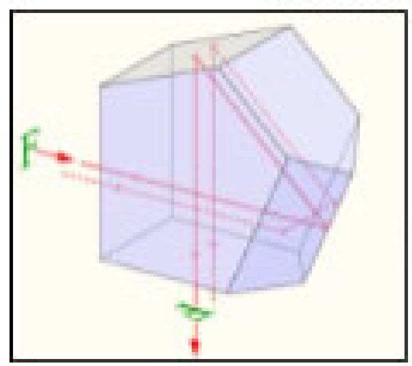
Porro-Abbe Prism



Schmidt-Pechan Prism



Penta Prism



Roof Penta Prism

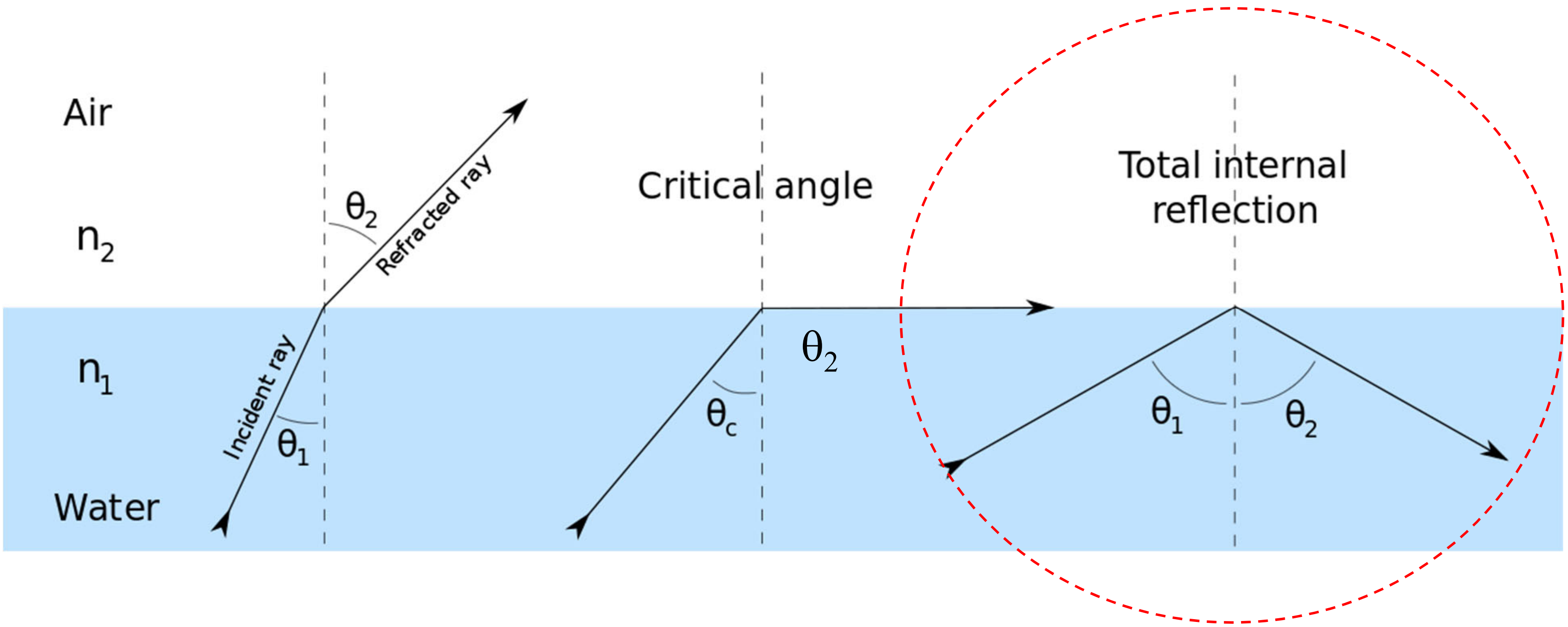
20

Using prism as mirror. Using total reflection or mtetal coatings to avoid dispersion

alter the direction of light beams, to offset the beam, and to rotate or flip images

W.Wang

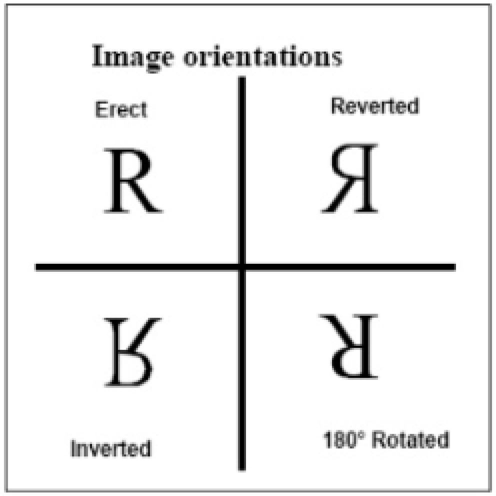
Total Reflection

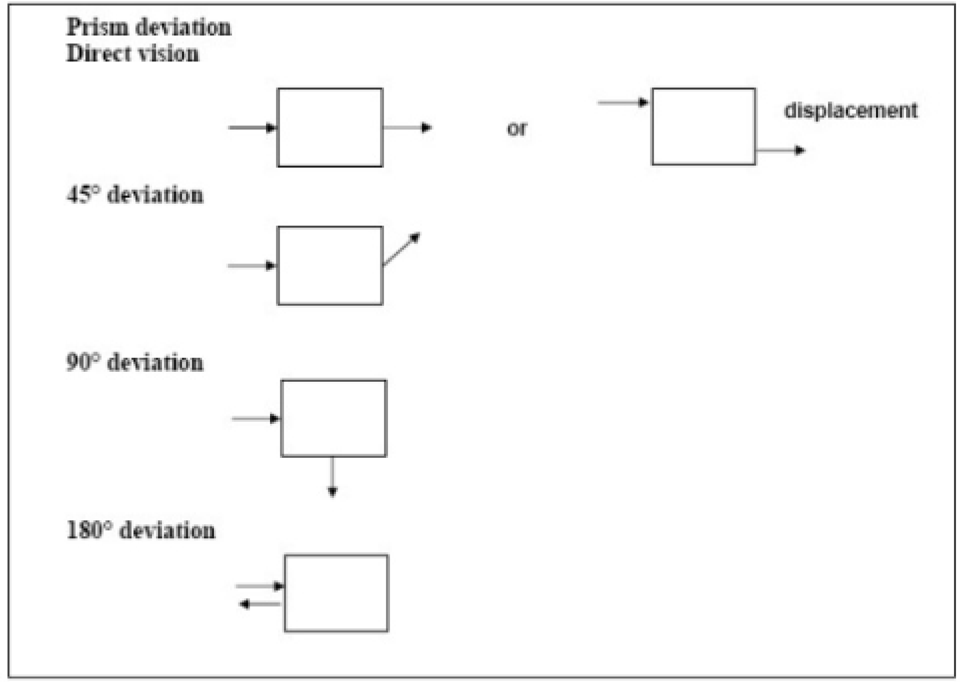


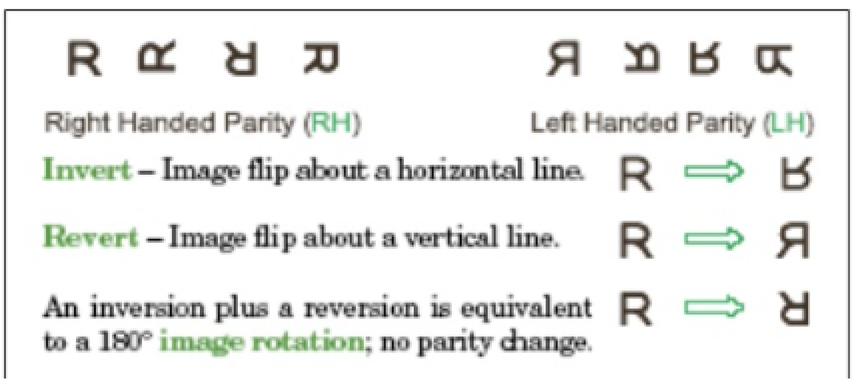
$n_1 \sin\theta_1 = n_2 \sin\theta_2$ $n_1 > n_2$

$\theta_2 = 90^o$ ➡ $\theta_c = \theta_1 = \sin^{-1}\dfrac{n_2}{n_1}$

Reflecting Prisms







All kinds of mirrored images

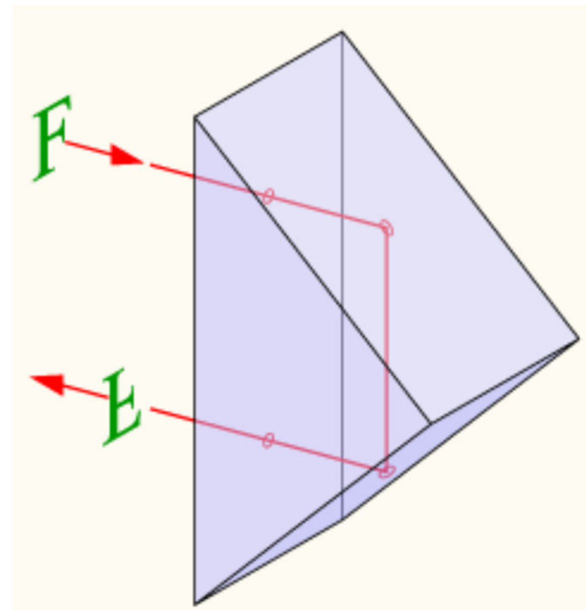
14

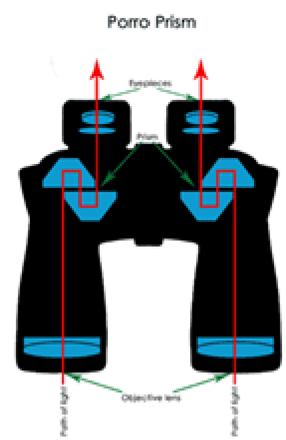
Single right-angle triangular prism

The right-angle triangular prism is the simplest type of optical prism. It has two rightangled triangular and three rectangular faces. As a reflective prism, it has two modes of operation.

1. The light beam enters orthogonal to one of the small rectangular faces, is reflected by the large rectangular face, and exits orthogonal to the other small rectangular face. The direction of the beam is altered by 90°, and the image is reflected left-to-right, as in an ordinary plane mirror.

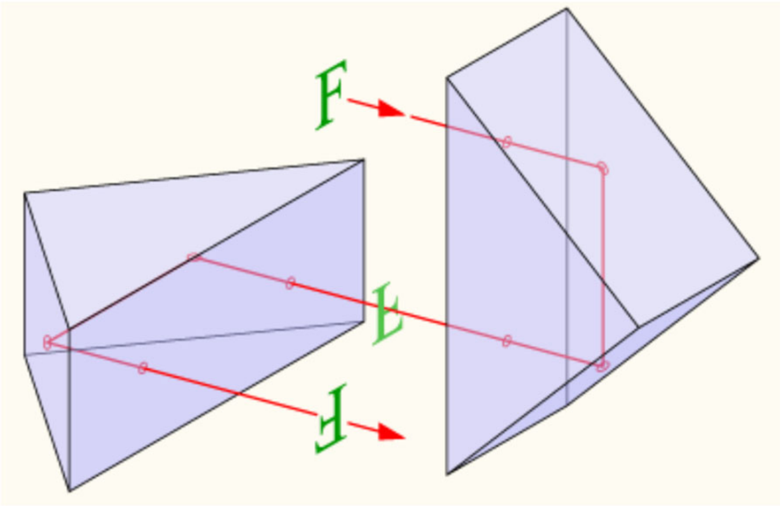
2. The light beam enters orthogonal to the large rectangular face, is reflected twice by the small rectangular faces, and exits again orthogonal to the large rectangular face. The beam exits in the opposite direction and is offset from the entering beam. The image is rotated 180°, and by two reflections the left-to-right relation is not changed (see figure to the right). In both modes, there is **no dispersion** of the beam, because of normal incidence and exit. **As the critical angle is approximately 41° for glass in air, we are also guaranteed that there will be total internal reflection, since the angles of incidence is always 41°.**

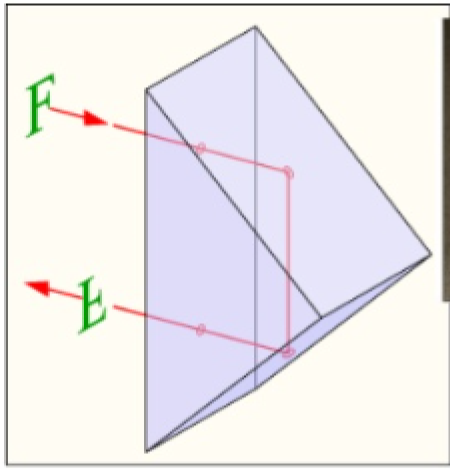




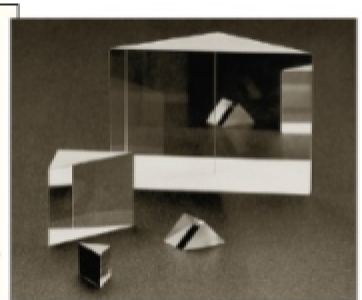
Combinations of right-angle triangular prisms

In the simplest configuration, the two prisms are rotated 90° with respect to each other, and offset so that half of their large rectangular faces coincide. Often, the two prisms are cemented together, and the sharp ends of the prisms may be truncated to save space and weight. The net effect of the configuration is that the beam will traverse both prisms through four reflections. The net effect of the prism system is a beam parallel to but displaced from its original direction, both horizontally and vertically.





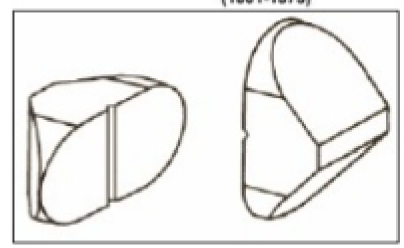
Porro Prism



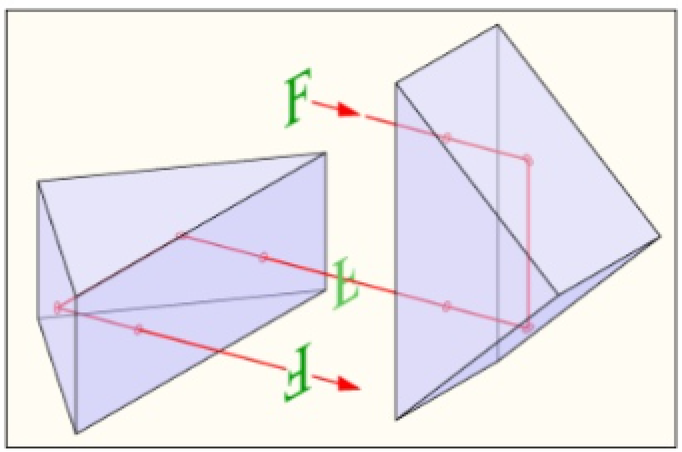
Porro prism is a right-angle prism in which the light enters and exits normal to the large rectangular phase preventing dispersion.

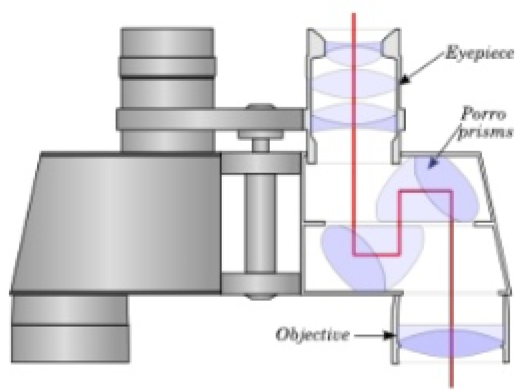
Ignazio Porro
(1801-1875)

Porro prism are used in pairs in binoculars



Double Porro Prism





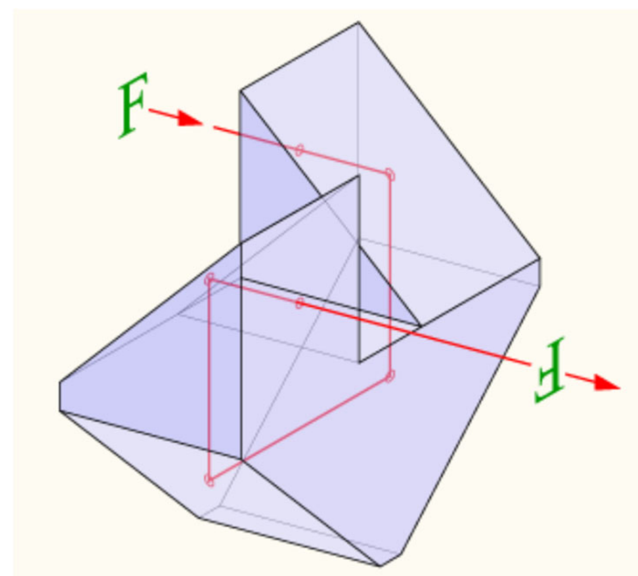
Porro prism are fabricated with rounded corners to save space and weight.

17

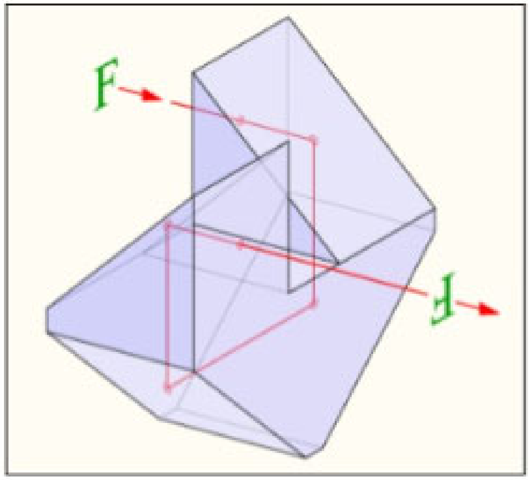
Combinations of right-angle triangular prisms

An alternative configuration is shown in the figure to the right. The exits beam again travels in the same direction as the input beam, but is offset in the horizontal direction.

In the configuration, the image rotated 180°, and given the even number of reflections, the handedness of the image is unchanged.



Reflecting Prisms



A Porro-Abbe prism is sometimes called a double right angle prism.

Two of the are used to make an erected system for telescopes and some binoculars..

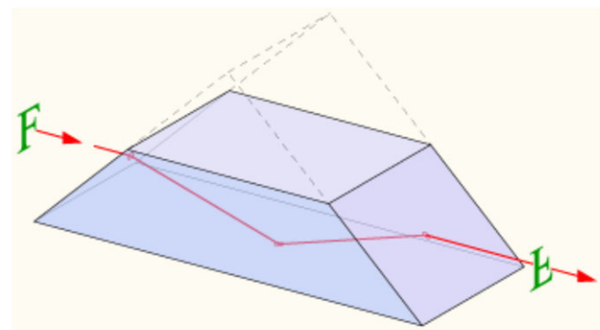

Ignazio Porro
(1801-1875)

Ernst Karl Abbe 1840-1905

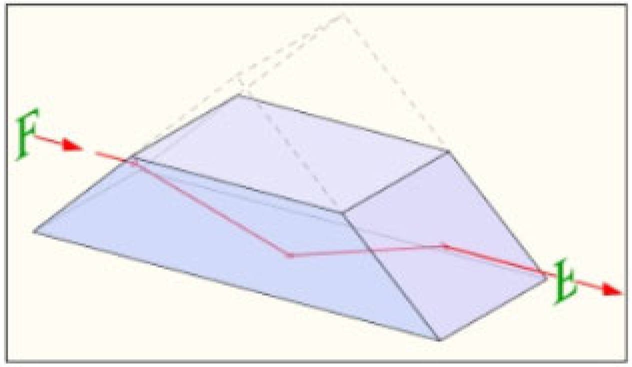
Porro-Abbe Prism

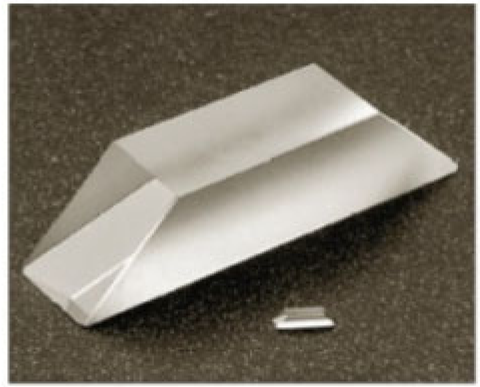
Truncated right-angle prism

A truncated right-angle (Dove) prism may be used to invert an image. A beam of light entering one of the sloped faces of the prism at an angle of incidence of 45°, undergoes total internal reflection from the inside of the longest (bottom) face, and emerges from the opposite sloped face. Images passing through the prism are flipped, and because only one reflection takes place, the image's handedness is changed to the opposite sense. Dove prisms have an interesting property that when they are rotated along their longitudinal axis, the transmitted image rotates at twice the rate of the prism. This property means they can rotate a beam of light by an arbitrary angle, making them useful in beam rotators, which have applications in fields such as interferometry, astronomy, and pattern recognition.

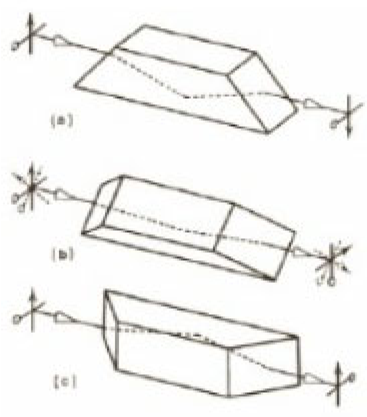


Differential interference contrast

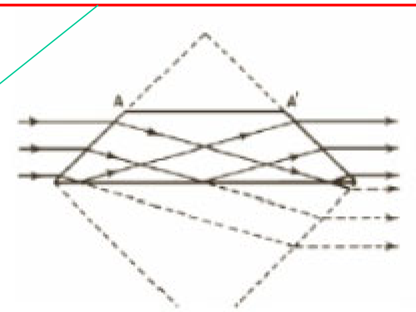


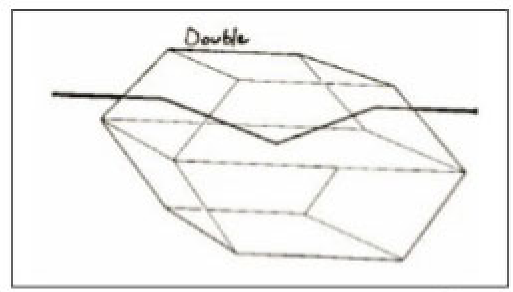


Dove Prism invented by H.W. Dove



In the Dove Prism for a rotation of θ around optical axis we obtain a 2 θ image rotation.



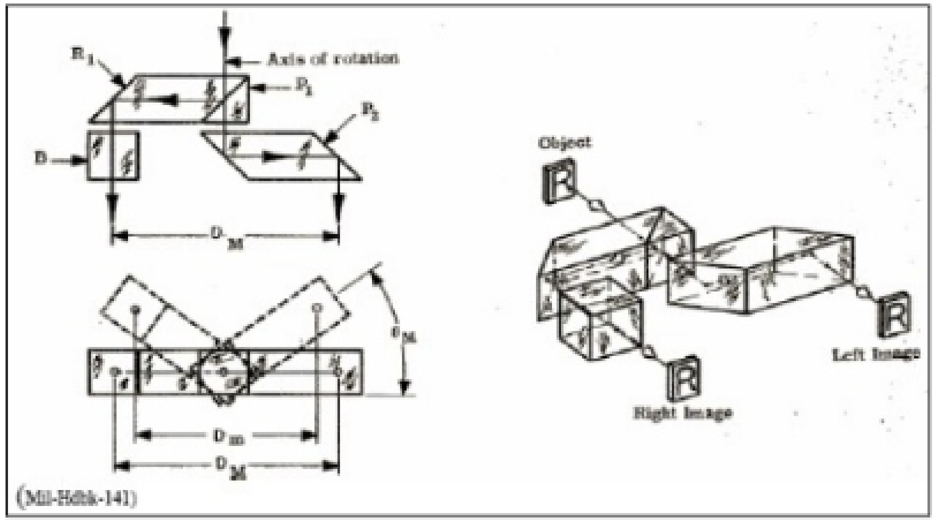


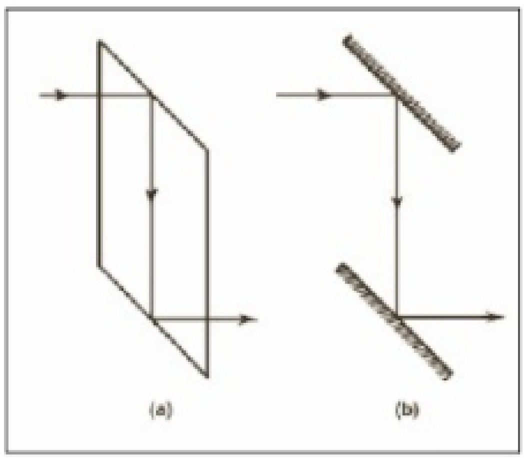
Double Dove Prisms

20

Rhomboid Prism

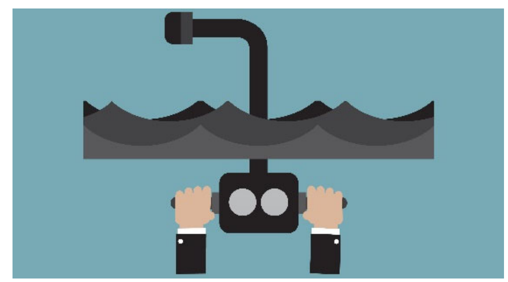
The Rhomboid Prism deviates the light, but does not change the angle even if the prism is rotated about all axes.





(a) The Rhomboid Prism
(b) and its mirrors equivalent

Both displace the optical axis without deviation or reorientation of the image.



periscope

W. Wang

Pechan Rotator Prism

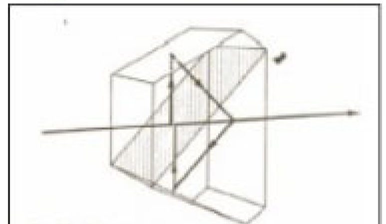
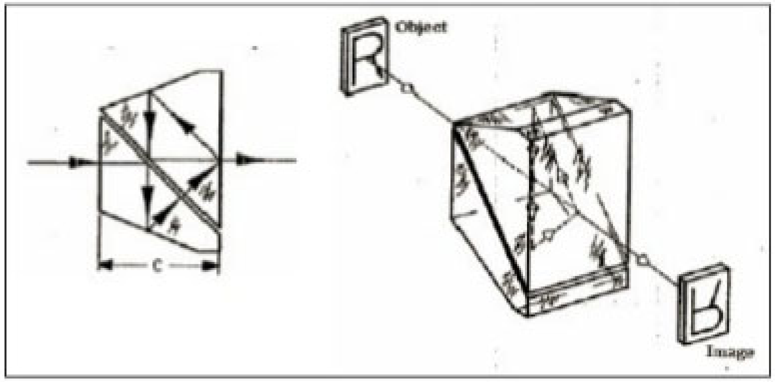
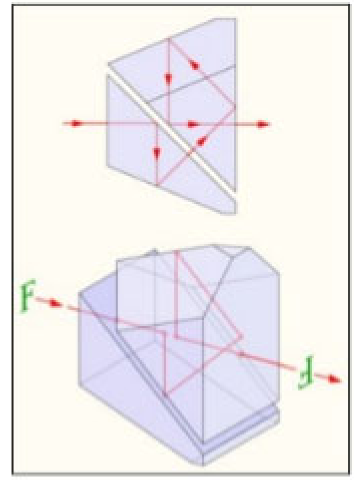
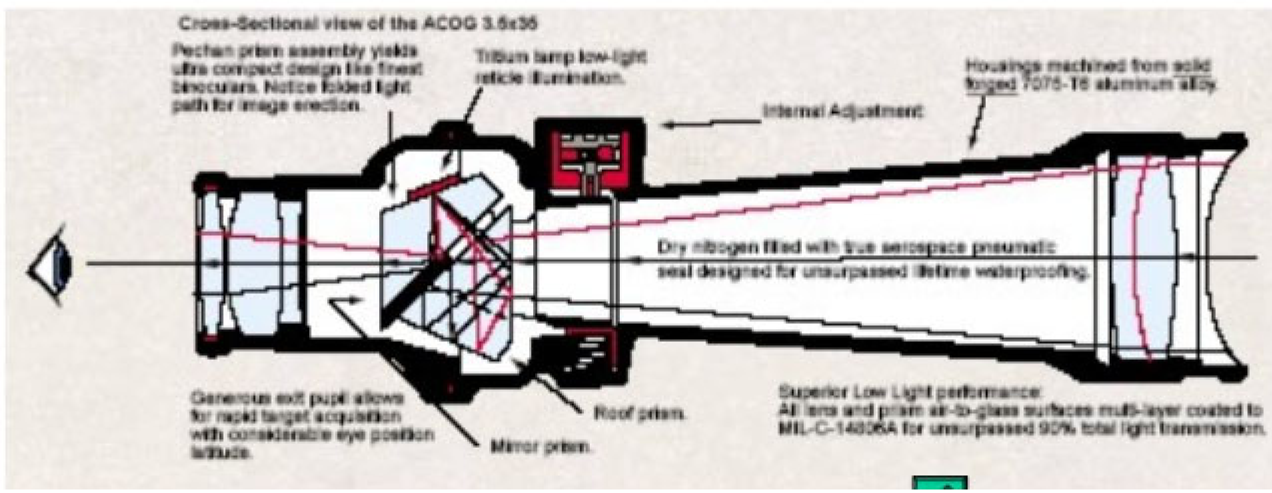


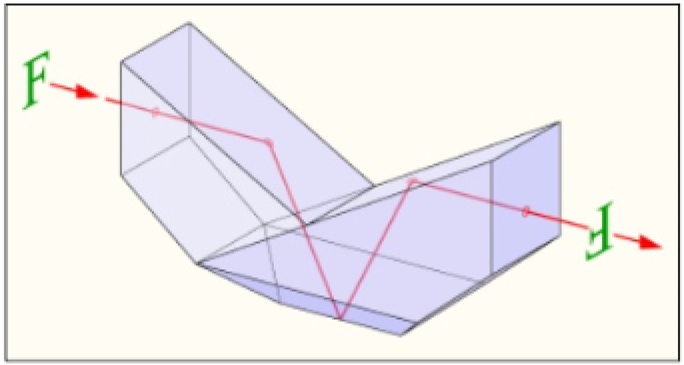
Fig. 22 Pechan rotator







Reflecting Prisms

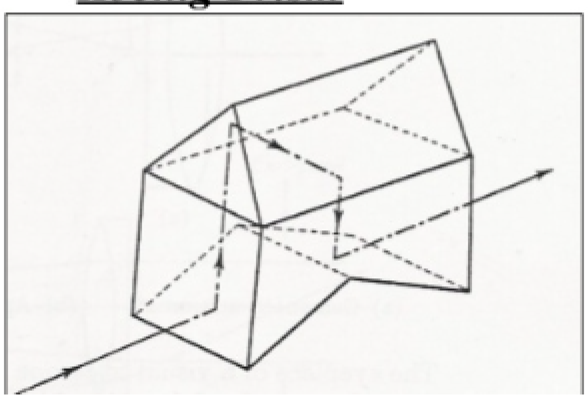


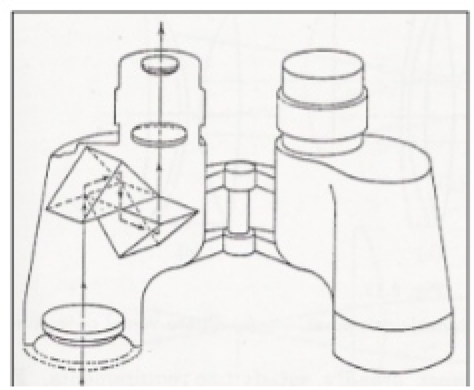
Ernst Karl Abbe 1840-1905

Abbe-Koenig Prism

The Abbe-Koenig prism is a reflecting prism used to rotate the image by 180º is used in binoculars and some telescopes. The prism is named after Ernst Abbe and Albert Koenig.

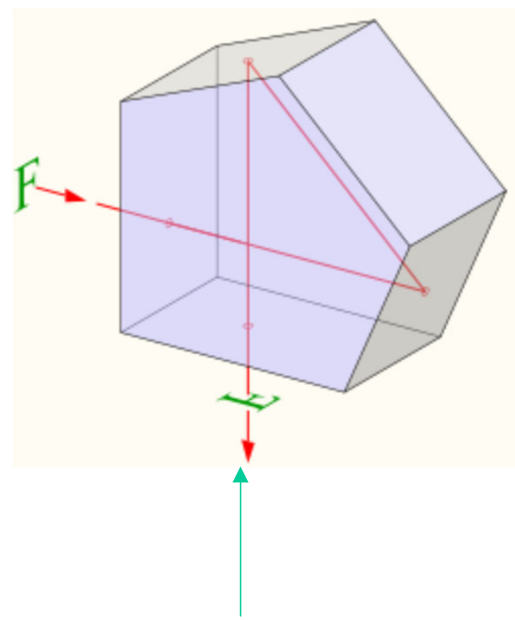
Koenig Prism





Pentaprism

In a pentaprism the light beam enters orthogonal to one of the two orthogonal rectangular faces, is reflected by the two neighboring faces, and exits orthogonal to the face that is orthogonal to the entry face. Thus, the direction of the beam is altered by 90°, and as the beam is reflected twice, the prism allows the transmission of an image through a right angle without inverting it. During the reflections inside the prism, the angles of incidence are less than the critical angle, so there is no total internal reflection. Therefore, the two faces have to be coated to obtain mirror surfaces. The two orthogonal transmitting faces are often coated with an antireflection coating to reduce reflections. The fifth face of the (90°, 112.5°, 112,5°, 112.5°) prism is not used, but truncates what would otherwise be a sharp angle of 25° joining the two mirror faces. This fifth face is usually smaller than the two mirror faces, in order to let the mirror faces receive all beams entering the input face.

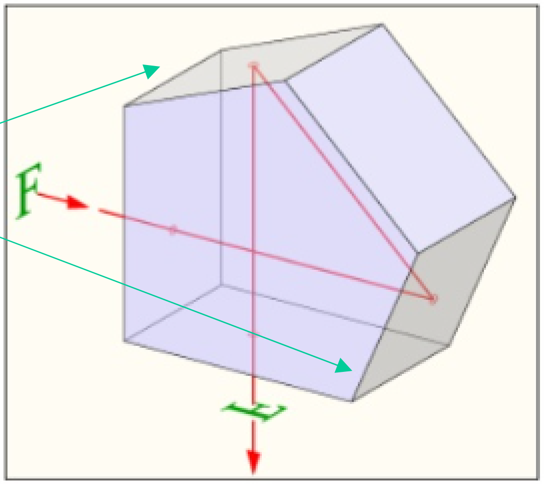


Same direction image

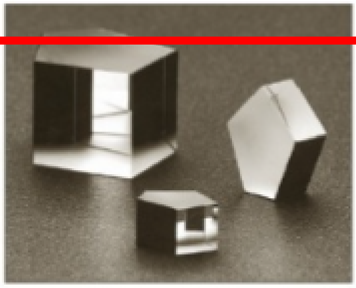
Reflecting Prisms

Penta Prism is a five (penta) sided prism that deflects the light by 90º without inverting or reversing it. The two reflecting sides are coated (mirrors).

It is used for range finding, alignment, surveying optical tooling.

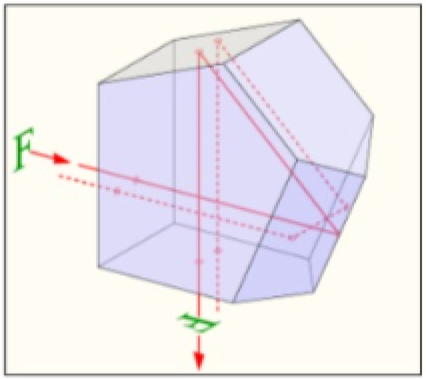


Penta Prism



In the RoofPenta Prism one of the reflecting surface is replaced by a "roof" constitute of two surfaces that intersect at an angle of 90º. The reflecting sides are coated (mirrors).

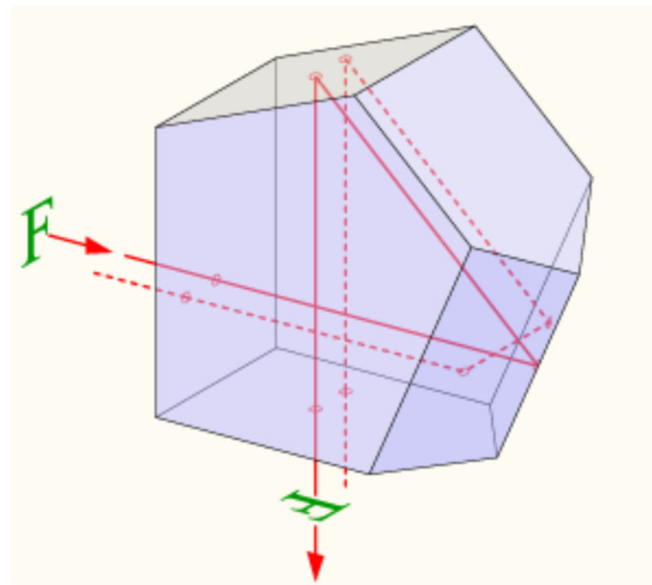
It is used in the viewfinder of a single lens reflecting camera.

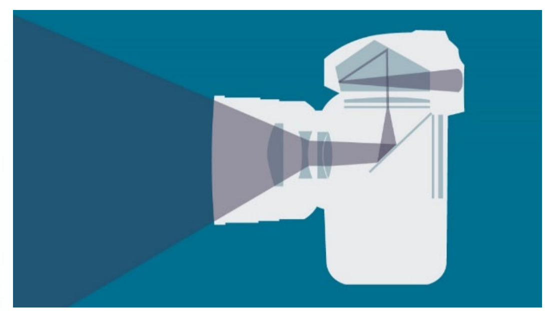


16

Roofed Pentaprims

A roofed pentaprism is a combination of an ordinary pentaprism and a right-angle triangular prism. The triangular prism substitutes one of the mirror faces of the ordinary pentaprism. Thus, the handedness of the image is changed. This construction is commonly used in the viewfinder of single-lens reflex cameras.

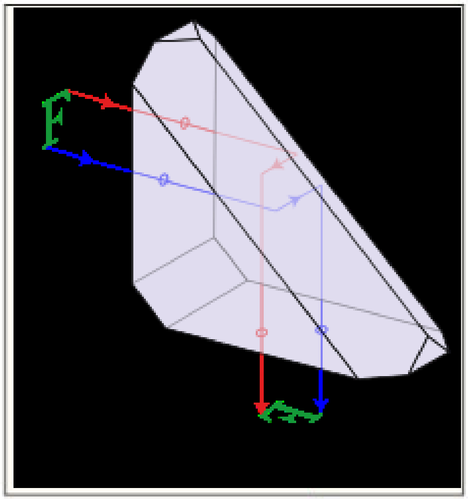




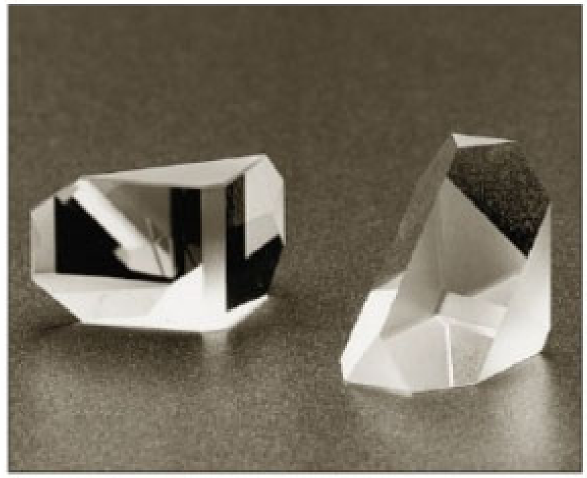
Nikon F3 prims viewfinder

Roof Prims

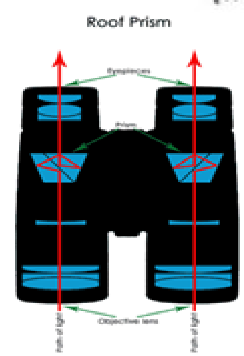
Reflecting Prisms

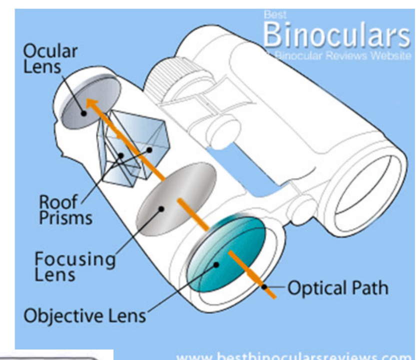


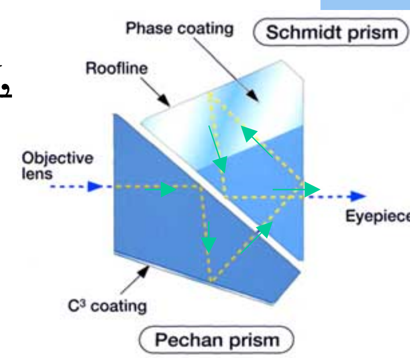
Amici-roof Prism



Giovanni Amici
(1786-1863)



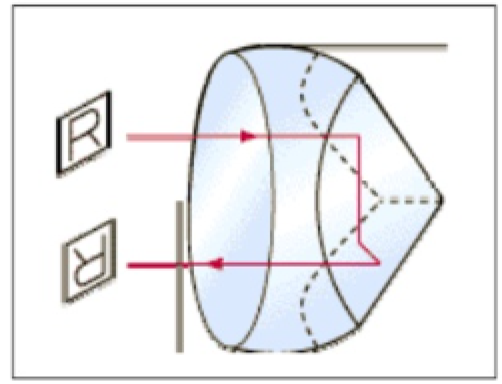




A **roof prism** is a combination of two right angled triangular prisms. <u>It consists of a simple right-angled prism with a right-angled "roof" prism on its longest side, as shown in the figure to the left. Total internal reflection from the roof prism flips the image laterally, while the handedness of the image is unchanged</u>. It is used to deviate a beam of light by 90° and simultaneously inverting the image, e.g., as an image erection system in the eyepiece of a telescope.

Reflecting Prisms

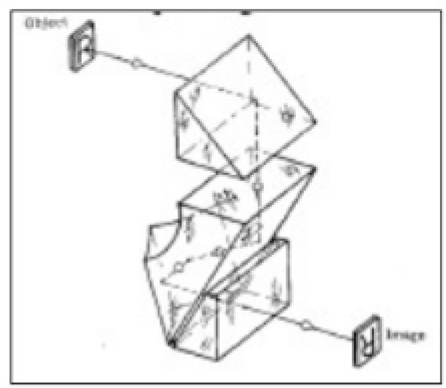
Corner Cube, Corner Reflector



The Corner Cube has three mutually perpendicular surfaces and a hypotenuse face. The light enters through the hypotenuse is reflected by each of the three surfaces in turn and emerges through the hypotenuse surface parallel to the entering ray.

Carl Zeiss Prism System

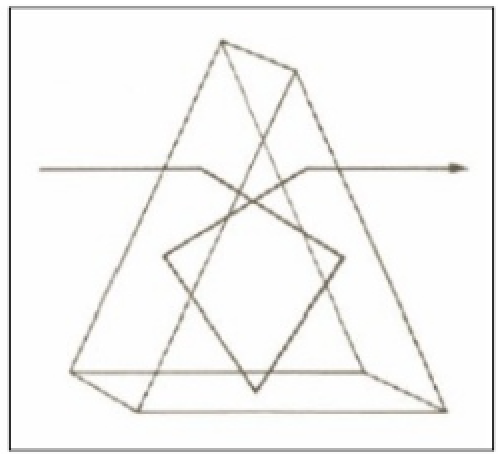
The system is composed by three single prisms which invert and rotate the image, but does not deviate the line of sight. The line of sight is displaced by a distance depending on the relative position of the first and second prism.



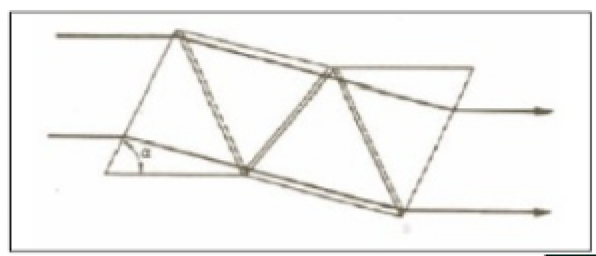
Carl Friedrich Zeiss
(1816-1888)

22

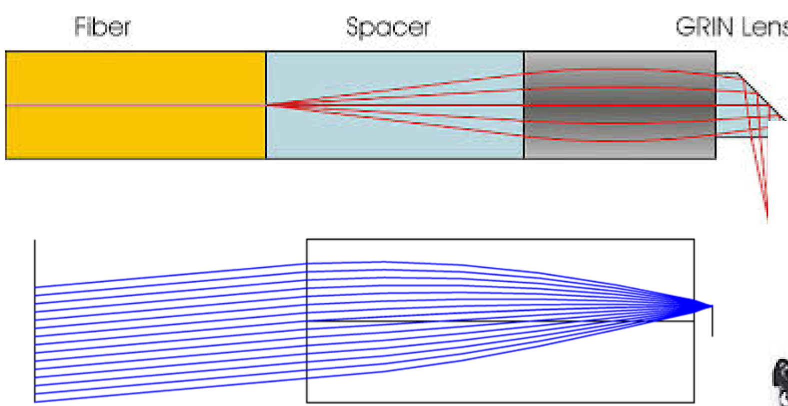
Schmidt Rotator Prism (Folded Dove Prism)



Schmidt Type Rotator Tunnel Prism

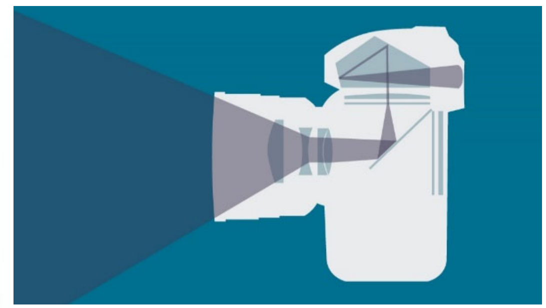


Mirror Application

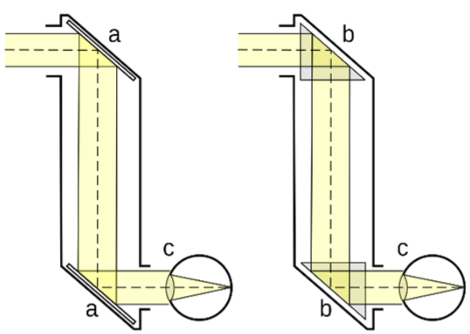


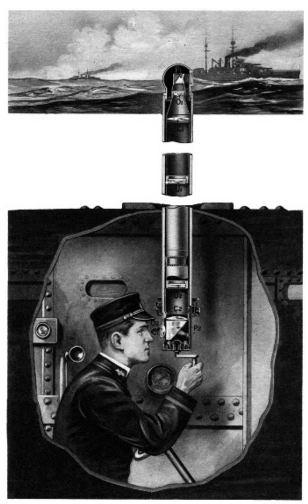
OCT beam bending mirror

Nikon F3 prims viewfinder



Roofed pententaprim

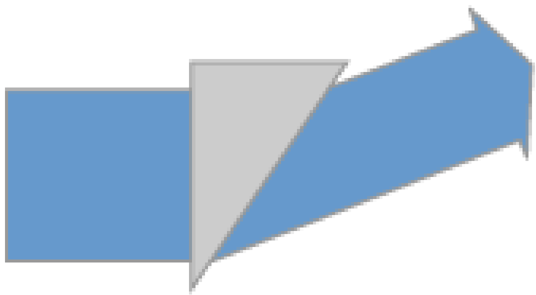




W. Wang

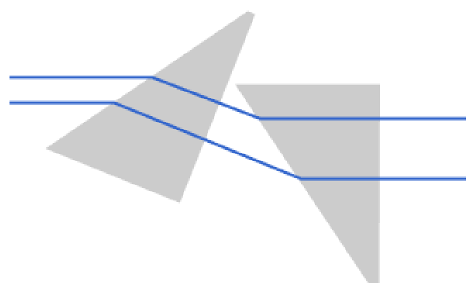
Rhomboid Prism in periscope

Beam Reduction and Expander

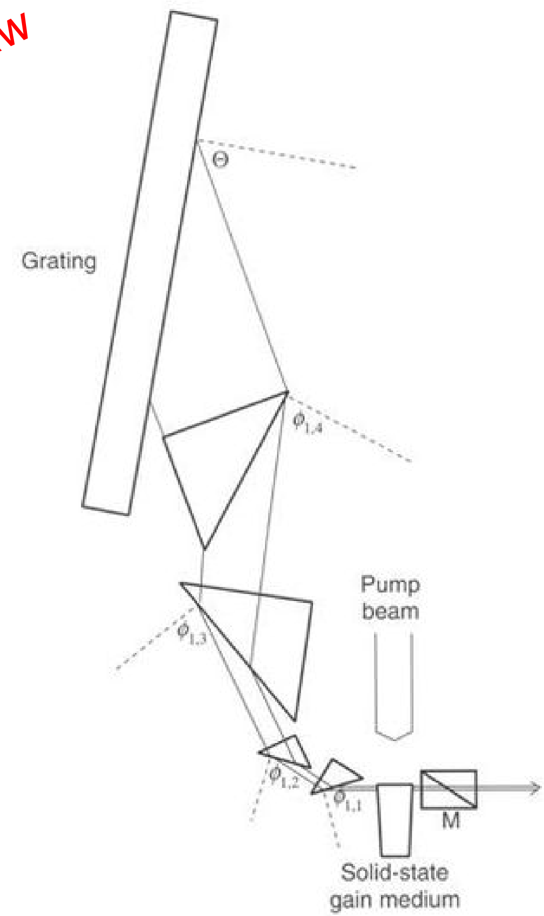


An anamorphic prism. The output beam is substantially narrower than the input beam

Use Smell's Law



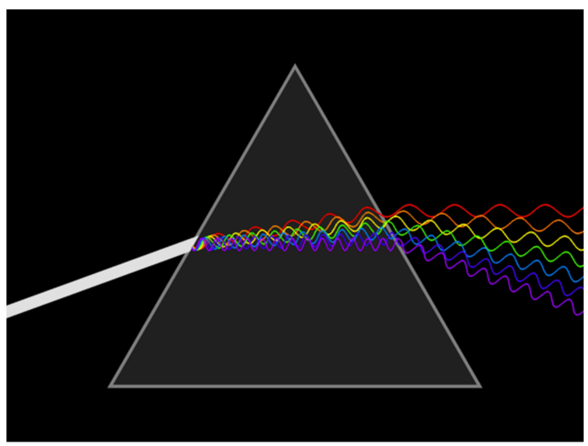
An anamorphic prism pair with refractive index of 1.5, where Brewster's angle is used on one side of each prism, and normal incidence on the other one. Two parallel beams passing through the prisms are shown. Their distance changes, and likewise their beam radii in the direction of the plane are changed. The prism pair thus works as a beam expander if the input beam comes from the left side. Of course, the beam radius in the direction perpendicular to the drawing plane is not changed.



Long-pulse tunable laser oscillator utilizing a multiple-prism beam expander

Prism

- Dispersing white light
- Deflect and steering light
- Mirrors
- Beam expander and reducer
- Used in coupling light into integrated optical system
- Beam splitter and combiner
- Polarizer and phase shifter



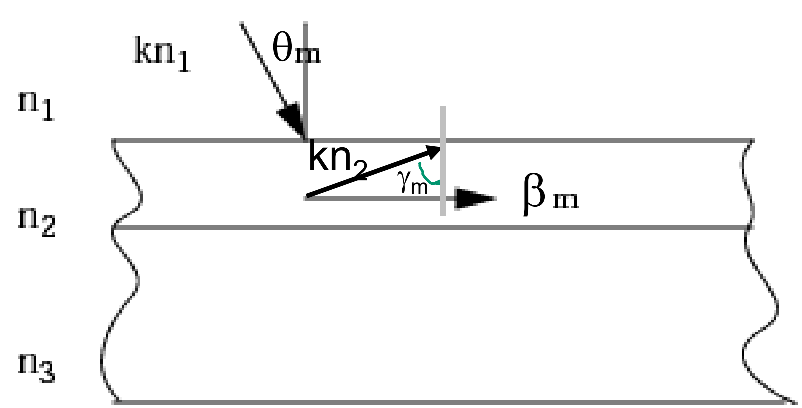
Prism Coupler

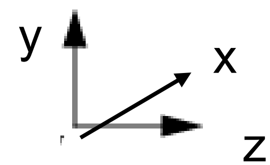
One of the ways to coupl the ligh into the waveguide is to utilize a prism.

The prsim coupler allows light to be coupled at an oblique angle. For this to happen, the components of phase velocities of the waves in the z direction be the same in both the waveguide and the incident beam

Two main mechanisms in creating coupling:

1. Evanescent wave
2. Coupling length is long enough ($\pi/2$)





$\beta_m \sim kn_2 \sin\gamma_m$

Thus, a phase-matching condition must be satisfied in z direction, which requires

Tangential E fields are continuous!!!

$\beta_m = kn_2 \sin\gamma_m = kn_1 \sin\theta_m = \frac{2\pi}{\lambda_o} n_1 \sin\theta_m$

Index inside waveguide n_2 > air n_1, θ_m always 90^o

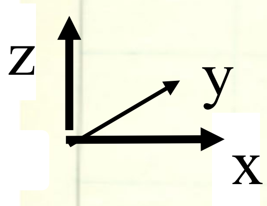
However, we know for a waveguided mode, $\beta_m > kn_1$, this leads to the result that $\sin\theta_m > 1$.

One solution to the problem of phase matching is to use a prism.

$\beta_m = kn_2 \sin\gamma_m = kn_1 \sin\theta_m$

Check out the derivation on evanescent wave page 106

Total reflection



Axes labels are different from last page

$\theta_2 = 90°$ (refraction angle = 90°)

$k_1 \sin\theta_1 = k_2 \sin\theta_2$ (use Snell's law)

or phase matching condition

$$\boxed{\theta_{critical} = \sin^{-1}\frac{k_2}{k_1}}$$

if $\theta_1 > \theta_{critical}$

Then $\sin\theta_2 > 1$

since $k_2^2 = k_{2z}^2 + k_{2x}^2$

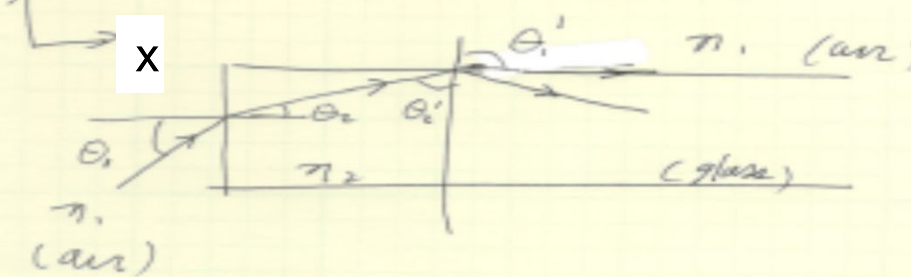
(z direction propagation constant) $k_{2z}^2 = k_2^2 - k_{2x}^2$

$$k_{2z}^2 = k_2^2 - (k_2 \underbrace{\sin\theta_2}_{>1})^2$$

$e^{jk_{2z}z}$ $\quad k_{2z}^2 < 0$

k_{2z} = imaginary

waveguide theory



$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$n_2 \sin\theta_2' = n_1 \sin\theta_1'$$

$\theta_1' = 90°$ ∴ total reflection occurs

then $\theta_2' = \theta_{critical}$

no wave travel outside the glass substrate, therefore glass substrate becomes a wave guide.

However if $\theta_2' > \theta_{critical}$

Then $n_2 \sin\theta_2' = n_1 \underbrace{\sin\theta_1'}_{>1}$

$1 = \underbrace{\sin^2\theta_1'}_{>1} + \boxed{\underbrace{\cos^2\theta_1'}_{<0}}$

since $k_1^2 = k_{1z}^2 + k_{1x}^2$

$k_1^2 = k_1^2\sin^2\theta_1' + \boxed{k_1^2\cos^2\theta_1'}$

$k_{1z}^2 = k_1^2 - k_{1x}^2 = k_1^2 - (k_1 \sin\theta_1')^2 \Rightarrow k_{1z}^2 < 0$

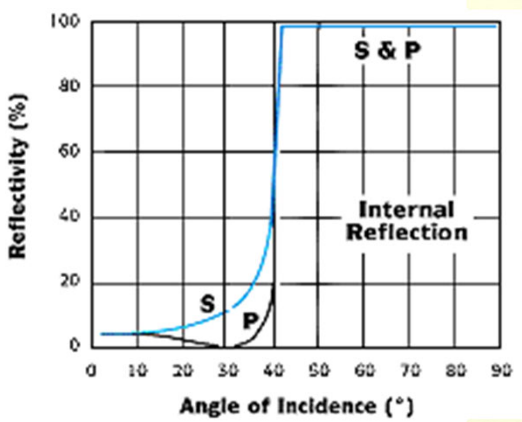
k_{1z} = imaginary

$$E e^{jk_{1x}x + jk_{1z}z}$$

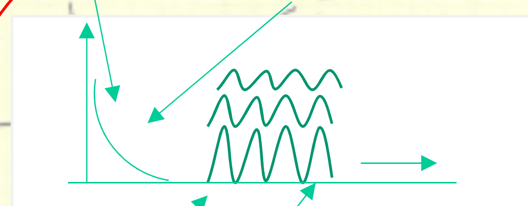
$$\mathrm{Re}\left\{ \left(E e^{jk_{1x}X}\right) e^{\overset{\text{imaginary}}{j k_{1z}} z} \right\}$$

$$= E_0 \cos k_{1x}X \; e^{-j\alpha_z z}$$

where α_z = ~~imaginary~~ k_{1z} = imaginary



$e^{-j\alpha_z z}$



$\cos k_{1x}X$

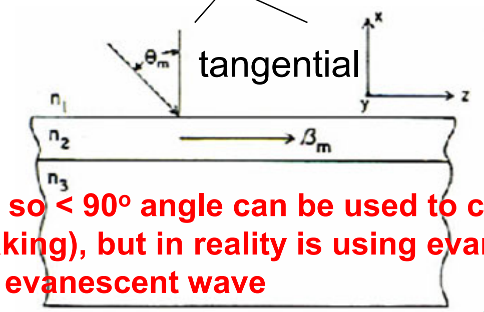
Evanescent wave

use for sensing and wave coupling

cosk_{1x}x at different z

Prism Coupler

If prism spacing is small enough so that tail of waveguide modes overlap the tail of the prism mode, there is a coherent coupling of energy from prism mode to the mth waveguide mode when θ_m is choosen so trhat $\beta_p = \beta_m$ (phase matching condition based on E field is contious). The condition for matching of the β terms is given by



$$\beta_p = \frac{2\pi}{\lambda_o} n_p \sin\theta_m = \beta_m$$

Change air with high index material so < 90° angle can be used to coupled the light in (hypothetically speaking), but in reality is using evanescent wave that matches waveguide evanescent wave

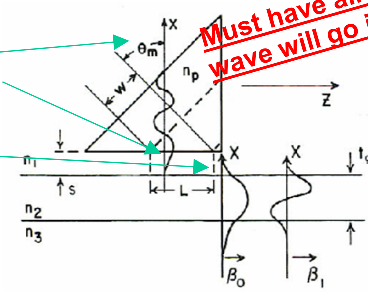
a single prism can be used to couple to many different modes by merely changing the angle of incidence of the optical beam.

The modes in the waveguide are only weakly coupled to the mode in the prism. Hence, negligible perturbation of the basic mode shapes occurs. Of course, the condition

$$\theta_m > \theta_c = \operatorname{asin}\frac{n_1}{n_p}$$

Key is making sure both prism modes and propagating modes exist also we can generate evanescent wave for coupling!

Must have air gap or wave will go into prims



must be satisified if total internal reflection is to occur in the prims where θ_c is the critical angle.

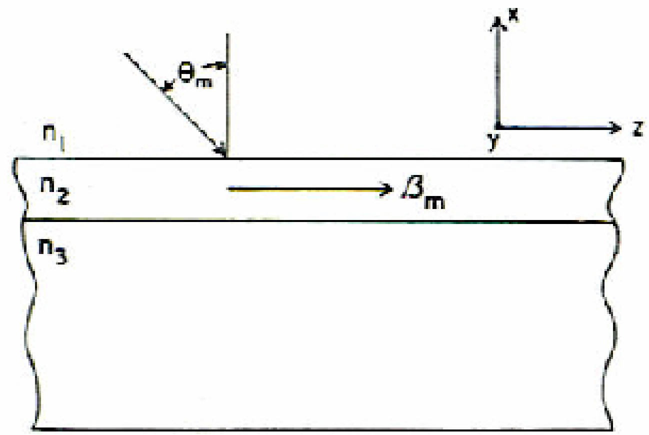


Fig. 7.6. Diagram of an attempt to obliquely couple light into a waveguide through its surface

Make sure beam width after refraction < prism's base width

Angle creates total internal reflection inside prism

$L = W/\cos\theta_m$

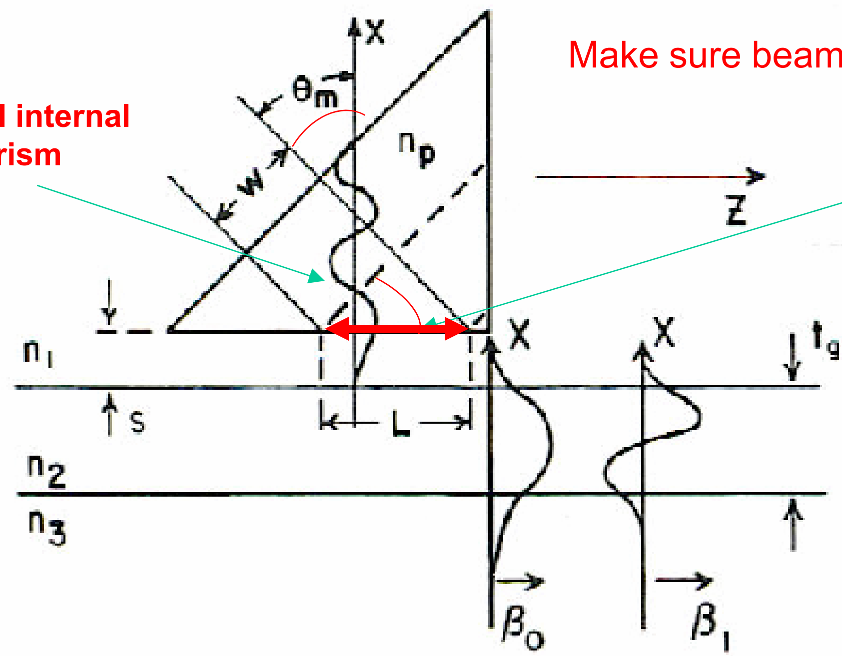
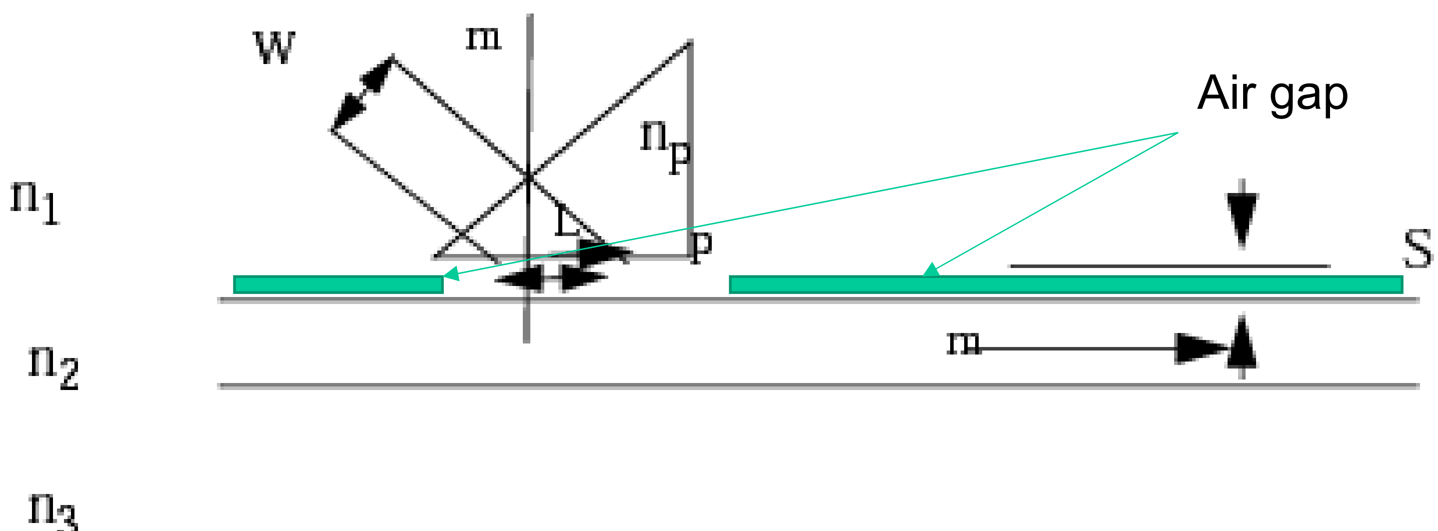


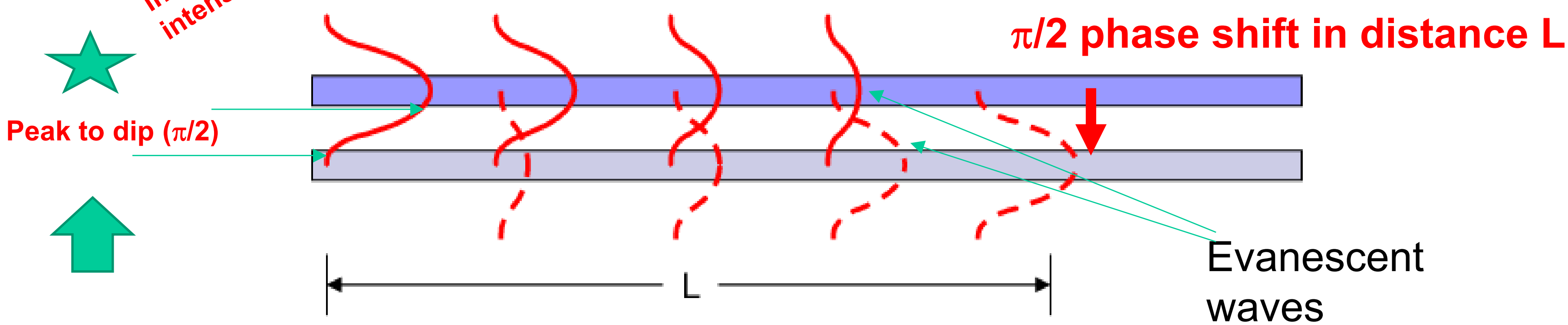
Fig. 7.7. Diagram of a prism coupler. The electric field distributions of the prism mode and the $m = 0$ and $m = 1$ waveguide modes in the x direction are shown

This condition for complete coupling assumes that the amplitude of the electric field is uniform over the entire width W of the beam. In praticial case this is never true. Also the trailing edge of the beam must exactly inte5rsect the righ-angle corner of the prism. If the intersects too far to the right, some of the incident power will be either reflected or transmitted directly into the waveguide and will not enter the prism mode. If the beam is incident too far to the left, some of the power coupled into the waveguide will be coupled back out into the prsim.



Coupled-Mode Theory

Transfer max intensity to max intensity requires $\pi/2$



$$\kappa L = \frac{\pi}{2}$$ Phase shift in L

κ : Coupling efficient (overlap integral between the prism mode and the waveguide mode)

$$L = \frac{W}{\cos\theta_m} = \frac{\pi}{2\kappa}$$

For a given L, the coupling coefficient required for complete coupling:

$$\kappa = \frac{\pi \cos\theta_m}{2W}$$

See next page for derivation

Because of the size of the prism, the interaction betwen prsim and waveguide mode can occur only over the length L. The thoery of weakly coupled mode indicates that a complete interchange of energy between pahse matched modes occurs if the interaction length in the z direction satifies the relation

$$kL = \frac{\pi}{2}$$

where k is coupling coefficient. The coefficient depend on n1, np and n2, which determine the shape of the mode tails and on the prism spcaing.

The length required for complete coupling is given by

$$L = \frac{W}{\cos\theta_m}$$

For a given length, the coupling coefficient required for complete coupling is thus given by

$$\kappa = \frac{\pi \cos\theta_m}{2W}$$

Why prism coupler?

The advantage of prism coupler is that it can be use as an **input and output coupling devices. (very important more throughput!!!!!)**

If **more than one mode is propagating in the guide**, light is coupled in and out at specific angles corresponding to each mode.

If a gas laser is used, the best method for coupling is using either prism or grating coupler

Disadvantage is is that **mechanical pressure must be applied to prism during each measurement sot hat spacing between prism and waveguide remains constant to get consistent coupling coefficient.**

Other disadvantage is **prism coupler index must be greater than the waveguide.**

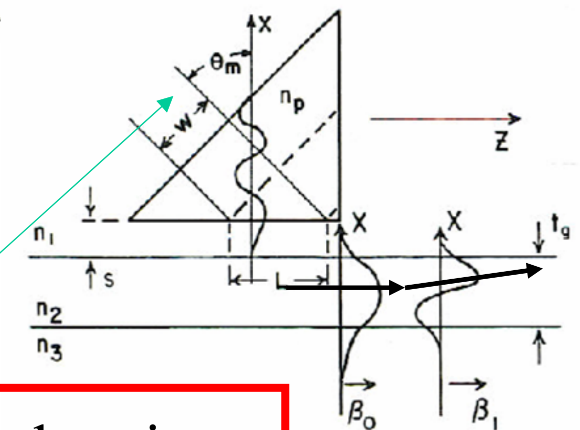
Another disadvantage is that **the incident beam must be highly collimated** because of the angular dependence of the **coupling efficiency on the lasing mode**.

For most semiconductor waveguides, $\beta_m \sim kn_2$ -> Difficult to find prism materials

Table 7.1. Practical prism materials for beam couplers

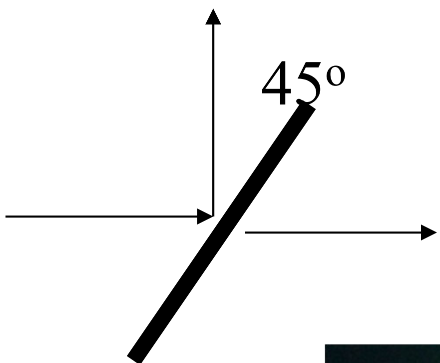
Material	Approximate refractive index	Wavelength range
Strontium titanat	2.3	visible – near IR
Rutile	2.5	visible – near IR
Germaium	4.0	IR

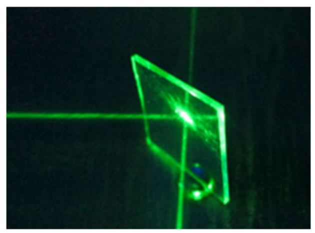
Higher the n_p better because larger the range or smaller the θ_m



$$\boxed{\beta_p = \frac{2\pi n_p}{\lambda_o} cos\theta_m = \beta_m \sim kn_2 sin\gamma_m}$$

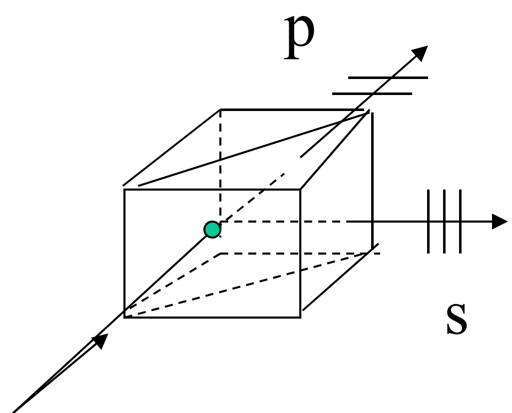
Beam Splitters





Aluminum coated beam splitter.

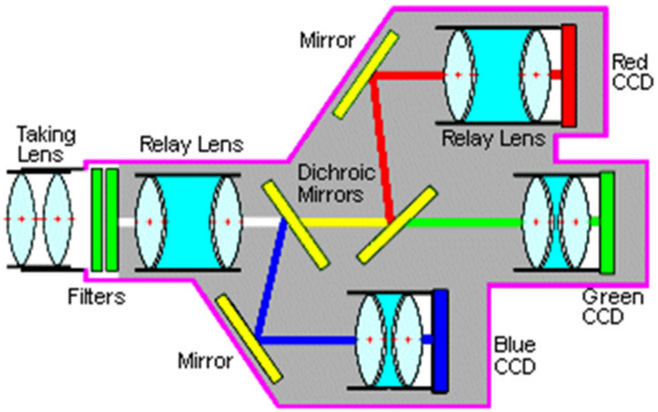
Reflective
Beam splitter
- Specific operating wavelength
- Split ratio
- 45° incident angle
- Unsually unpolarized

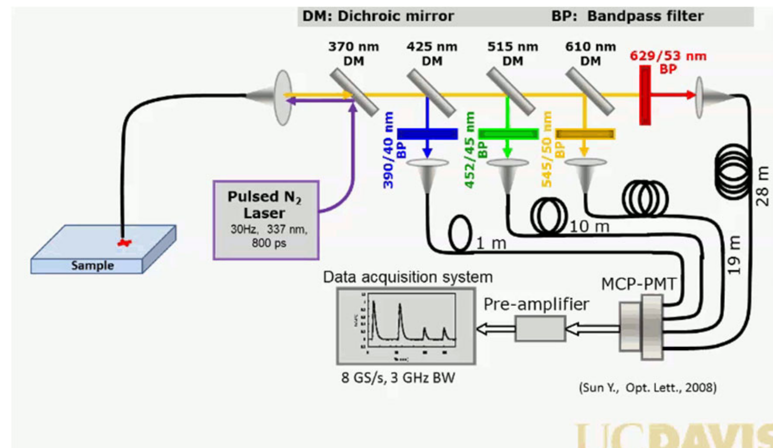


Cube
Beam splitter
- Broadband
- Split ratio
- Polarized or nonpolarized

Index matching adhesive between two prisms

A third version of the beam splitter is a dichroic mirrored prism assembly which uses dichroic optical coatings to **divide an incoming light beam into a number of spectrally distinct output beams.** Such a device was used in multi-tube color television cameras, in the three-film Technicolor movie cameras as well as modern, three-CCD cameras. It is also used in the 3 LCD projectors to separate colors and in ellipsoidal reflector spotlights to eliminate heat radiation.





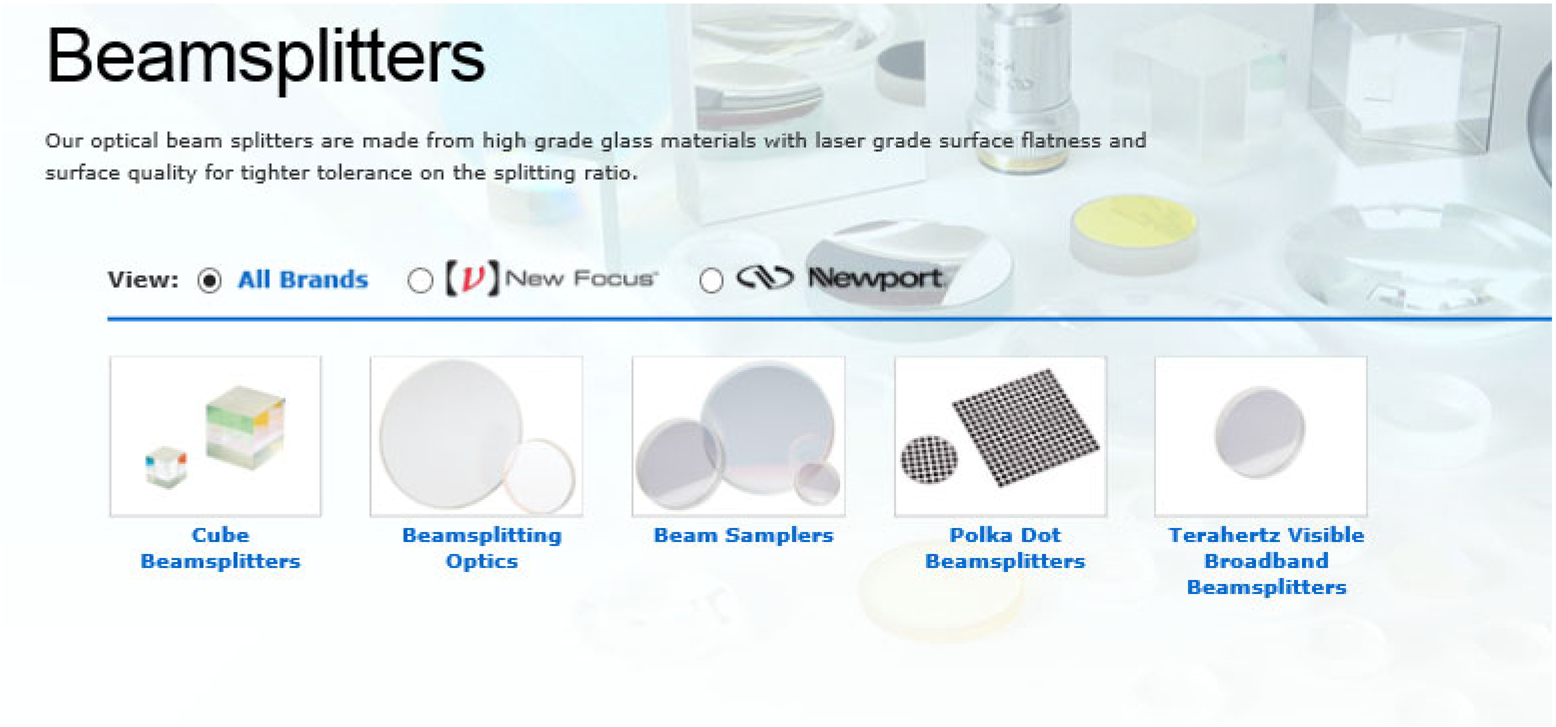
Beam splitters are also used in stereo photography to shoot stereo photos using a single shot with a non-stereo camera. The device attaches in place of the lens of the camera. Some argue that "image splitter" is a more proper name for this device. Beam splitters with single mode fiber for PON networks use the single mode behavior to split the beam. The splitter is done by physically splicing two fibers "together" as an X.

Other way to create the split or filtering?

Can it be done ?

- Use total reflection angle
- Use Brewster angle (polarization dependent)

Beam Splitters



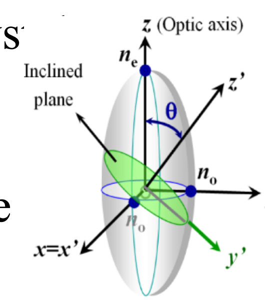
Examples of Cubic Beam Splitters

Birefringence

Birefringence is the optical property of a material having a refractive index that depends on the polarization and propagation direction of light.[1] These optically anisotropic materials are said to be birefringent (or birefractive). The birefringence is often quantified as the maximum difference between refractive indices exhibited by the material. Crystals with asymmetric crystal structures are often birefringent, as are plastics under mechanical stress (e.g. polycarbonate) or naturally created chiral nematic structure (e.g. cellophane)

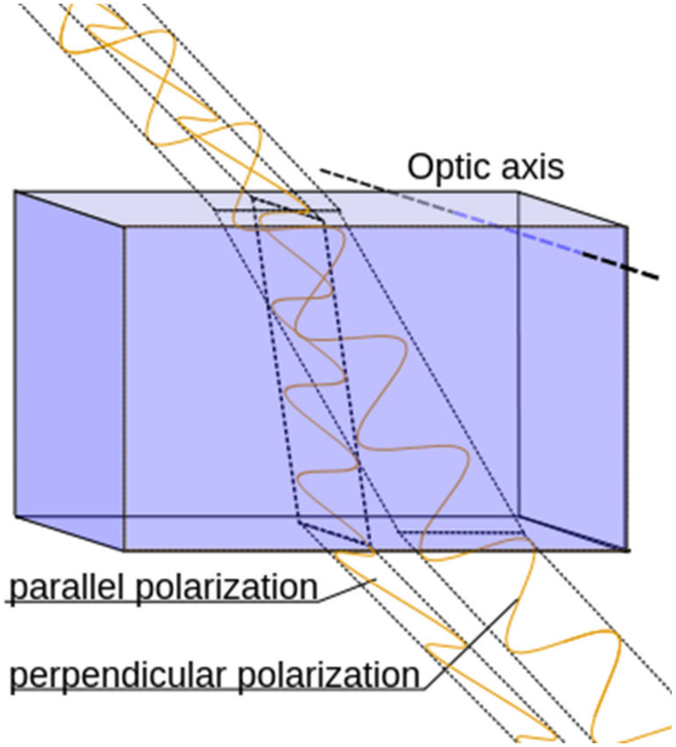
cellophane

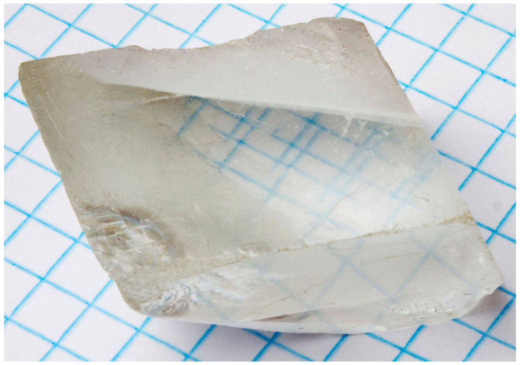
Birefringence is responsible for the phenomenon of double refraction whereby a ray of light, when incident upon a birefringent material, is split by polarization into two rays taking slightly different paths. This effect was first described by the Danish scientist Rasmus Bartholin in 1669, who observed it[2] in calcite, a crys having one of the strongest birefringences. However it was not until the 19th century that Augustin-Jean Fresnel described the phenomenon in terms of polarization, understanding light as a wave with field components in transverse polarizations (perpendicular to the direction of the wave vector).



(wiipedia)

Birefringence

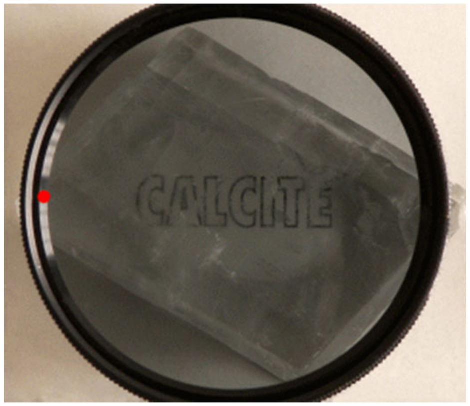




A calcite crystal laid upon a graph paper with blue lines showing the double refraction

Incoming light in the parallel (s) polarization sees a different effective index of refraction than light in the perpendicular (p) polarization, and is thus refracted at a different angle.

W. Wang

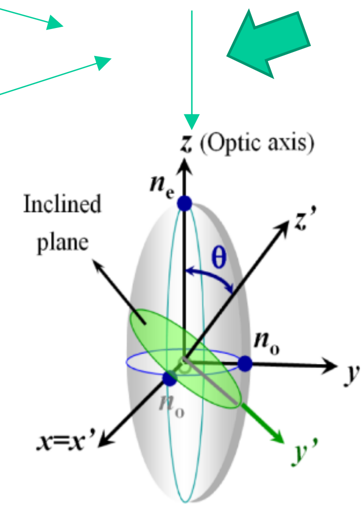


Doubly refracted image as seen through a calcite crystal, seen through a rotating polarizing filter illustrating the opposite polarization states of the two images

Beam propagation in anisotropic crystals

<u>Optic axis of a crystal is the direction in which a ray of transmitted light suffers no birefringence</u> (double refraction). Light propagates along that axis with a speed independent of its polarization

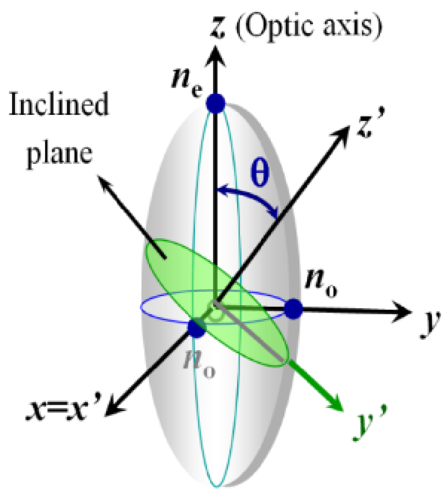
If wave coming in optical axis, then both x and y components are in ordinary direction so there is no refractive index difference.



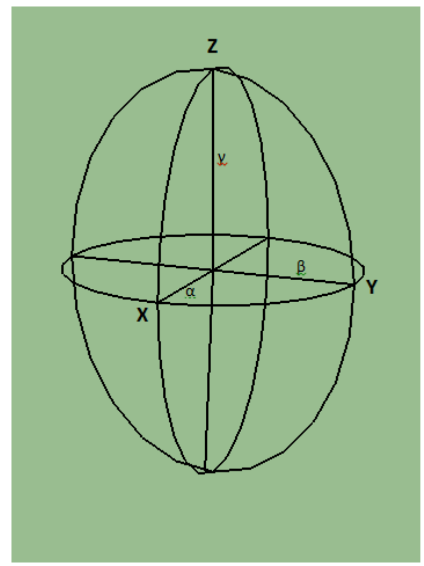
However, if the light beam is not parallel to the optical axis, then, when passing through the crystal the beam is split into two rays: the ordinary and extraordinary, to be mutually perpendicular polarized.

Uniaxial and Biaxial Crystal and

A crystal which has only **one optic axis is called uniaxial crystal**. An uniaxial crystal is isotropic within the plane orthogonal to the optical axis of the crystal. A crystal which has **only two optic axis is called biaxial crystal.**



The refractive index of the ordinary ray is constant for any direction in the crystal, and of the extraordinary ray is variable and depends on the direction. In a uniaxial crystal for the direction parallel to the optical axis the refractive indices are equal.



A biaxial crystal is an optical element which has two optic axes. However, these optical directions and refractive indices are different from the crystallographic axes on the nature of the crystal system.

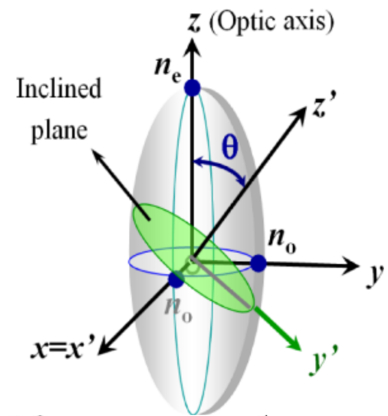
What is the Difference Between Uniaxial and Biaxial Crystals?

Uniaxial vs Biaxial Crystals	
A uniaxial crystal is an optical element that has a single optic axis.	A biaxial crystal is an optical element that has two optic axes.
Negative Form	
A negative uniaxial crystal has the refraction index of o-ray (n_o) larger than that of the e-ray (ne).	A negative biaxial crystal has its β closer to γ than to α.
Splitting the Light Beam	
When a light beam passes through a uniaxial crystal, the light beam splits into two rays named as ordinary ray (o-ray) and the extraordinary ray (E-ray).	When a light beam passes through a biaxial crystal, the light beam splits into two rays being both are extraordinary rays (e-rays).
Positive Form	
A positive uniaxial crystal has the refraction index of e-ray (n_e) is smaller than that of the e-ray (ne).	A positive biaxial crystal has its β closer to α than to γ.
Examples	
Quartz, calcite, rutile, etc.	All the monoclinic, triclinic an orthorhombic crystal systems

Uniaxial crystals

The refractive index of the ordinary ray is constant for any direction in the crystal, and of the extraordinary ray is variable and depends on the direction.

In a uniaxial crystal for the direction parallel to the optical axis the refractive indices are equal.



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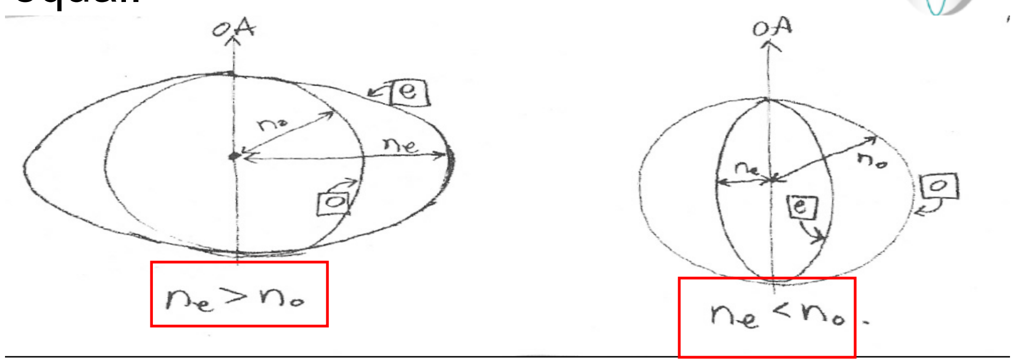


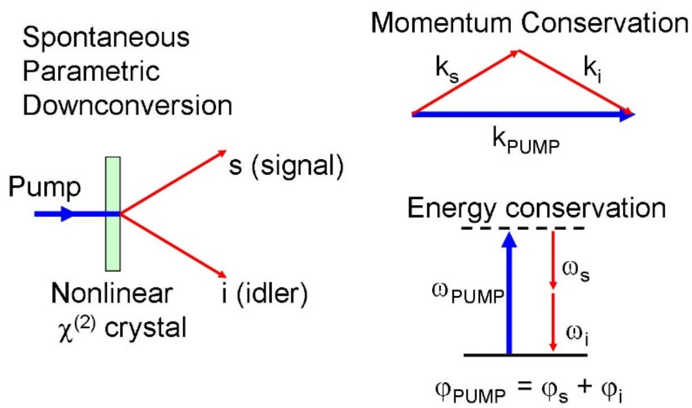
Positive birefringence

Negative birefringence

BBO is negative uniaxial crystal

Fast and Slow Axis

Direction having a low refractive index is the fast axis; at right angles to it is the slow axis, with a high index of refraction.



$$\boldsymbol{k}_p = \boldsymbol{k}_s + \boldsymbol{k}_i$$

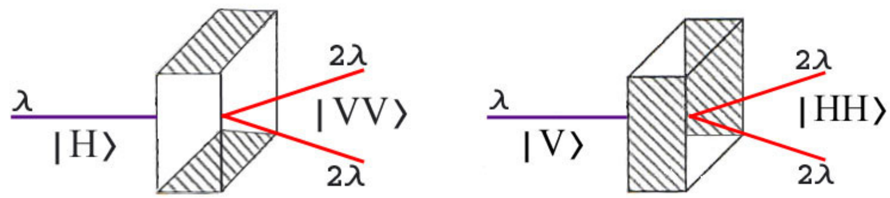
$$n_p\omega_p = 2n_s\omega_s\cos\theta_s$$

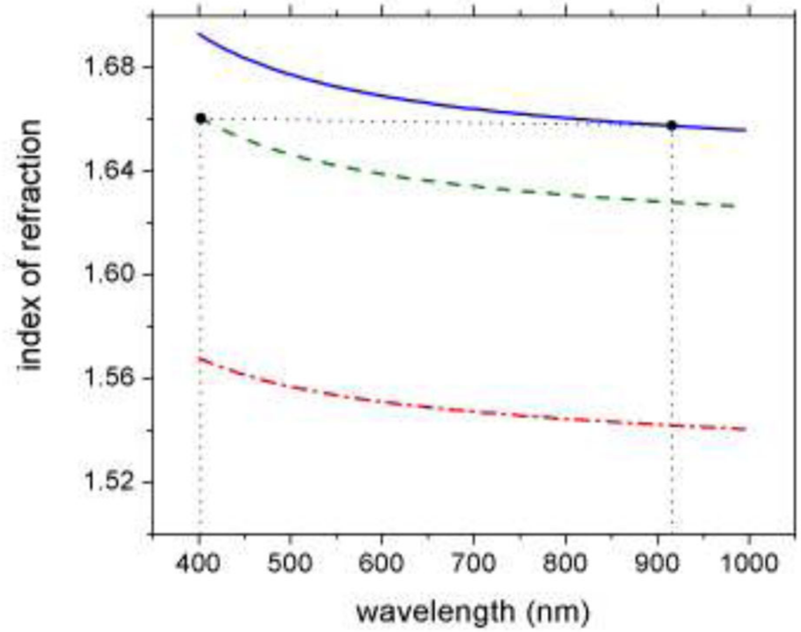
for $\omega_s = \omega_p/2$

$$n_p = n_s\cos\theta_s$$

For collinear down-conversion the index of refraction at wavelengths differing by a factor of two must be equal. For isotropic material it is impossible, but by using birefringent material we can achieve this condition via the different indices of refraction of orthogonal linear polarizations.

Type I cut

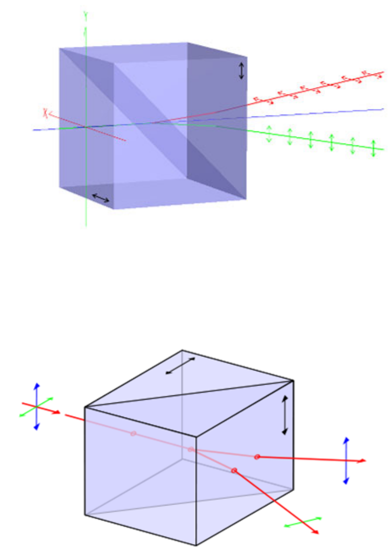




Index of refraction of BBO (negative uniaxial crystal): blue solid curve: ordinary polarization; red/dash-dot curve: full extraordinary polarization; green/dash curve: extraordinary polarization at the phase matching angle (29°)

Uniaxial crystals, at 590 nm

Material	Crystal system	n_o	n_e	Δn
barium borate BaB_2O_4	Trigonal	1.6776	1.5534	-0.1242
beryl $Be_3Al_2(SiO_3)_6$	Hexagonal	1.602	1.557	-0.045
calcite $CaCO_3$	Trigonal	1.658	1.486	-0.172
ice H_2O	Hexagonal	1.309	1.313	+0.004
lithium niobate $LiNbO_3$	Trigonal	2.272	2.187	-0.085
magnesium fluoride MgF_2	Tetragonal	1.380	1.385	+0.006
quartz SiO_2	**Trigonal**	**1.544**	**1.553**	**+0.009**
ruby Al_2O_3	Trigonal	1.770	1.762	-0.008
rutile TiO_2	**Tetragonal**	**2.616**	**2.903**	**+0.287**
sapphire Al_2O_3	Trigonal	1.768	1.760	-0.008
silicon carbide SiC	Hexagonal	2.647	2.693	+0.046
tourmaline (complex silicate)	Trigonal	1.669	1.638	-0.031
zircon, high $ZrSiO_4$	Tetragonal	1.960	2.015	+0.055
zircon, low $ZrSiO_4$	Tetragonal	1.920	1.967	+0.047



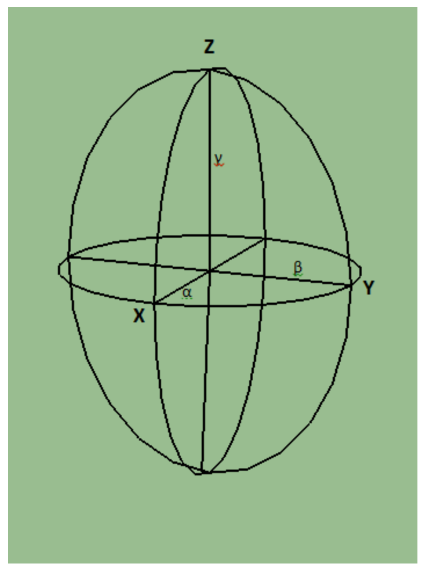
Wallaston polarizer made of an uniaxial crystal calcite

Biaxial Crystal

A biaxial crystal is an optical element which has two optic axes. **When a light beam passes through a biaxial crystal, the light beam splits into two fractions, being both fractions are extraordinary waves (two e-rays).** These waves have different directions and different speeds. The crystalline structures such as orthorhombic, monoclinic, or triclinic are biaxial crystal systems.

The Refractive Indices for a Biaxial Crystal are as follows:

1. The smallest refractive index is α (the corresponding direction is X)
2. The intermediate refractive index is β (the corresponding direction is Y)
3. The largest refractive index is γ (the corresponding direction is Z)



The Indicatrix of a Biaxial Crystal. (Indicatrix is an imaginary ellipsoidal surface whose axes represent the refractive indices of a crystal for light following different directions concerning the crystal axes)

Biaxial Crystal

However, these optical directions and refractive indices are different from the crystallographic axes on the nature of the crystal system.

- Orthorhombic crystal system –optical directions correspond to crystallographic axes. Ex: X,Y or Z directions (α, β and γ refractive indices) may parallel to any of the crystallographic axes (a, b or c).
- Monoclinic crystal system – one of the X, Y and Z directions (α, β and γ refractive indices) is parallel to the b crystallographic axis while other two direction is not parallel to any crystallographic direction.
- Triclinic crystal system –none of the optical directions coincide with crystallographic axes.

There are two types of biaxial crystals such as, negative biaxial crystal and positive biaxial crystals. Negative biaxial crystals have their β closer to γ than to α. Positive biaxial crystals have their β closer to α than to γ.

Biaxial crystals, at 590 nm

Material	Crystal system	n_α	n_β	n_γ
Borax $Na_2(B_4O_5)(OH)_4 \cdot 8(H_2O)$	Monoclinic	1.447	1.469	1.472
epsom saltMgSO$_4 \cdot 7(H_2O)$	Monoclinic	1.433	1.455	1.461
mica, biotiteK(Mg,Fe) 3AlSi 3O 10(F,OH) 2	Monoclinic	1.595	1.640	1.640
mica, muscovite $KAl_2(AlSi_3O_{10})(F,OH)_2$	Monoclinic	1.563	1.596	1.601
olivine $(Mg,Fe)_2SiO_4$	Orthorhombic	1.640	1.660	1.680
perovskite $CaTiO_3$	Orthorhombic	2.300	2.340	2.380
topaz $Al_2SiO_4(F,OH)_2$	Orthorhombic	1.618	1.620	1.627
Ulexite $NaCaB_5O_6(OH)_6 \cdot 5(H_2O)$	Triclinic	1.490	1.510	1.520

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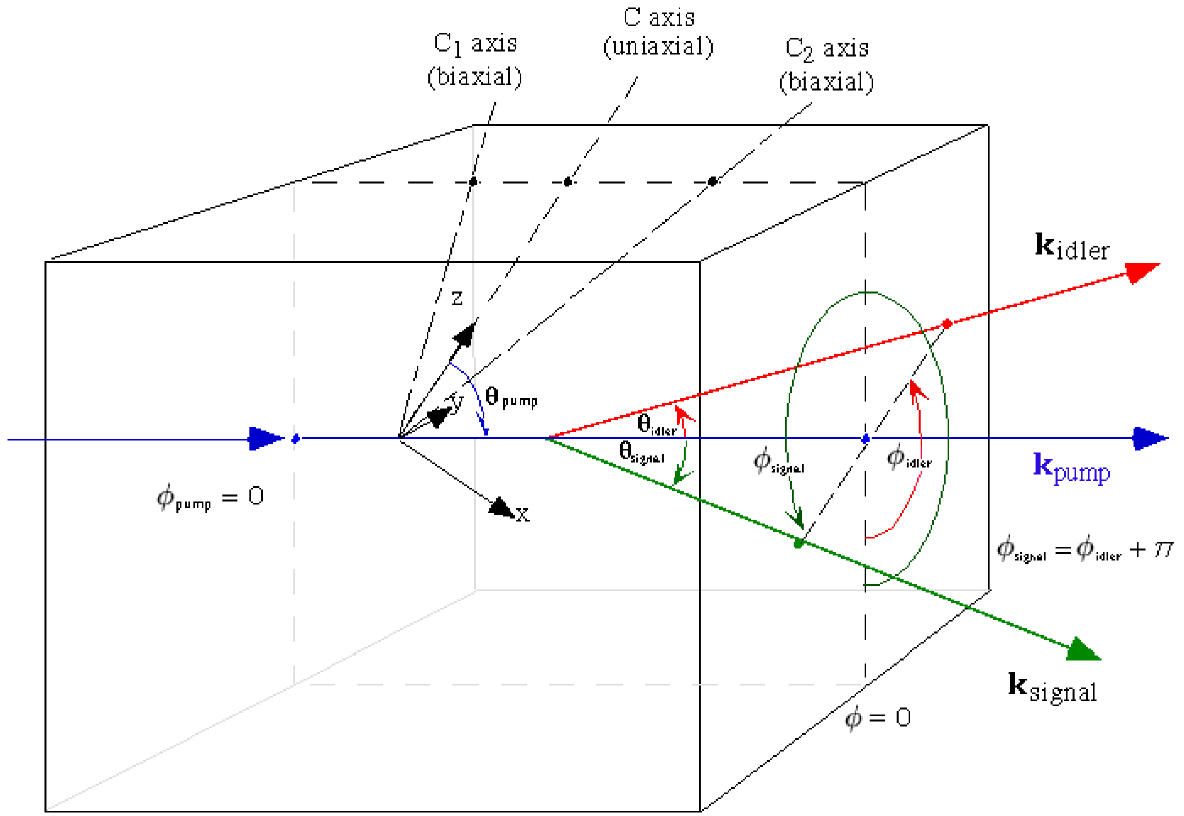
Examples	
Quartz, calcite, rutile, etc.	All the monoclinic, triclinic an orthorhombic crystal systems

Calculating Characteristics of Noncollinear Phase Matching in Uniaxial and Biaxial Crystals

N. Boeuf, D. Branning, I. Chaperot, E. Dauler, S. Guérin, G. Jaeger, A. Muller, A. Migdall

Originally published in: Opt. Eng. **39**(4), 1016-1024 (April 2000).

http://units.nist.gov/Divisions/Div844/facilities/cprad/index.html



Liquid Crystal

Liquid crystals have the **ordering properties of solids but they flow like liquids**. Liquid crystalline materials have been observed for over a century but were not recognized as such until 1880s. In 1888, **Friedrich Reinitzer** (picture) is credited for the first systematic description of the liquid crystal phase and reported his observations when he prepared cholesteryl benzoate, the first liquid crystal.

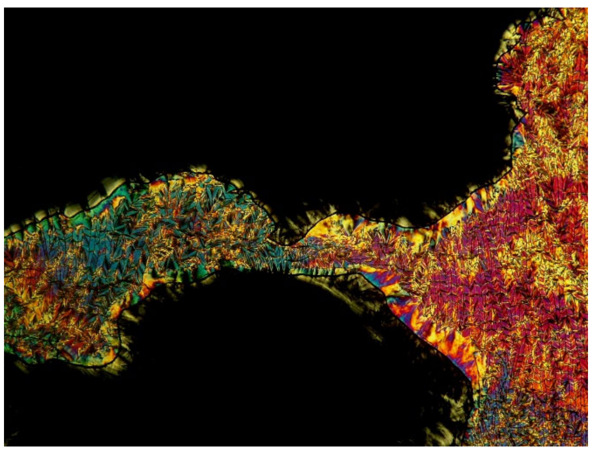
Liquid crystals are composed **of moderate size organic molecules which tend to be elongated, like a cigar.** At high temperatures, the molecules will be oriented arbitrarily, as shown in the figure below, forming an isotropic liquid. Because of their elongated shape, under appropriate conditions, the molecules exhibit orientational order such that all the axes line up and form a so-called nematic liquid crystal. The molecules are still able to move around in the fluid, but their orientation remains the same. Not only orientational order can appear, but also a positional order is possible. Liquid crystals exhibiting some positional order are called smectic liquid crystals. In smectics, the molecular centers of mass are arranged in layers and the movement is mainly limited inside the layers.

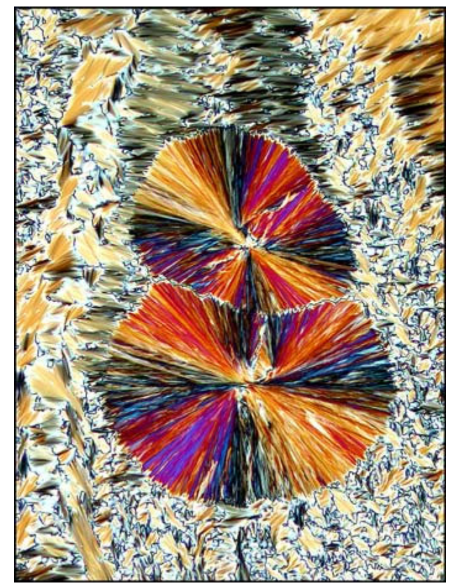
a b c

Isotropic **Smectic** **Nematic** **chiral nematic**

Liquid Crystal

Liquid crystals consist of molecules – often rod-**shaped** – that organise themselves in the same direction but are still able to move about. It turns out that **liquid-crystal** molecules respond to an electrical voltage or thermal energy, which changes their orientation and alters the optical characteristics of the bulk material.





CB-5 molecule size ~ 20nm

Microscopic images of LC

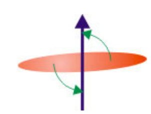
Reorientation of the molecules in electric fields

As a result of the **<u>uniaxial anisotropy</u>**, an electric field experiences a different dielectric constant when oscillating in a direction parallel or perpendicular to the director**<u>. The difference is called the dielectric anisotropy.</u>** If the dielectric constant along the director is larger than in the direction perpendicular to it, one speaks of positive anisotropy.

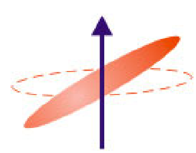
Due to the anisotropy, the dielectric displacement and the induced dipole moment are not parallel to the electric field, except when the director is parallel or perpendicular to the electric field. As a result, a torque is exerted on the director. For materials with positive anisotropy, the director prefers to align parallel to the electric field. Liquid crystals with a negative anisotropy tend to orient themselves perpendicularly to the electric field.

The effect of an electric field on a liquid crystal medium with positive anisotropy is illustrated in the pictures below. <u>Originally the orientation is almost horizontal. When an electric field with direction along the blue arrow is applied, a torque (represented in green) rising from the dielectric anisotropy, acts on the molecule</u>. The torque tends to align the molecule parallel to the field. When the field strength is increased, the molecule will reorient parallel to the field.

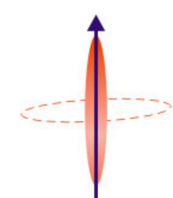
Original orientation



Situation in electric field



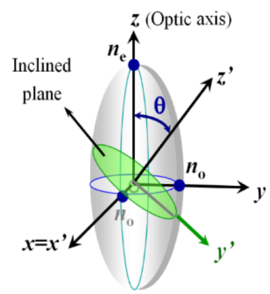
Result electric field



Result strong electric field

Optical birefringence

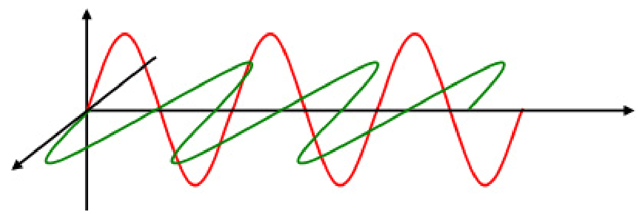
Optical waves can also reorient the liquid crystal director in an analogue manner as the electrically applied fields. In a display this can be neglected, since both the optical dielectric anisotropy and the intensity of the optical fields are typically much lower than those used in the static case. Therefore the optical transmission is mostly independent of the director calculations.



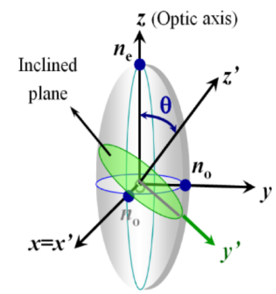
To understand the influence of birefringence on the propagation of light through a liquid crystal, the light must be represented by an electric field. This electric field is described by a wave vector in each point. At a certain time and location, the direction and the length of the vector correspond with the direction and magnitude of the electric field. For a plane wave propagating in a specific direction, the electric field vector in an isotropic medium describes an ellipse in the plane perpendicular to the propagation direction. This ellipse represents the polarization of the light. Some special cases are the linear polarization and the circular polarization where this ellipse is distorted to a straight line or a perfect circle. Generally each ellipsoidal polarization can be decomposed as a superposition of linear polarizations along two perpendicular axes. In an isotropic medium, both linear polarizations move with the same speed. The speed of the wave is determined by the refractive index of the medium.

For the uniaxial liquid crystal medium, an electric field feels a different refractive index when it oscillates in the plane perpendicular to the director or along the director. This uniaxial anisotropy of the refractive index is called birefringence. Birefringence allows to manipulate the polarization of the light propagating through the medium.

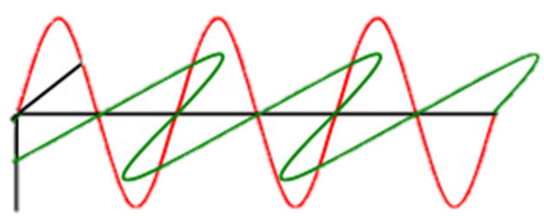
The elliptical polarization of light entering a liquid crystal medium must be decomposed into two linear polarizations called the ordinary and the extra-ordinary mode. Along these two directions, the two linearly polarized modes feel a different refractive index. Therefore, they propagate through the liquid crystal with a different speed as illustrated in the picture below.



Light propagation in a birefringent medium



In the isotropic medium, the two parts propagated with the same speed. Combining them back together will result in the same polarization ellipse as the original. In birefringent media, the different speed of the ordinary and extra-ordinary waves results in a phase difference between the two modes (= retardation). At the end of the medium this phase difference between the two oscillations will result in a different polarization ellipse.



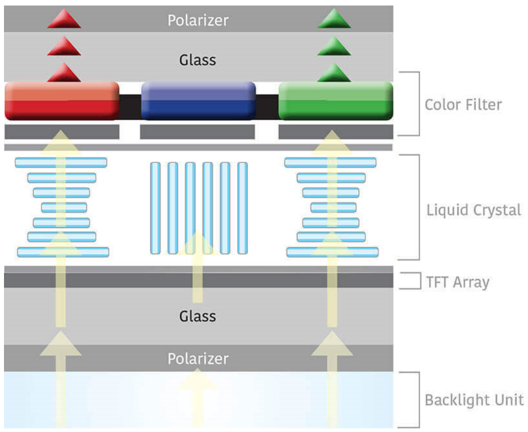
Light propagation in an isotropic medium

Application

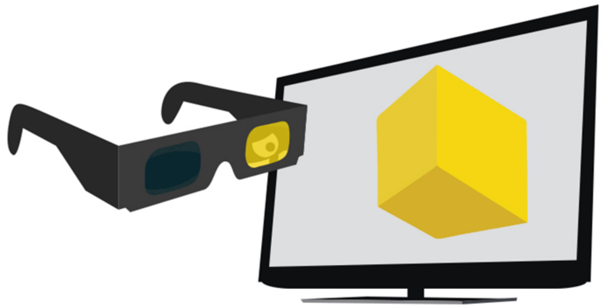
Birefringence is used in many optical devices. Liquid crystal displays, the most common sort of flat panel display, cause their pixels to become lighter or darker through rotation of the polarization (circular birefringence) of linearly polarized light as viewed through a sheet polarizer at the screen's surface. Similarly, light modulators modulate the intensity of light through electrically induced birefringence of polarized light followed by a polarizer. The Lyot filter is a specialized narrowband spectral filter employing the wavelength dependence of birefringence. Wave plates are thin birefringent sheets widely used in certain optical equipment for modifying the polarization state of light passing through it.

Birefringence also plays an important role in second harmonic generation and other nonlinear optical components, as the crystals used for this purpose are almost always birefringent. By adjusting the angle of incidence, the effective refractive index of the extraordinary ray can be tuned in order to achieve phase matching which is required for efficient operation of these devices.

Application



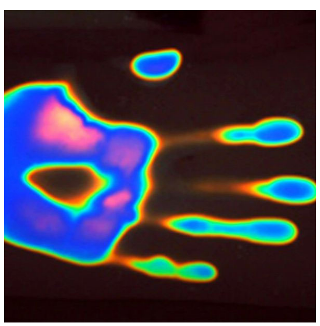
TFT Liquid Crystal Display

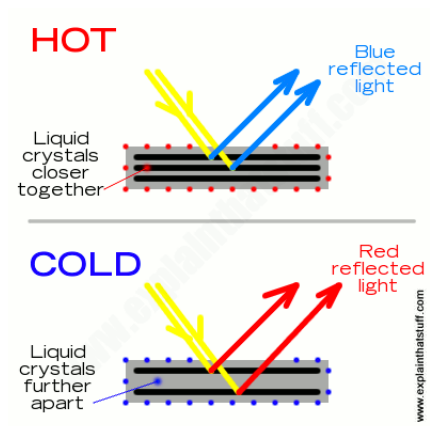


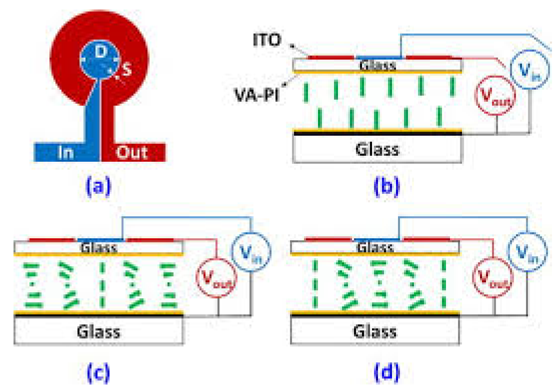
3D glasses
(active shutter 3D systems)

Leuco dyes

Use in Thermal printer papers and certain pH indicators







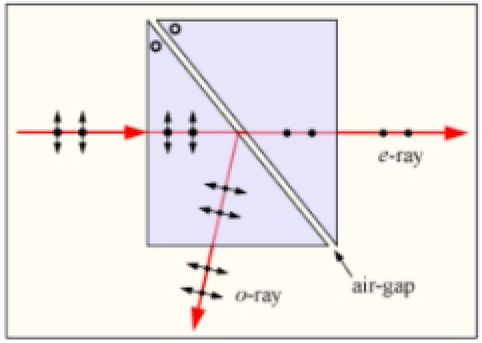
liquid crystal lens

w.wang

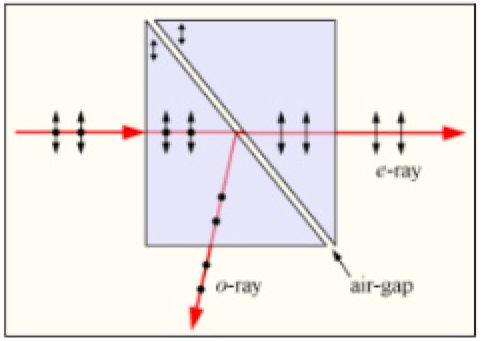
Thermochromic liquid crystals (TLC)

Polarizing Prims

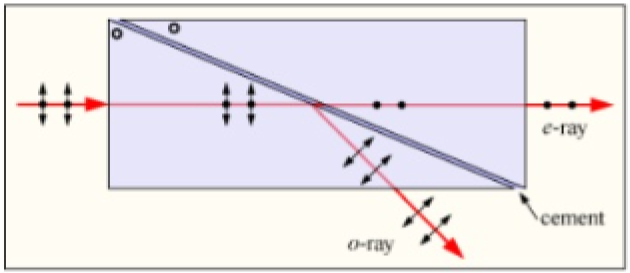
Polarizing Prisms



A Glan-Foucault prism deflects polarized light transmitting the s-polarized component. The optical axis of the prism material is perpendicular to the plane of the diagram.



A Glan-Taylor prism reflects polarized light at an internal air-gap, transmitting only the p-polarized component. The optical axes are vertical in the plane of the diagram.



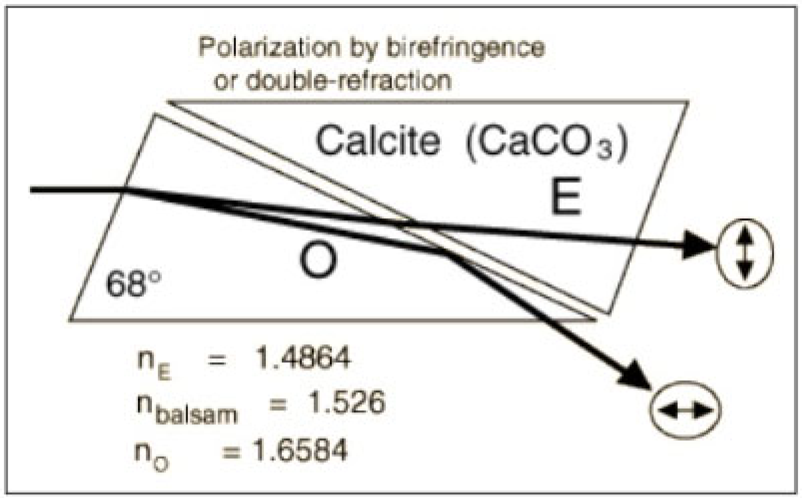
A Glan-Thompson prism deflects the p-polarized ordinary ray whilst transmitting the s-polarized extraordinary ray. The two halves of the prism are joined with Optical cement, and the crystal axis are perpendicular to the plane of the diagram.

Nicol Prisms

Polarization can be achieved with crystalline materials which have a different index of refraction in different planes. Such materials are said to be birefringent or doubly refracting.

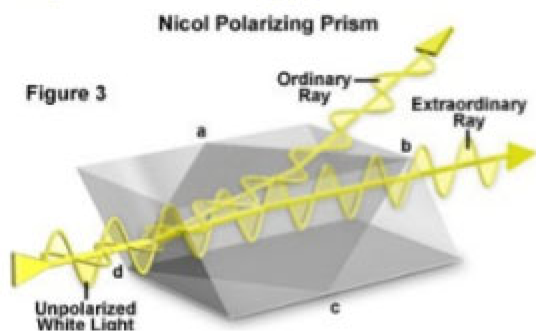
The Nicol Prism is made up from two prisms of calcite cemented with Canada balsam. The ordinary ray can be made to totally reflect off the prism boundary, leving only the extraordinary ray..

Nicol Prism



http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html

http://microscopy.fsu.edu



Nicol Prisms

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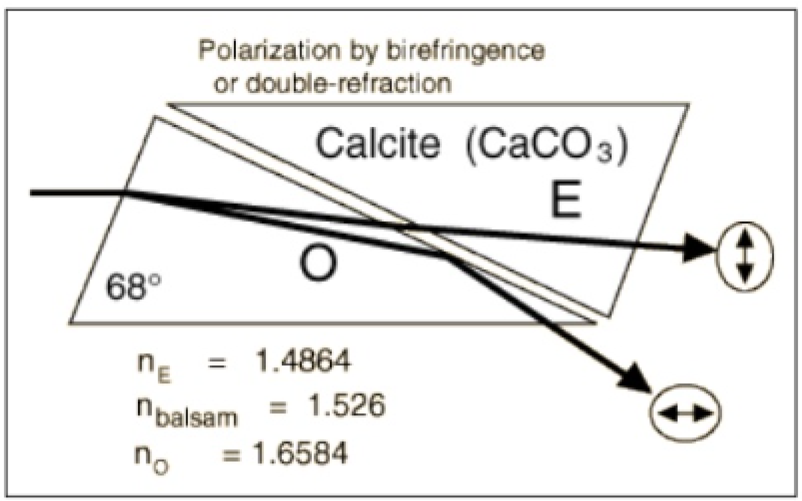
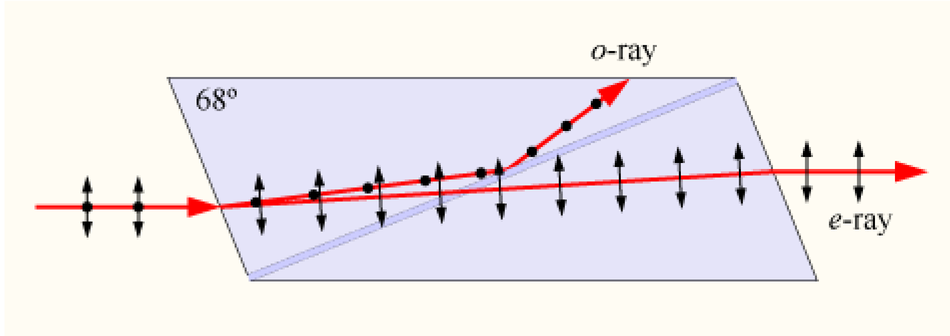
<u>Nicol Prism</u>



The Nicol Prism is made up from two prisms of calcite cemented with Canada balsam. The ordinary ray can be made to totally reflect off the prism boundary, leving only the extraordinary ray..

http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html



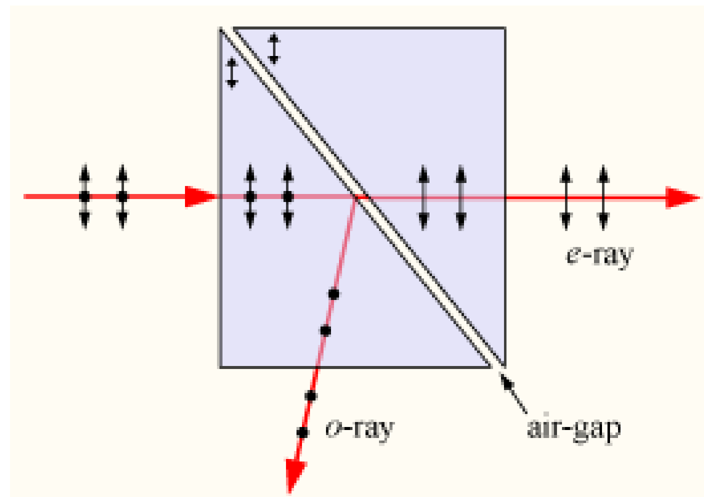
A **Nicol prism** is a type of polarizer, an optical device used to produce a polarized beam of light from an unpolarized beam. See polarized light. It was the first type of polarizing prism to be invented, in 1828 by William Nicol (1770–1851) of Edinburgh. It consists of a rhombohedral crystal of Iceland spar (a variety of calcite) that has been cut at an angle of 68° with respect to the crystal axis, cut again diagonally, and then rejoined as shown using, as a glue, a layer of transparent Canada balsam.

Unpolarized light enters through the left face of the crystal, as shown in the diagram, and is split into two orthogonally polarized, differently directed, rays by the birefringence property of the calcite. One of these rays (the *ordinary* or *o*-ray) experiences a refractive index of $n_o = 1.658$ in the calcite and it undergoes total internal reflection at the calcite-glue interface because its angle of incidence at the glue layer (refractive index $n = 1.55$) exceeds the critical angle for the interface. It passes out the top side of the upper half of the prism with some refraction as shown. The other ray (the *extraordinary* ray or *e*-ray) experiences a lower refractive index ($n_e = 1.486$) in the calcite, and is not totally reflected at the interface because it strikes the interface at a sub-critical angle. The *e*-ray merely undergoes a slight refraction, or bending, as it passes through the interface into the lower half of the prism. It finally leaves the prism as a ray of plane polarized light, undergoing another refraction as it exits the far right side of the prism. The two exiting rays have polarizations orthogonal (at right angles) to each other, but the lower, or *e*-ray, is the more commonly used for further experimentation because it is again traveling in the original horizontal direction, assuming that the calcite prism angles have been properly cut. The direction of the upper ray, or *o*-ray, is quite different from its original direction because it alone suffers total internal reflection at the glue interface as well as a final refraction on exit from the upper side of the prism.

Nicol prisms were once widely used in microscopy and polarimetry, and the term "using crossed Nicols" (abbreviated as **XN**) is still used to refer to the observing of a sample placed between orthogonally oriented polarizers. In most instruments, however, Nicol prisms have been replaced by other types of polarizers such as Polaroid sheets and Glan–Thompson prisms.

Glan–Taylor prism (Brewster angle)

A **Glan–Taylor prism** is a type of prism which is used as a polarizer or polarizing beam splitter.[1] It is one of the most common types of modern polarizing prism. It was first described by Archard and Taylor in 1948.[2]

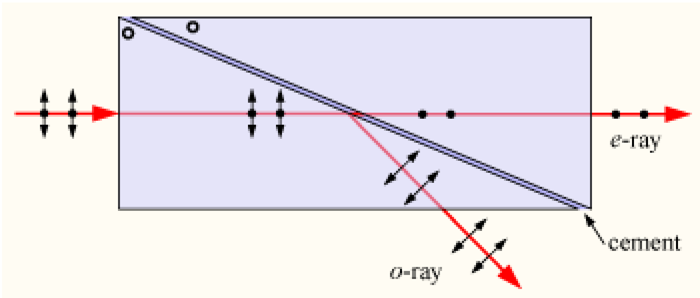


The prism is made of two right-angled prisms of calcite (or sometimes other birefringent materials) which are separated on their long faces with an air gap. The optical axes of the calcite crystals are aligned parallel to the plane of reflection. Total internal reflection of *s*-polarized light at the air-gap ensures that only *p*-polarized light is transmitted by the device**. Because the angle of incidence at the gap can be reasonably close to Brewster's angle, unwanted reflection of *p* polarized light is reduced**, giving the Glan–Taylor prism better transmission than the Glan–Foucault design.[1][3] Note that while the transmitted beam is 100% polarized, the reflected beam is not. The sides of the crystal can be polished to allow the reflected beam to exit, or can be blackened to absorb it. The latter reduces unwanted Fresnel reflection of the rejected beam.

A variant of the design exists called a **Glan–laser prism**. This is a Glan–Taylor prism with a steeper angle for the cut in the prism, which decreases reflection loss at the expense of reduced angular field of view.[1] These polarizers are also typically designed to tolerate very high beam intensities, such those produced by a laser. The differences may include using calcite which is selected for low scattering loss, improved polish quality on the faces and especially on the sides of the crystal, and better antireflection coatings. Prisms with irradiance damage thresholds greater than 1 GW/cm^2 are commercially available.

Glan–Thompson prism

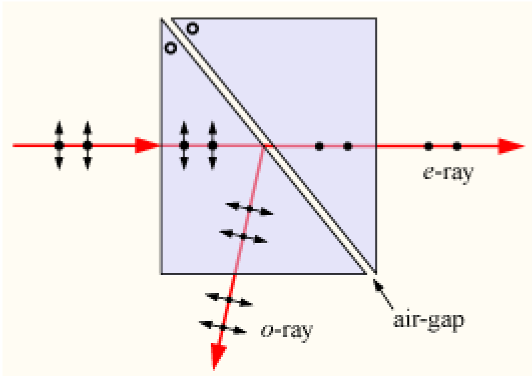
A **Glan–Thompson prism** is a type of polarizing prism similar to the Nicol and Glan–Foucault prisms. It consists of two right-angled calcite prisms that are cemented together by their long faces. The optical axes of the calcite crystals are parallel and aligned perpendicular to the plane of reflection. Birefringence splits light entering the prism into two rays, experiencing different refractive indices; the *p*-polarized ordinary ray is totally internally reflected from the calcite-cement interface, leaving the *s*-polarized extraordinary ray to be transmitted. The prism can therefore be used as a polarizing beam splitter. Traditionally Canada balsam was used as the cement in assembling these prisms, but this has largely been replaced by **synthetic polymers**.[1] Compared to the similar Glan–Foucault prism, the Glan–Thompson has **a wider acceptance angle**, but a much lower limit of maximum irradiance (due to optical damage limitations of the cement layer).



Glan–Foucault prism

A Glan–Foucault prism (also called a Glan–air prism) is a type of prism which is used **as a polarizer**. It is similar in construction to a Glan–Thompson prism, except that two right-angled calcite prisms are spaced with an air-gap instead of being cemented together.[1] Total internal reflection of p-polarized light at the air gap means that only s-polarized light is transmitted straight through the prism.

Compared to the Glan–Thompson prism, the Glan–Foucault has a narrower acceptance angle over which it will work, but because it uses an air-gap rather than cement, much higher irradiances can be used without damage. The prism can thus be used with laser beams. The prism is also shorter (for a given usable aperture) than the Glan–Thompson design, and the deflection angle of the rejected beam can be made close to 90°, which is sometimes useful. Glan–Foucault prisms are not typically used as polarizing beamsplitters because while the transmitted beam is 100% polarized, the reflected beam is not.

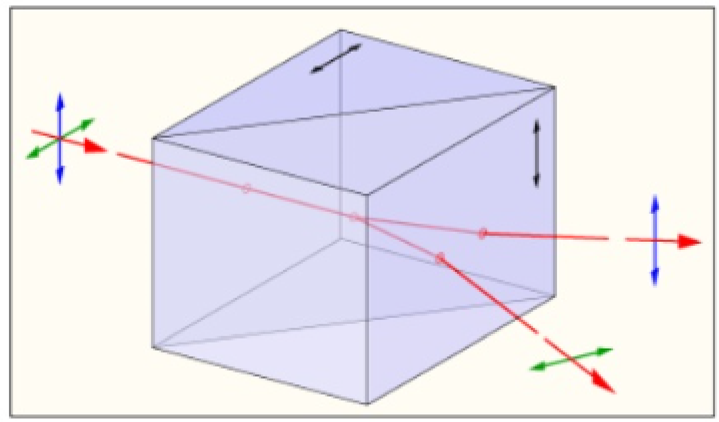


The Glan–Taylor prism is very similar, except that the crystal axes and transmitted polarization direction are orthogonal to the Glan–Foucault design. This yields higher transmission, and better polarization of the reflected light.[2] Calcite Glan–Foucault prisms are now rarely used, having been mostly replaced by Glan–Taylor polarizers and other more recent designs.

Yttrium orthovanadate (YVO4) prisms based on the Glan–Foucault design have superior polarization of the reflected beam and higher damage threshold, compared with calcite Glan–Foucault and Glan–Taylor prisms.[3] YVO4 prisms are more expensive, however, and can accept beams over a very limited range of angles of incidence.

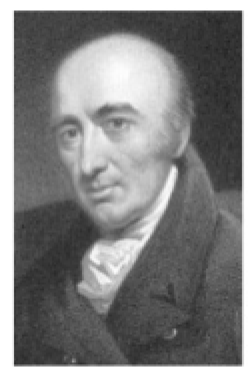
Wollaston Prisms

Polarization can be achieved with crystalline materials which have a different index of refraction in different planes. Such materials are said to be birefringent or doubly refracting.



Wollaston Prism

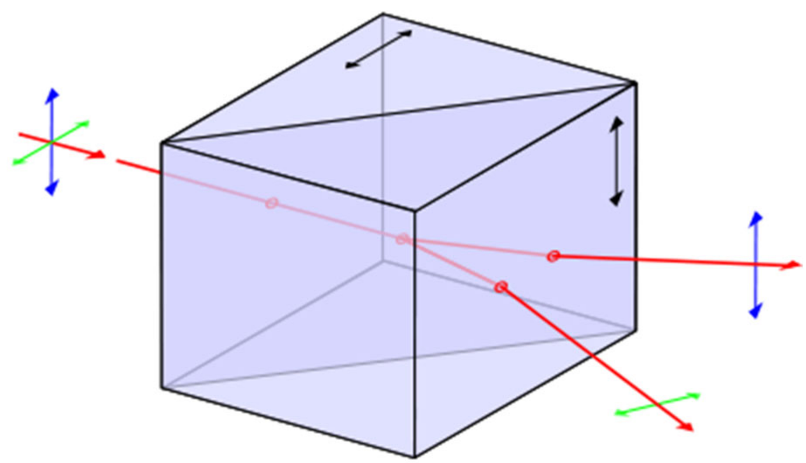
William Hyde Wollaston 1766-1828



http://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html

Wollaston prism

A Wollaston prism is an optical device, invented by William Hyde Wollaston, that manipulates polarized light. It separates randomly polarized or unpolarized light into two orthogonal linearly polarized outgoing beams.



The Wollaston prism consists of two orthogonal calcite prisms, cemented together on their base (traditionally with Canada balsam) to form two right triangle prisms with perpendicular optic axes. Outgoing light beams diverge from the prism, giving two polarized rays, with the angle of divergence determined by the prisms' wedge angle and the wavelength of the light. Commercial prisms are available with divergence angles from 15° to about 45°.

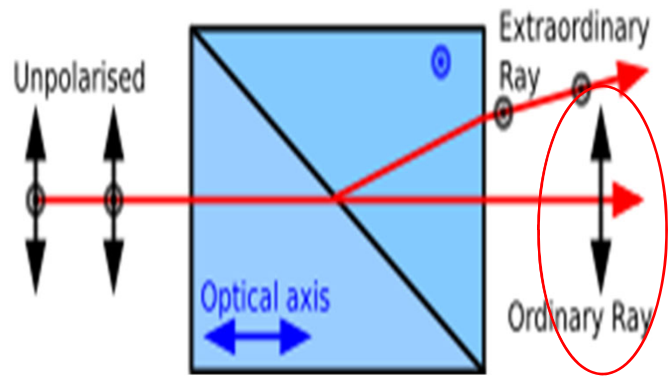
Nomarski prism

- A Nomarski prism is a modification of the Wollaston prism that is used in **differential interference contrast microscopy**. It is named after its inventor, Polish physicist Georges Nomarski. Like the Wollaston prism, the Nomarski prism consists of two birefringent crystal wedges (e.g. quartz or calcite) cemented together at the hypotenuse (e.g. with Canada balsam). One of the wedges is identical to a conventional Wollaston wedge and has the optical axis oriented parallel to the surface of the prism. The second wedge of the prism is modified by cutting the crystal in such a manner that the optical axis is oriented obliquely with respect to the flat surface of the prism. The Nomarski modification causes the light rays to come to a focal point outside the body of the prism, and allows greater flexibility so that when setting up the microscope the prism can be actively focused.

Rochon prism

A Rochon prism is a type of polariser. It is made from two prisms of a birefringent material such as calcite, which are cemented together.[1]

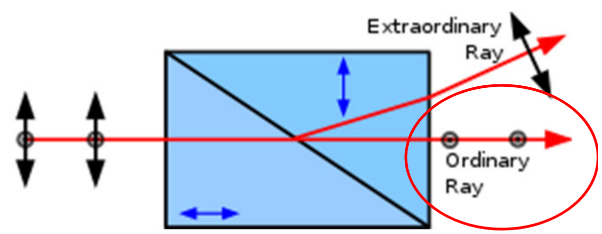
The Rochon prism was invented by and is named after Abbé Alexis Marie Rochon. It is in many ways similar to the Wollaston prism, but **one ray (the ordinary ray) passes through the prism undeviated**. **The Sénarmont prism is similar but transmits the s-polarized ray undeviated**. In both the Rochon and the Sénarmont prisms the undeviated ray is ordinary on both sides of the interface. Rochon prisms are commercially available, but for many applications other polarisers are



Sénarmont prism

The Sénarmont prism is a type of polariser. It is made from two prisms of a birefringent material such as calcite, usually cemented together.[1] The Sénarmont prism is named after Henri Hureau de Sénarmont. It is similar to the Rochon and Wollaston prisms.

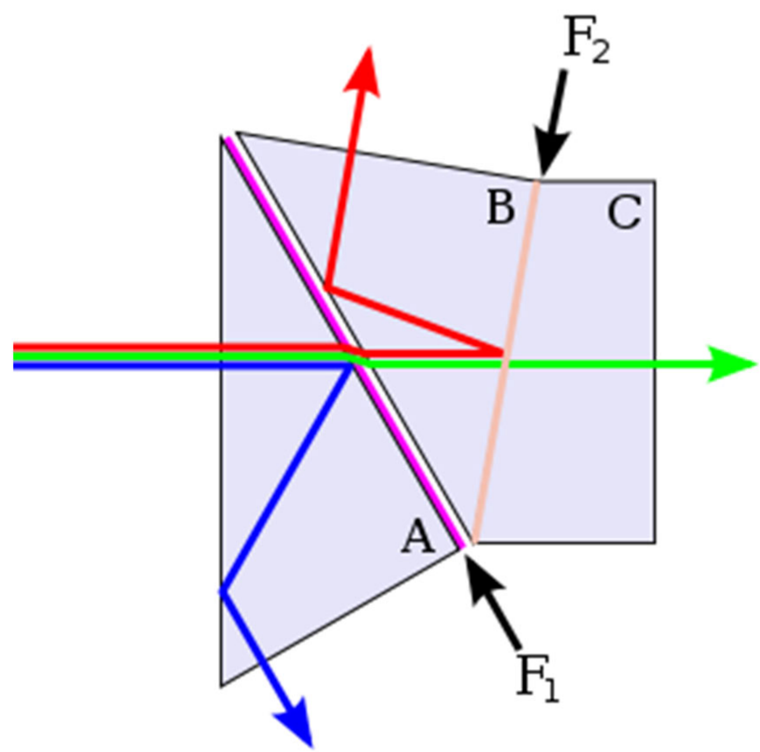
In the Sénarmont prism the s-polarized ray (i.e., the ray with polarization direction perpendicular to the plane in which all rays are contained, called the plane of incidence) passes through without being deflected, while the p-polarized ray (with polarization direction in the plane of incidence) is deflected (refracted) at the internal interface into a different direction. Both rays correspond to ordinary rays (o-rays) in the first component prism, since both polarization directions are perpendicular to the optical axis, which is the propagation direction. In the second component prism the s-polarized ray remains ordinary (o-ray, polarized perpendicular to the optical axis), while the p-polarized ray becomes extraordinary (e-ray), with a polarization component along the optical axis. As a consequence, the s-polarized ray is not deflected since the effective refractive index does not change across the interface. The p-polarized wave, on the other hand, is refracted because the effective refractive index changes upon changing from o-ray to e-ray.



The Sénarmont prism is similar in construction and action to the Rochon prism, as in both polarizers the ray that is not deflected is the o-ray after the internal interface, while the deflected ray is the e-ray. However, in the Rochon prism, it is the p-polarized ray that remains an o-ray on both sides of the interface, and is therefore not deflected, while the s-polarized ray changes from o-ray to e-ray and is therefore deflected.

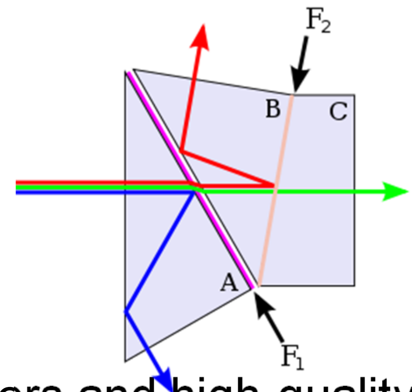
Dichroic Prisms

A dichroic prism is a prism that splits light into two beams of differing wavelength (colour). A trichroic prism assembly combines two dichroic prisms to split an image into 3 colours, typically as red, green and blue of the RGB colour model. They are usually constructed of one or more glass prisms with dichroic optical coatings that selectively reflect or transmit light depending on the light's wavelength. That is, certain surfaces within the prism act as dichroic filters. These are used as beam splitters in many optical instruments. (See: Dichroism, for the etymology of the term.) wikipedia



A trichroic prism assembly

Application



One common application of dichroic prisms is in some camcorders and high-quality digital cameras. A trichroic prism assembly is a combination of two dichroic prisms which are used to split an image into red, green, and blue components, which can be separately detected on three CCD arrays.

A possible layout for the device is shown in the diagram. A light beam enters the first prism (A), and the blue component of the beam is reflected from a low-pass filter coating (F1) that reflects blue light (high-frequency), but transmits longer wavelengths (lower frequencies). The blue beam undergoes total internal reflection from the front of prism A and exits it through a side face. The remainder of the beam enters the second prism (B) and is split by a second filter coating (F2) which reflects red light but transmits shorter wavelengths. The red beam is also totally internally reflected due to a small air-gap between prisms A and B. The remaining green component of the beam travels through prism C.

The trichroic prism assembly can be used in reverse to combine red, green and blue beams into a coloured image, and is used in this way in some projector devices. Assemblies with more than 3 beams are possible.

Thin Film Beam Splitters

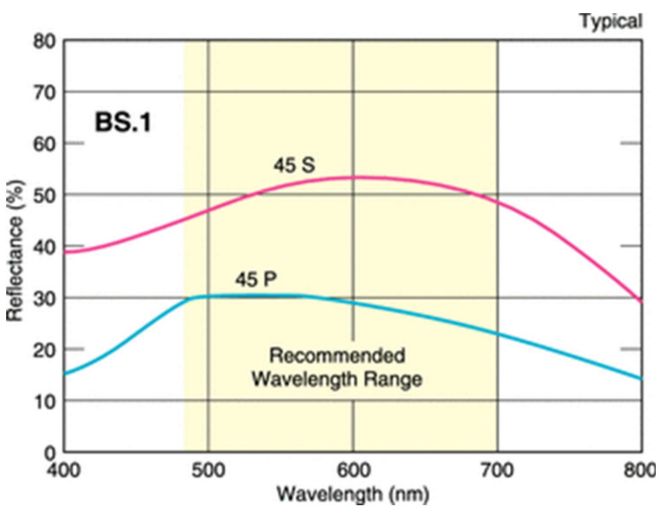
Beamsplitting Optics

Our standard beamsplitters are designed to provide general purpose laser beamsplitting and combining for visible through near infrared applications.

Example of Dielectric Thin Film Beam Splitter

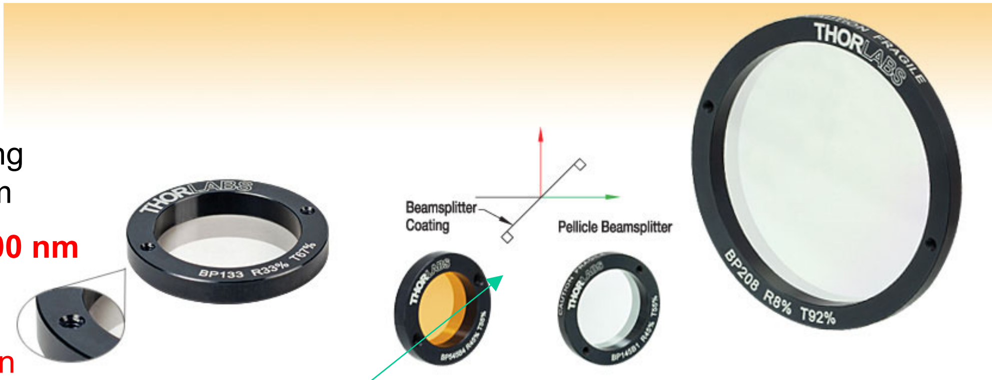
Newport N-BK7 Broadband Dielectric Beamsplitter, 50.8mm, λ/10, 480-700 nm

Model	20B20BS.1
Material	Grade A N-BK7
Diameter	2.00 in. (50.8 mm)
Antireflection Coating	480-700 nm
Angle of Incidence	45 °
Surface Quality	15-5 scratch-dig
Damage Threshold	500 W/cm^2 CW, 0.5 J/cm^2 with 10 nsec pulses, typical
Efficiency	R_{avg} < 0.75% @ 400-700 nm
Clear Aperture	> central 80% of diameter
Coating Code	BS.1
Wedge	30 ±15 arc min
Thickness	0.37 in. (9.4 mm)
Thickness Tolerance	±0.38 mm
Diameter Tolerance	+0/-0.13 mm
Chamfers	0.38-1.14 mm face width
Chamfers Angle/Tolerance	45° ±15°
Cleaning	Non-abrasive method, acetone or isopropyl alcohol on lens tissue recommended see Care and Cleaning of Optics



Thorlab uncoated Thin film Pellicle Beam Splitters

- **Eliminates Ghosting**
- No Chromatic Aberration with Uncollimated Beams
- Uncoated for Beam Sampling or Coated for Beamsplitting
- Versions Available for Wavelengths from 300 nm to 5 μm



8:92 (R:T) Pellicle Beamsplitters, Uncoated: 400 - 2400 nm

Specification

Membrane Material Nitrocellulose

Membrane Thickness 2 μm, 5 μm for 300-400 nm Version

Index of Refraction (nd) 1.5 (@ 550 nm)

Surface Quality 40-20 Scratch-Dig

Transmitted Wavefront Errora λ/2 (Typical)

Reflected Wavefront Errora <λ (Typical)

Frame Thickness 3/16" (4.8 mm)

Inner Diameter, I.D. Ø1/2" Size: Ø1/2" (12.7 mm)

Ø1" Size: Ø1" (25.4 mm)

Ø2" Size: Ø2" (50.8 mm)

Outer Diameter, O.D. Ø1/2" Size: Ø0.75" (19.1 mm)

Ø1" Size: Ø1.38" (34.9 mm)

Ø2" Size: Ø2.38" (60.3 mm)

Mounting Hole Spacingb Ø1/2" Size: 0.63" (15.9 mm)

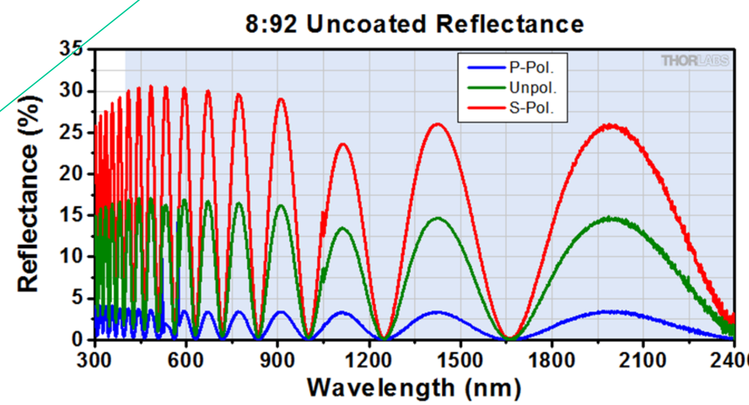
Ø1" Size: 1.19" (30.2 mm)

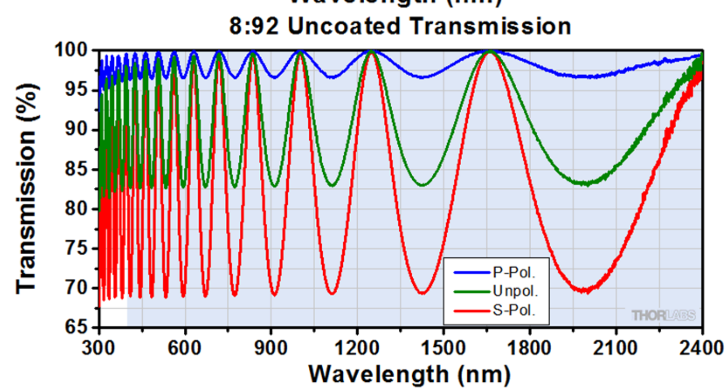
Ø2" Size: 2.19" (55.6 mm)

Temperature Range -40 to 70 °C

a.Angle of Incidence = 45o

b.Denoted by "A" in the figure to the right





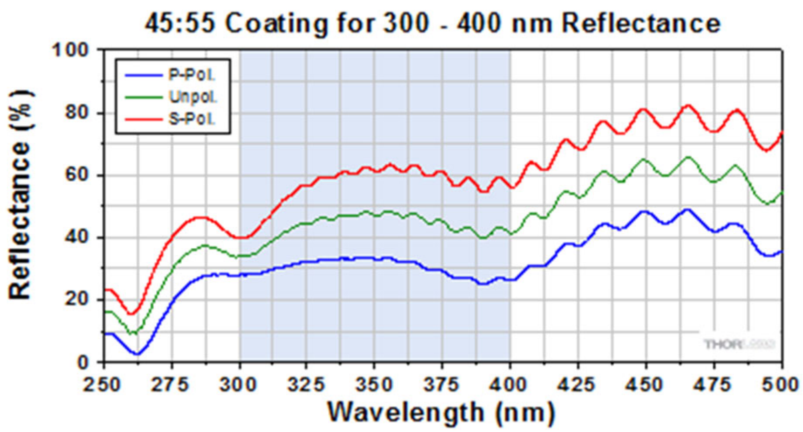
Pellicle beamsplitters display sinusoidal oscillations when plotting the splitting ratio as a function of wavelength. This is caused by thin film interference effects. In the plot above, the shaded region represents the specified operating range.

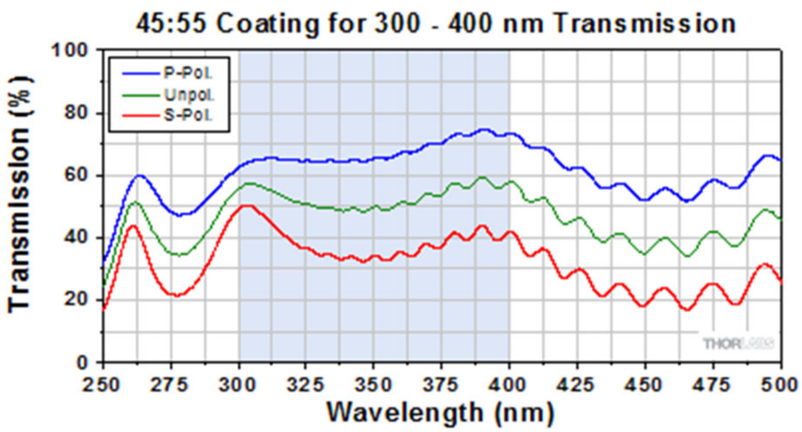
W.Wang

Thorlab coated Thin film Pellicle Beam Splitters

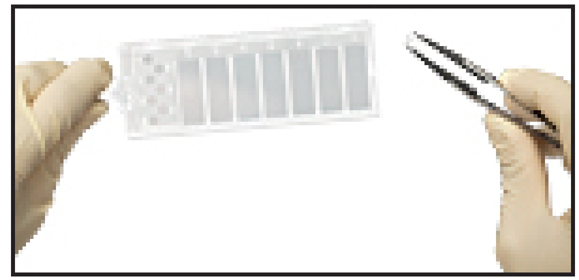
45:55 (R:T) Pellicle Beamsplitters, Coating: 300 - 400 nm

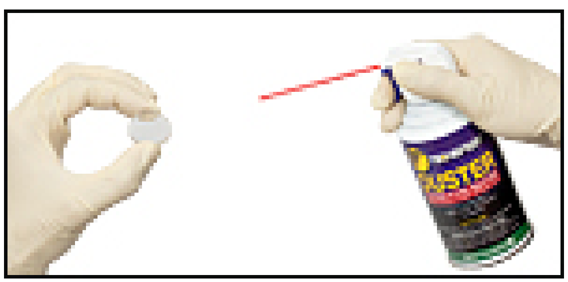
- Eliminates Ghosting
- No Chromatic Aberration with Focused Beams
- Minimal Change in Optical Path Length
- Ø1/2", Ø1", and Ø2" Versions Available
- Surface Quality: 40-20 (Scratch-Dig)



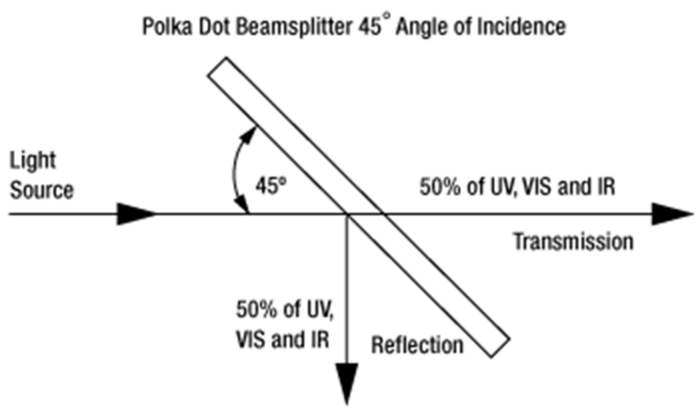


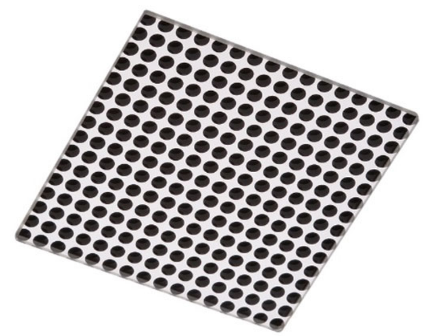
Cleaning Procedure





Polka Dots Beam Splitter





The metal coating is applied in a regularly repeating array, which lends the beamsplitter its "polka dot" appearance, as shown to the right. Light is reflected by the metal-coated portion of the beamsplitter and transmitted through the uncoated portion of the beamsplitter. To maximize the reflected intensity, light should be incident on the coated side of the beamsplitter. The square dots have 0.0040" (100 µm) [UVFS, B270, and CaF_2] or 0.0042" (107 µm) [ZnSe] sides. The spacing between the dots is 0.0022" (56 µm) [UVFS, B270, and CaF_2] or 0.0018" (46 µm) [ZnSe] in all directions.

Polka dot beamsplitters are typically used at a 45° angle relative to the incident beam as shown in the diagram above. Our polka dot beamsplitters transmit 50% ± 5% (±10% for ZnSe) when a beam is larger than 2 mm in diameter.

Pellicle mirror

A very thin half-silvered mirror used in photography is often called a pellicle mirror. To reduce loss of light due to absorption by the reflective coating, so-called "**swiss cheese**" beam splitter mirrors have been used. Originally, these were sheets of highly polished metal perforated with holes to obtain the desired ratio of reflection to transmission. Later, metal was sputtered onto glass so as to form a discontinuous coating, or small areas of a continuous coating were removed by chemical or mechanical action to produce a very literally "half-silvered" surface.

Light transmission for different substrate

•50:50 Beamsplitting Over Broad Transmission Range
◦UVFS: 250 nm to 2.0 µm
◦B270: 350 nm to 2.0 µm
◦CaF_2: 180 nm to 8.0 µm
◦ZnSe: 2.0 to 11.0 µm

•Four Substrate Options: UV Fused Silica, B270 Glass, Calcium Fluoride (CaF_2), or Zinc Selenide (ZnSe)

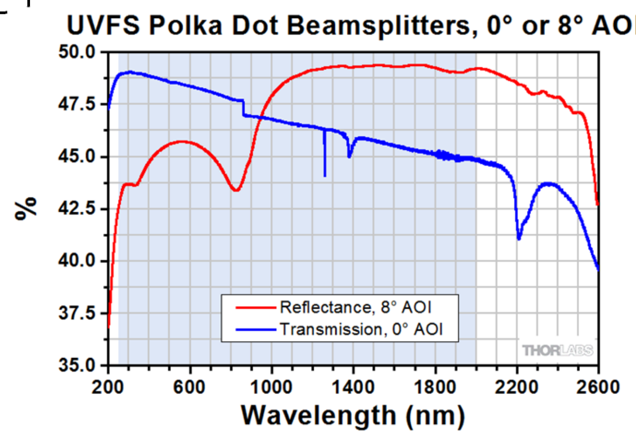
W.Wang

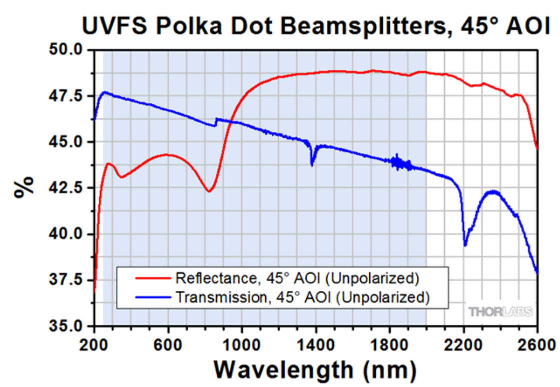
Example of Polka Dots Beam Splitter

UV Fused Silica Polka Dot Beamsplitters: 250 nm - 2.0 µm

Specifications

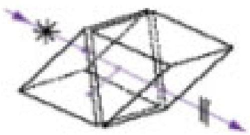
Available Sizes	Ø1", Ø2", or 1" Square
Beamsplitting Ratio	50% ± 5%
Minimum Beam Diameter for 50/50 Split	2 mm
Material	UV Fused Silica
Wavelength Range	250 nm - 2.0 µm
Coating Pattern	Square-Coated Apertures 0.0040" (100 µm) Sides, 0.0022" (56 µm) Spacing
Clear Aperture	>90% Diameter (Round Optics) >90% Length and Height (Square Optics)
Thickness	1.5 mm (Nominal)
Dimensional Tolerance	+0.0 / -0.5 mm
Angle of Incidence	0 to 45°





Glan Thompson Polarizers

- Wide acceptance angles
- Low power applications



Glan Thompson Polarizers can be used over wide acceptance angles. They are available with field angles of either 15° or 26°. This makes them ideal for use with diverging or converging beams. They can be used with lower power lasers or broadband light sources.

The two calcite prisms are cemented together with index matching cement. The side faces are unpolished, and covered with a black paint to absorb the reflected component.

Specifications

Material: Optical grade calcite
Wavelength Range: 350-2500 nm
Peak Transmission: 90%
Extinction Ratio: 10^5
Beam Deviation: <3 arcmin
Flatness: λ/8 at 589 nm
Surface Quality: 20-10

Catalog Number	Diameter (mm)	Length (mm)
43-6581	25.4	27.0
43-6599	25.4	32.0
43-6923	31.8	40.0
43-6931	31.8	52.0
43-6949	38.1	52.0

Dimension Tolerance: ±0.1mm
Laser Damage Threshold: 1 W/cm²

Glan Thompson Polarizers

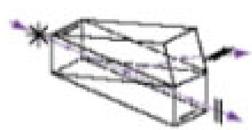
Catalog Number	Clear Aperture (mm)	Field Angle (deg.)	Price US
43-6581	10.0	15	$535.00
43-6599	10.0	26	$635.00
43-6923	15.0	15	$1,200.00
43-6931	15.0	26	$1,595.00
43-6949	20.0	15	$2,321.00

Glan Thompson Beamsplitting Prisms

Glan Thompson Beamsplitting Prisms are ideal in applications where it is essential that the transmitted p-polarized light is undeviated in its path. Unlike standard Glan Thompson Polarizers where the s-polarized ordinary ray is reflected and absorbed, these Beamsplitting Prisms have an additional escape window to allow transmission of the ordinary ray. The escape window is designed such that the beam emerges normal to it ensuring that there is no chromatic dispersion. The p-polarized extraordinary ray is transmitted undeviated from its original path. Ealing Glan Thompson Beamsplitting Prisms have an angular deviation between the two beams of 44° which is not wavelength dependent.

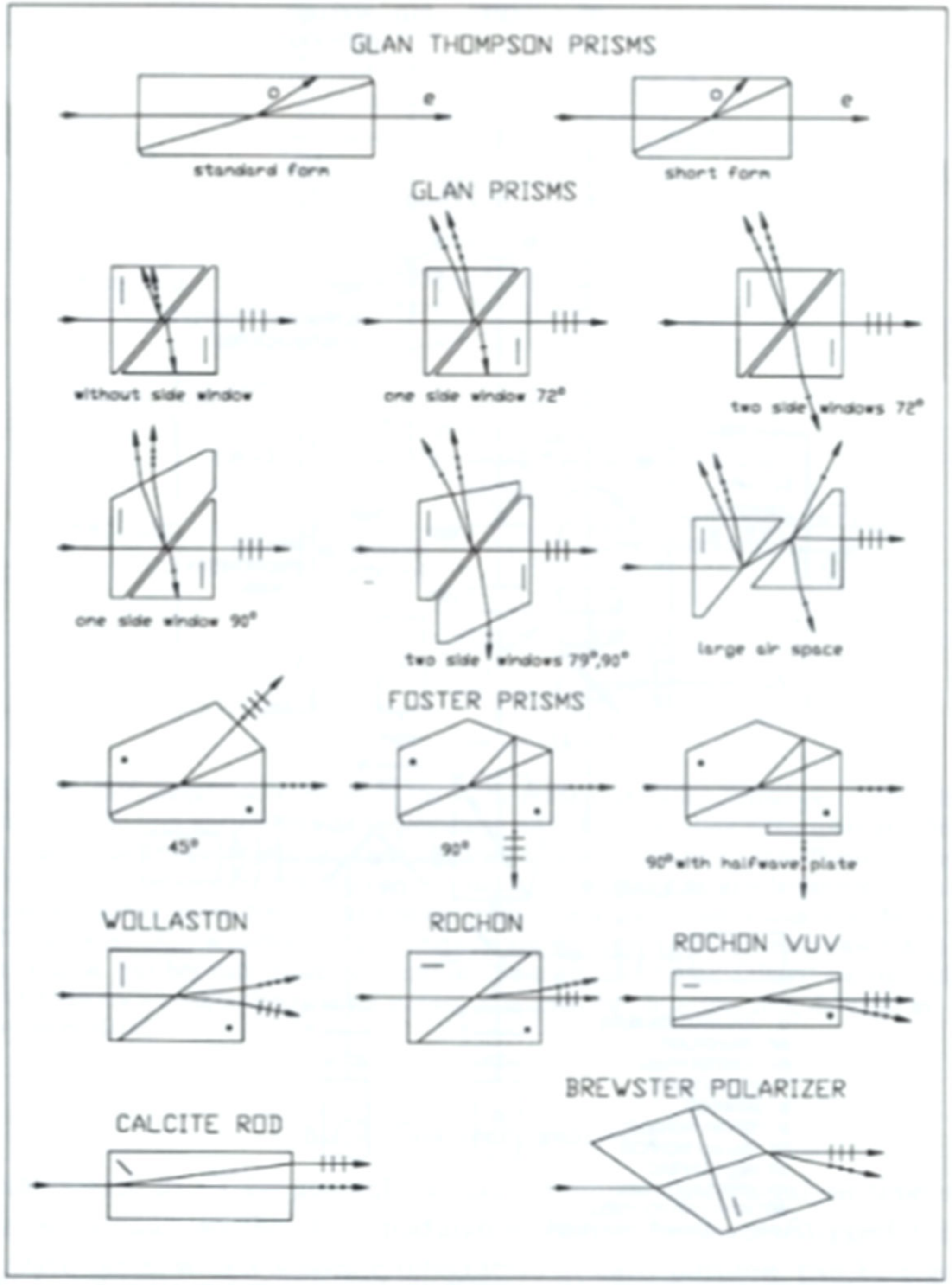
Specifications

Material: Optical calcite
Wavelength Range: 350-2500 nm
Peak Transmission: 90%
Extinction Ratio: 10^5
Surface Quality: 20-10
Beam Deviation: <3 mins
Dimensions: ±0.1mm
Laser Damage Threshold: 1 W/cm²

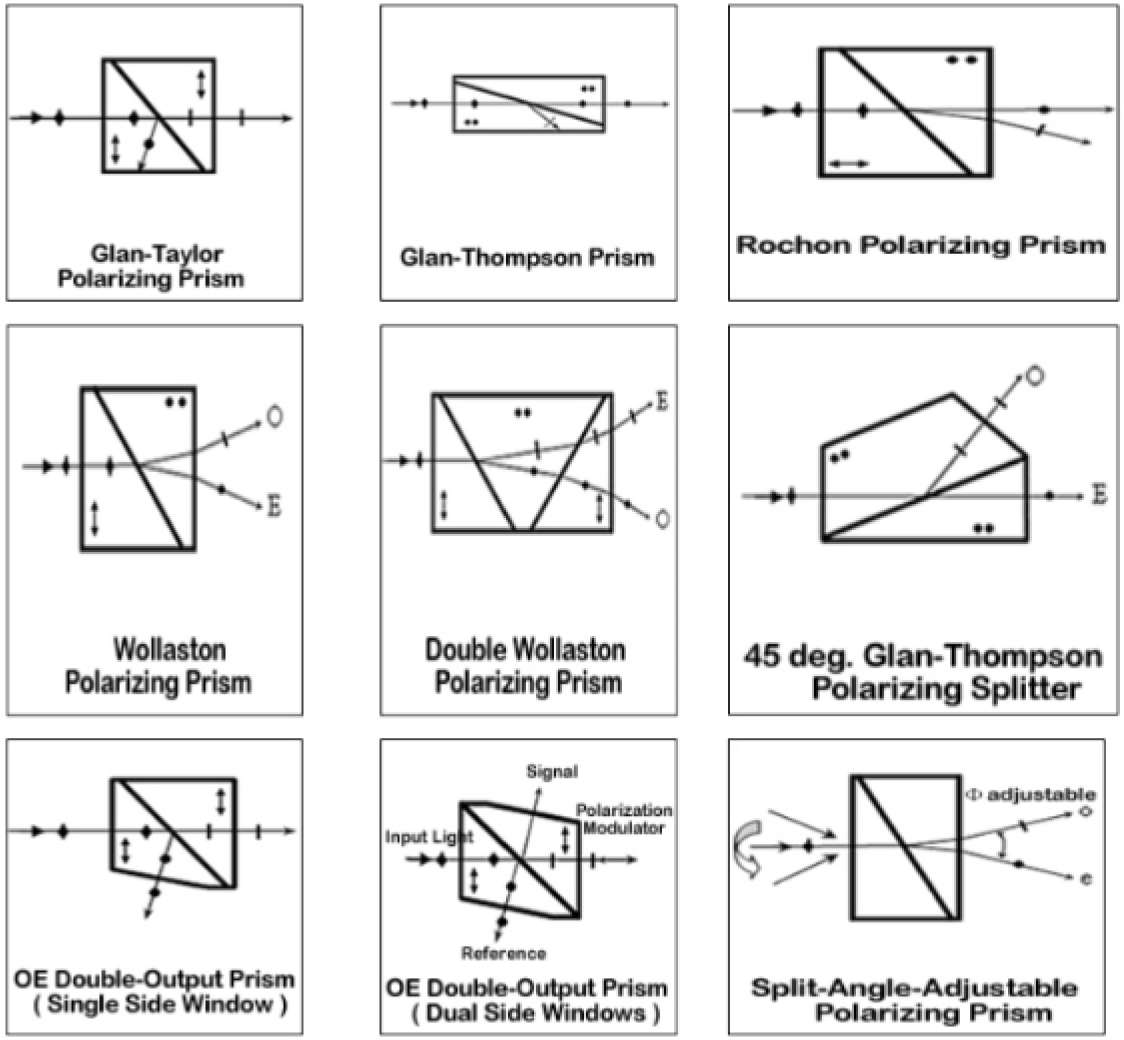


Glan Thompson Beamsplitting Prisms

Catalog Number	Aperture (mm)	Diameter (mm)	Length (mm)	Price US
43-6523	10.0	31.8	48	$855.00
43-6531	12.0	38.1	49	$1,070.00

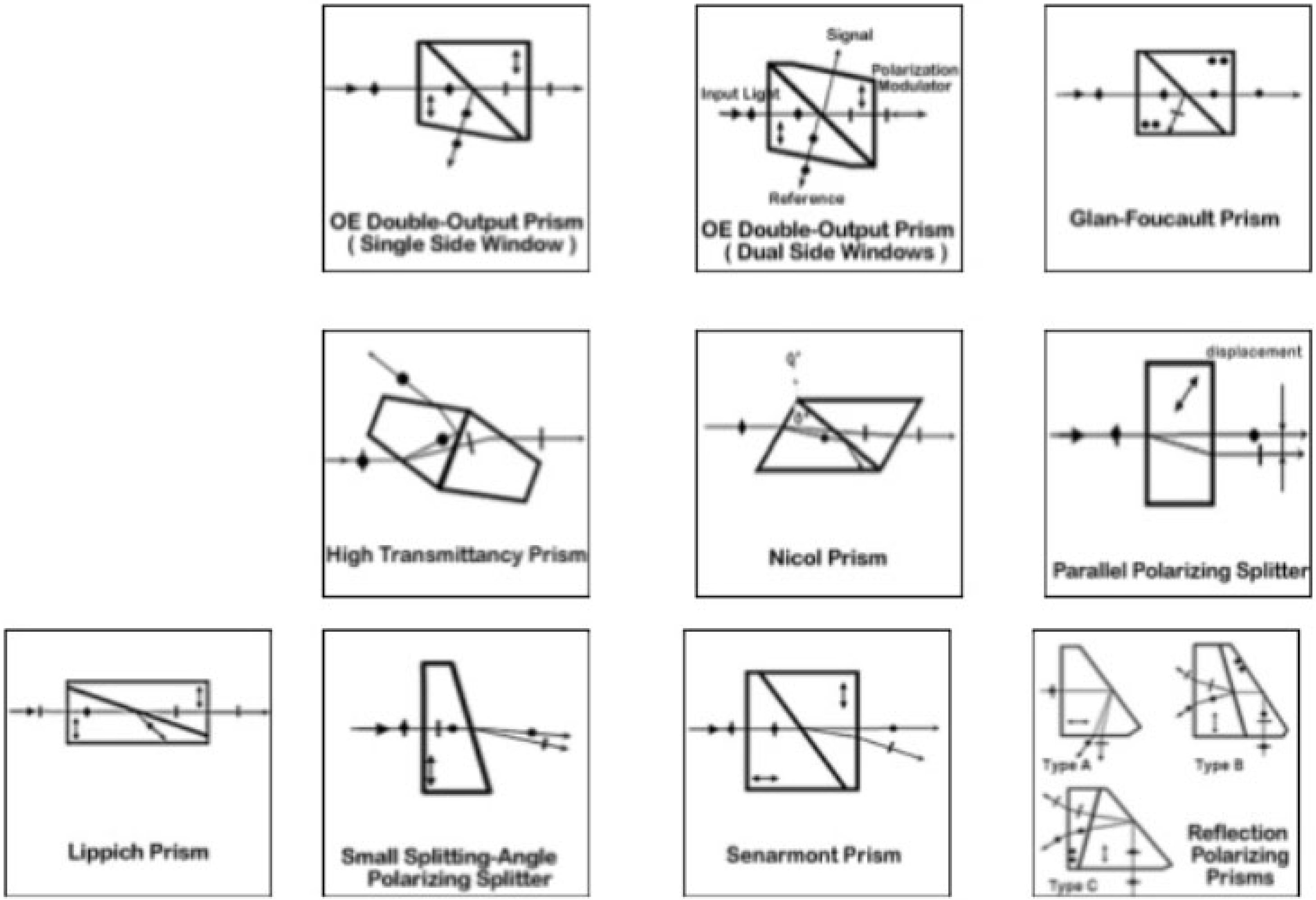

GLAN THOMPSON PRISMS
o
e
standard form
short form
GLAN PRISMS
without side window
one side window 72°
two side windows 72°
one side window 90°
two side windows 79°,90°
large air space
FOSTER PRISMS
45°
90°
90° with halfwave plate
WOLLASTON
ROCHON
ROCHON VUV
CALCITE ROD
BREWSTER POLARIZER

Polarizations Prisms Overview



http://www.unitedcrystals.com/POverview.html

Polarizations Prisms Overview



http://www.unitedcrystals.com/POverview.html

Design Project 2

- Using combination of prisms for optical beam manipulation:
- Spectrometer (dispersing light, birefringent effect)
- Stereoscope (3D imaging)
- Image coupling or anything you can come up with
- Show all calculation and design.
- Please follow the Memo requirement instruction for the Memo preparation:
  http://courses.washington.edu/me557/optics/memo.doc

Design Project 2

- This time do some literature search for ideas
- I like to see more sophisticated design and put more effort into your presentation and memo write-up

Design Project 2 (Alternative)

- Utilizing any of the wave theory concepts we have shown so far, such as polarization, complex refractive index, epsilon, interference, diffraction, phase and group velocity, dispersion, etc., design an optical device or demonstrate a concept in this project. This will be given a higher grade than if you just use the prism design.

Birefringent FTS

- Using birefringent effect to create interferometer
- Interferometer is used to lower the frequency so we can observed these high frequency signal in lower frequency in time or space domain.

Polarizer

Polarization states are linear, circular and elliptical according to
The path traced by electric field vectors in a propagating wave train.

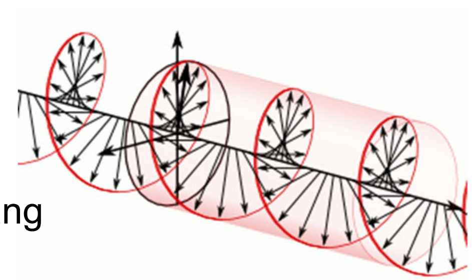
- Quartz (e.g. calcite),
- Polarized by reflection (Brewster angle from glass)
- metal film
- Dichroic (sheet-type polarizers are manufactured from an organic materials embedded into a plastic. When stretched, aligning molecules and causing them to be birefringent, and then dyed. Dye molecules selectively attach to aligned polymer molecules, so absorption is high in one and weak in other. (Polaroid)

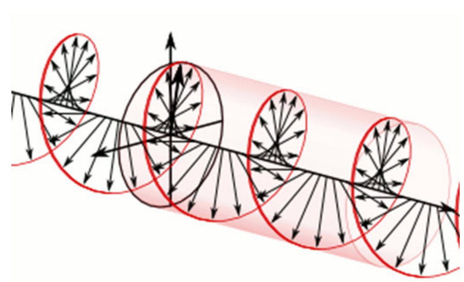
Polarizer

The second meaning of dichroic refers to a material in which light in different polarization states traveling through it experiences a different absorption coefficient. This is also known as diattenuation. When the polarization states in question are right and left-handed circular polarization, it is then known as circular dichroism. Since the left- and right-handed circular polarizations represent two spin angular momentum (SAM) states, in this case for a photon, this dichroism can also be thought of as Spin Angular Momentum Dichroism.

In some crystals, the strength of the dichroic effect varies strongly with the wavelength of the light, making them appear to have different colours when viewed with light having differing polarizations.[dubious – discuss] This is more generally referred to as pleochroism,[2] and the technique can be used in mineralogy to identify minerals. In some materials, such as herapathite (iodoquinine sulfate) or Polaroid sheets, the effect is not strongly dependent on wavelength.

W. Wang





Retarders

A retarder, or waveplate, is an optical device that resolves a light wave into two orthogonal linear polarization components and produces a phase shift between them. The resulting light wave generally is of a different polarization form.

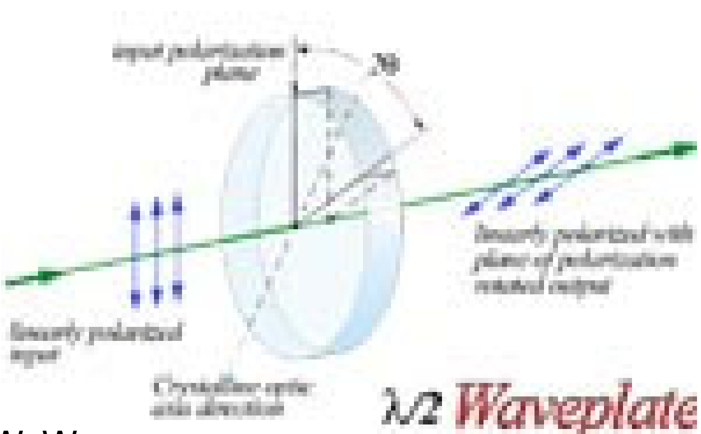
Ideally, retarders do not polarize, nor do they induce an intensity change in the light beam. They simply change its polarization form. Retarders are used in applications where control or analysis of polarization states is required.

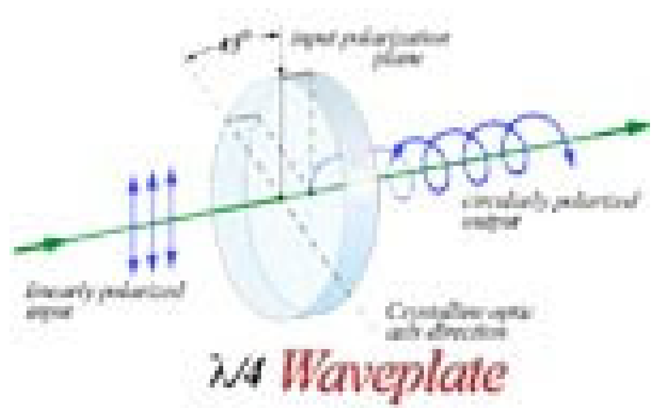
$$\phi = 2\pi n d/\lambda$$

Retardation plates

Retardation plates or phase shifters, including ¼ or ½ wave plates,
Are usually used primarily for synthesis and analysis of light
In various polarization states.

When combine with polarizer, it either rotates the polarization or
change linear polarized light into circularly polarized light.





Let's assume the real time-space *E* vector has x and y components:

$$E(z,t) = a\cos(\omega t - kz + \phi_a)\hat{x} + b\cos(\omega t - kz + \phi_b)\hat{y}$$

$E_y/E_x = Ae^{j\phi}$

linearly polarized: $\phi_b - \phi_a = 0..or\,\pi$ $\quad E_y = \pm(\frac{b}{a})E_x$

circularly polarized: $\phi_b - \phi_a = \pm\frac{\pi}{2}$ $\quad \frac{E_y}{E_x} = \frac{b}{a} = 1$

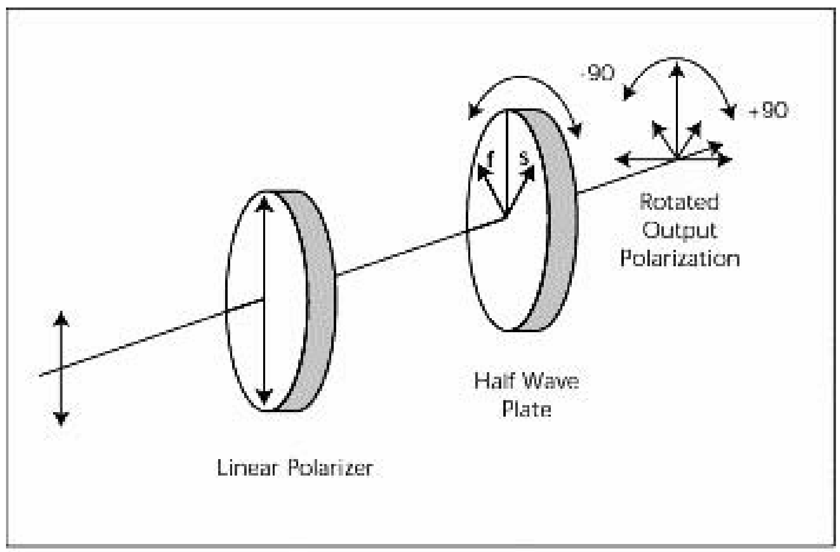
Elliptically polarized: $\phi_b - \phi_a = anything..except..0, \pi, \pm\frac{\pi}{2}$

$$\frac{E_y}{E_x} = \frac{b}{a} = anything$$

Half wave plate

A retarder that produces a λ/2 phase shift is known as a half wave retarder. Half wave retarders can rotate the polarization of linearly polarized light to <u>twice the angle between the retarder fast axis</u> and the plane of polarization.

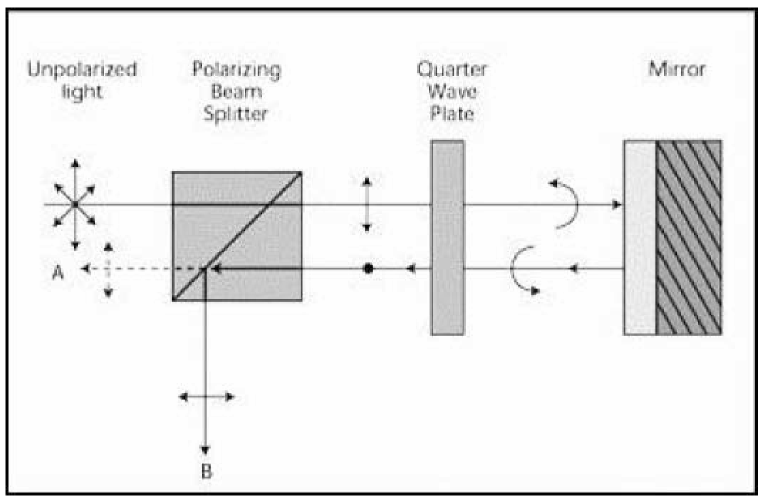
Placing the fast axis of a half wave retarder at 45° to the polarization plane results in a polarization rotation of 90°. Passing circularly polarized light through a half wave plate changes the "handedness" of the polarization.



A. Polarization rotation with a half wave plate

Quarter waveplate

If the orthogonal electric field components are equivalent, a phase shift λ/4 in one component will result in circularly polarized light. Retarders that cause this shift are known as quarter wave retarders. They have the unique property of turning elliptically polarized light into linearly polarized light or of transforming linearly polarized light into circularly polarized light <u>when the fast axis of the quarter wave plate at 45° to the incoming polarization plane.</u> (Light polarized along the direction with the smaller index travels faster and thus this axis is termed the fast axis. The other axis is the slow axis).



W.Wang

Jones Vectors

A monochromatic plane wave of frequency of w traveling in z direction is characterized by

$$E_x = a_x\, e^{j\phi x}$$
$$E_y =_{ay}\, e^{j\phi y} \qquad (10)$$

of x and y component of the electric fields. It is convenient to write these complex quantities in the form of a column matrix

$$J = \begin{bmatrix} E_x \\ E_y \end{bmatrix} \qquad (11)$$

Total intensity is $I = (|E_x|^2 + |E_y|^2)/2\eta$ Use the ratio a_y / a_x and phase difference $\phi_y - \phi_x$ to determines the orientation and shape of the polarization ellipse

Jones matrix

Consider the transmission of a plane wave of arbitrary polarization through an optical system that maintains the plane-wave nature of the wave, bu alters its polarization, the complex envelopes of the two electric-field components of the input E_{1x}, E_{1y} and those output waves, E_{2x}, E_{2y} can be express by weighted superposition,

$$\begin{bmatrix} E_{2x} \\ E_{2y} \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} E_{1x} \\ E_{1y} \end{bmatrix} \Rightarrow \quad J_2 = TJ_1 \qquad (12)$$

Linearly polarizer

$$T = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

Linearly polarized along x axis

Polarization rotator

$$T = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

Rotate linearly polarized light By an angle of θ

Wave retarder

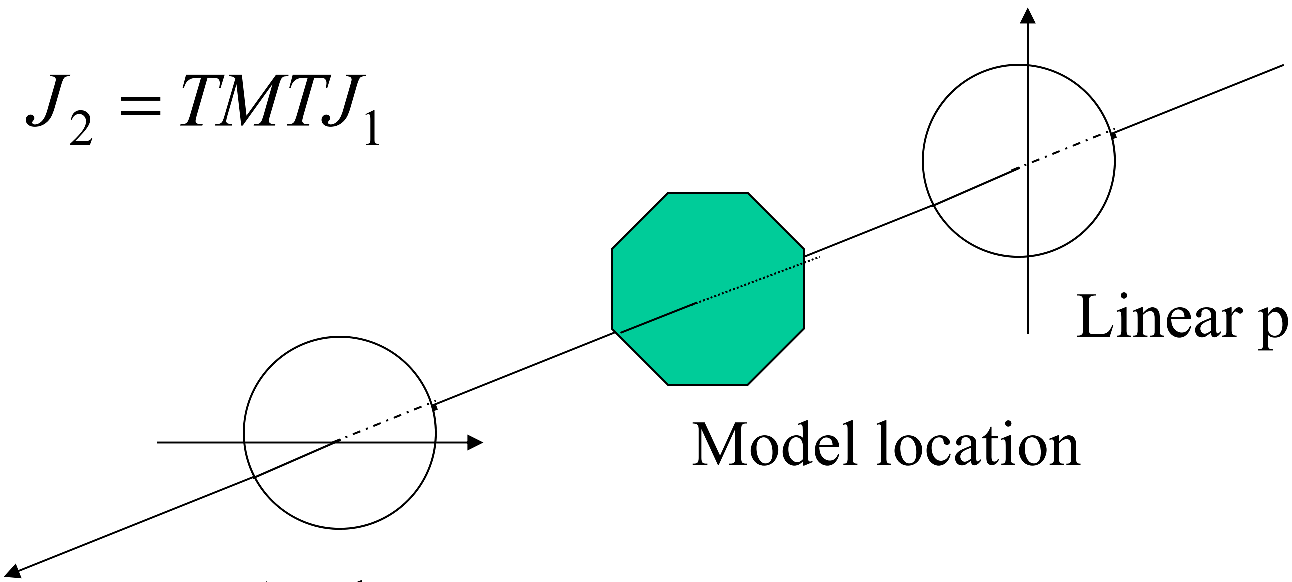
$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{-j\Gamma} \end{bmatrix}$$

Fast axis Along x axis

$\Gamma = \pi/2$ quarter wave retarder
$\Gamma = \pi$ half wave retarder
$\Gamma = 2n\pi d/\lambda$ where d is thickness of birefringent material

Plane Polariscope

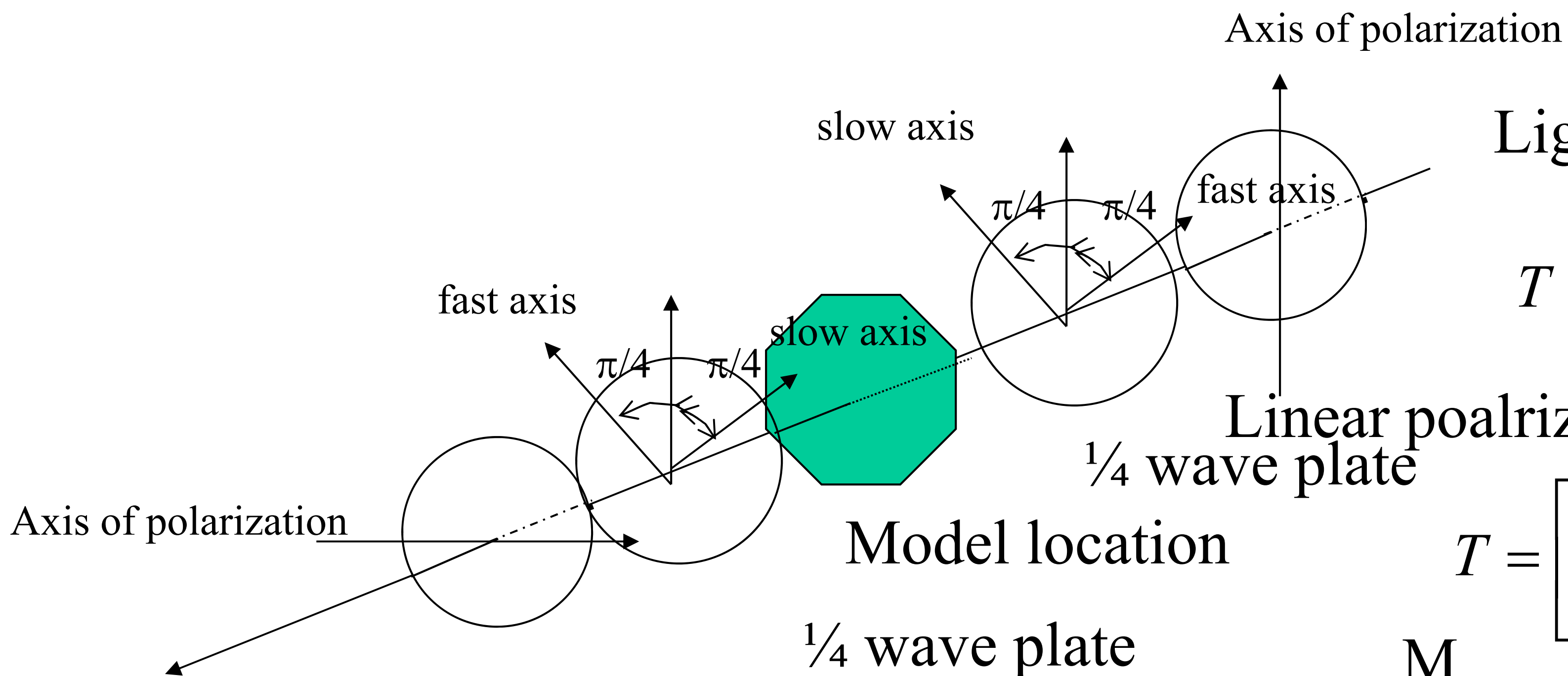
$$J_2 = TMTJ_1$$



$$T = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

Circular Polariscope



Light source

$$T = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Linear poalrizer

¼ wave plate

$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{-j\Gamma} \end{bmatrix}$$

Model location

M

¼ wave plate

$$T = \begin{bmatrix} 1 & 0 \\ 0 & e^{-j\Gamma} \end{bmatrix}$$

Analyzer
Linear polarizer

$$T = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

$$J_2 = TTMTTJ_1$$

Filters

Dichroism

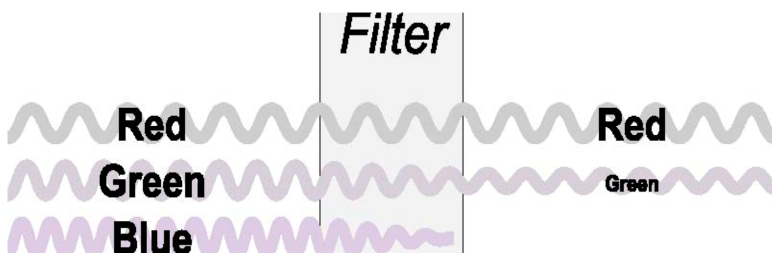
In optics, a dichroic material is either one which causes visible light to be split up into distinct beams of different wavelengths (colors) (not to be confused with dispersion), or one in which light rays having different polarizations are absorbed by different amounts. wikipedia

Filter

Neutral density- decrease intensity (broadband, metallic)

Interference filter – utilizing etalon created by thin layer(s) of dielectric or metallic coating to selectively filter certain Wavelength (narrow band)

Gel film– usually braodband, either passband or lowpass or highpass



Transmission: Beer-Lambert or Bouger's Law

Absorption by a filter glass varies with wavelength and filter thickness. Bouger's law states the logarithmic relationship between internal transmission at a given wavelength and thickness.

$$\log_{10}(\tau_1) / d_1 = \log_{10}(\tau_2) / d_2$$

Internal transmittance, τ_i, is defined as the transmission through a filter glass after the initial reflection losses are accounted for by dividing external transmission, T, by the reflection factor P_d.

$$\tau_i = T / P_d$$

The law that the change in intensity of light transmitted through an absorbing substance is related exponentially to the thickness of the absorbing medium and a constant which depends on the sample and the wavelength of the light. Also known as Lambert's law.

Example

The external transmittance for a nominal 1.0 mm thick filter glass is given as $T_{1.0}$ = 59.8 % at 330 nm. The reflection factor is given as P_d = 0.911. Find the external transmittance $T_{2.2}$ for a filter that is 2.2 mm thick.

Solution:

$\tau_{1.0} = T_{1.0} / P_d = 0.598 / 0.911 = 0.656$

$\tau_{2.2} = [\tau_{1.0}]^{2.2/1.0} = [0.656]^{2.2} = 0.396$

$T_{2.2} = \tau_{2.2} * P_d = (0.396)(0.911) = 0.361$

So, for a 2.2 mm thick filter, the external transmittance at 330 nm would be 36.1%

Beer–Lambert law

The absorbance of a beam of collimated monochromatic radiation in a homogeneous isotropic medium is proportional to the absorption path length, *l*, and to the concentration, *c*, or — in the gas phase — to the pressure of the absorbing species. The law can be expressed as:

$$A = \log_{10}\left(\frac{P_\lambda^0}{P_\lambda}\right) = \varepsilon\, c\, l$$

or

$$P_\lambda = P_\lambda^0\, 10^{-\varepsilon c l}$$

where the proportionality constant, ε, is called the molar (decadic) absorption coefficient. For *l* in cm and *c* in mol dm^{-3} or M, ε will result in dm^{-3} mol^{-1} cm^{-1} or M cm^{-1}, which is a commonly used unit. The SI unit of ε is m^2 mol^{-1}. Note that spectral radiant power must be used because the Beer–Lambert law holds only if the spectral bandwidth of the light is narrow compared to spectral linewidths in the spectrum.
See: absorbance, extinction coefficient, Lambert law

Transmittance and Absorbance

A spectrophotometer is an apparatus that measures the intensity, energy carried by the radiation per unit area per unit time, of the light entering a sample solution and the light going out of a sample solution. The two intensities can be expressed as transmittance: the ratio of the intensity of the exiting light to the entering light or percent transmittance (%T). Different substances absorb different wavelengths of light. Therefore, the wavelength of maximum absorption by a substance is one of the characteristic properties of that material. A completely transparent substance will have $I_t = I_0$ and its percent transmittance will be 100. Similarly, a substance which allows no radiation of a particular wavelength to pass through it will have $I_t = 0$, and a corresponding percent transmittance of 0.

Transmittance

$T = I_t / I_0$

% Transmittance: $\%T = 100\ T$

Absorbance

$A = \log10\ (I_0/I_t)$

$A = \log10\ (1/T) = -\log10\ (T)$

$A = \log10\ (100/\%T)$

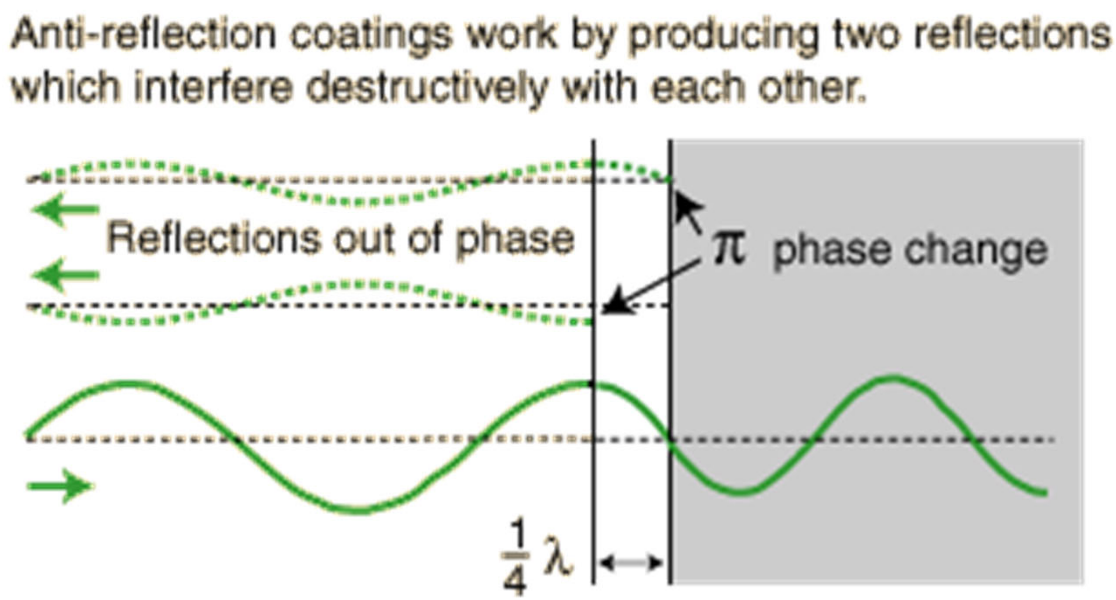
$A = 2 - \log10\ (\%T)$

Transmittance for liquids is usually written as: $T = I/I_0 = 10^{-\alpha l} = 10^{\Sigma lc'}$
Transmittance for gases is written as $T = I/I_0 = 10^{-\alpha}l = e^{-\sigma l N}$
I_0 and I are the intensity (or power) of the incident light and the transmitted light, respectively.
Absorbance for liquids is written as $A = -\log10 = (I/I_0)$
Absorbance for gases it is written as $A' = -\ln(I/I_0)$

Anti-Reflection Coatings

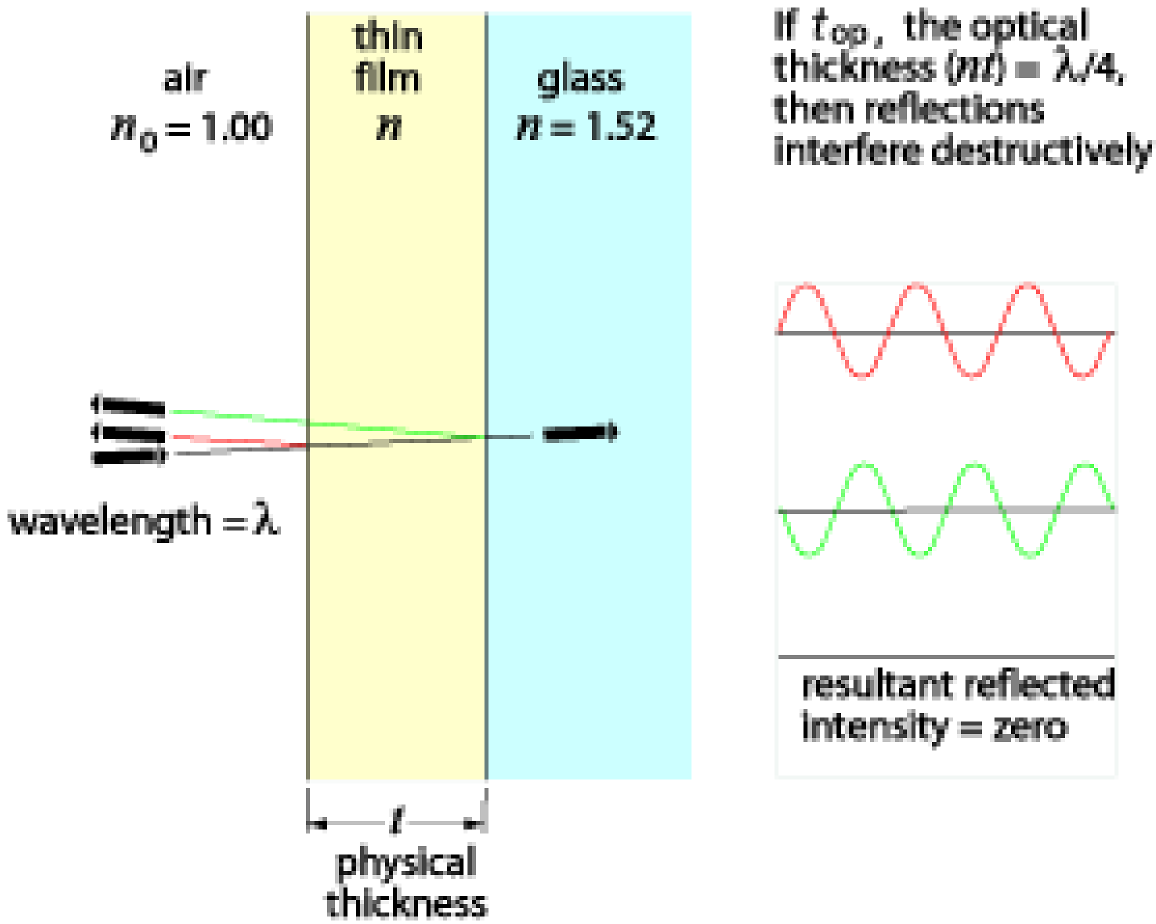
Without anti-reflection With anti-reflection

Anti-Reflection Coatings

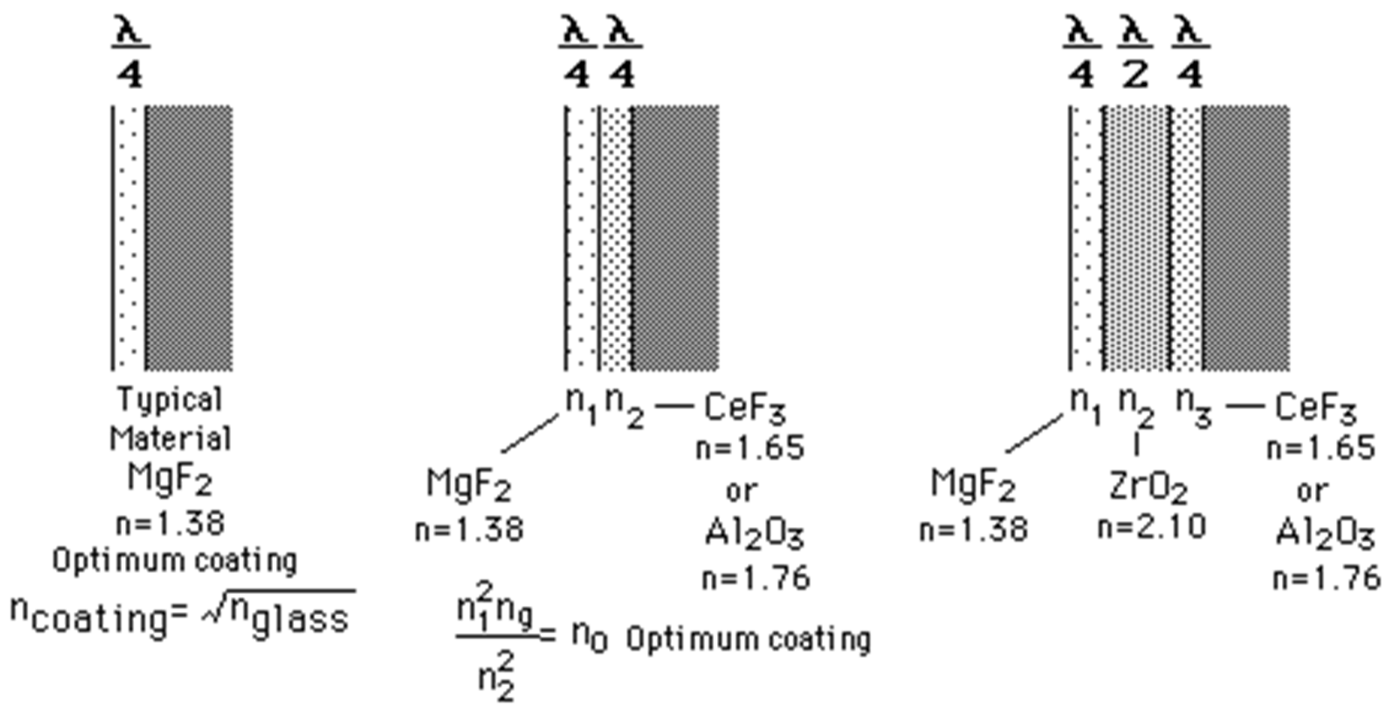


$$\phi = \frac{2\pi n d}{\lambda} = \pi \Rightarrow d = \lambda/4$$

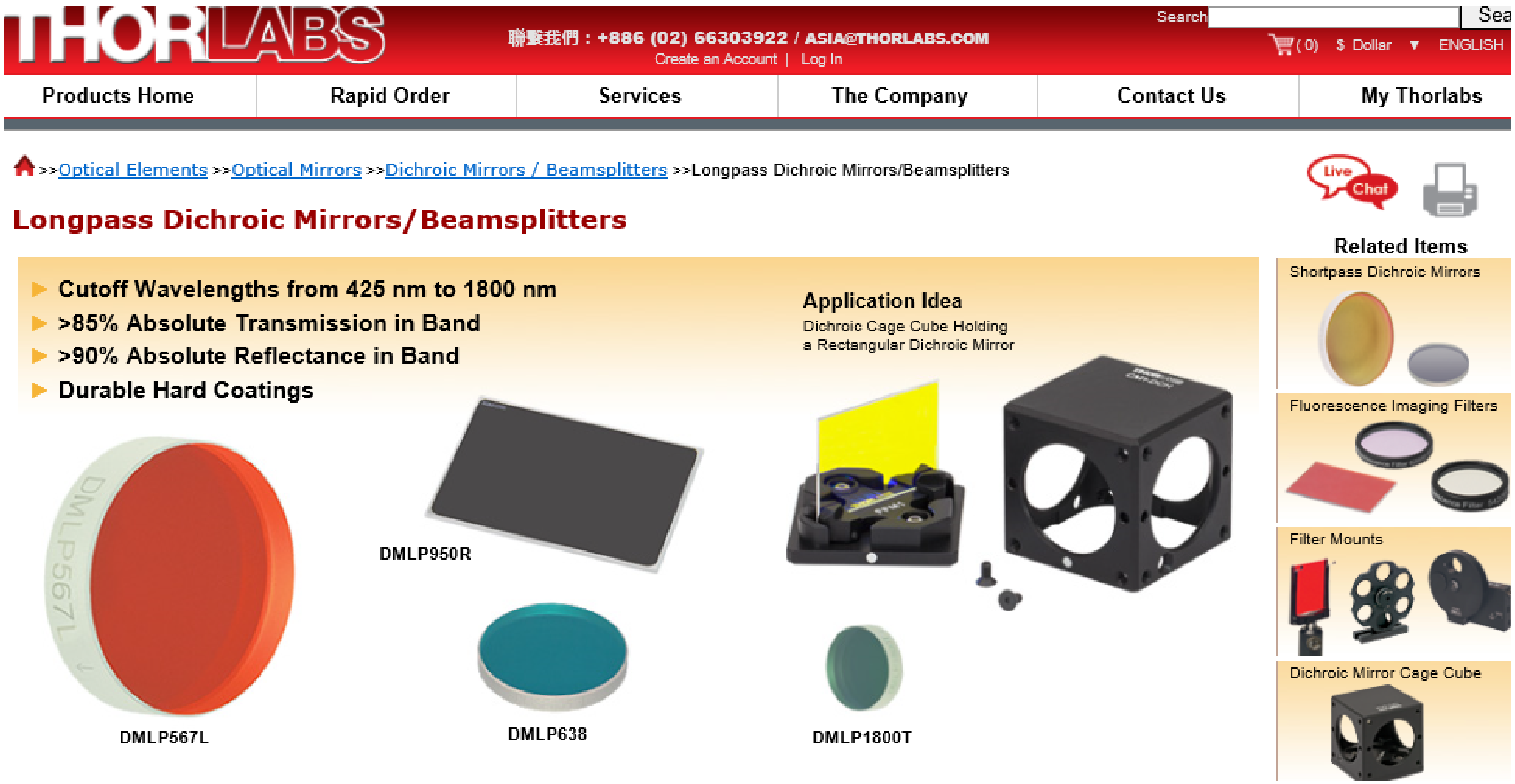
Antirefelction coating



Multi-Layer Anti-Reflection Coatings



Long pass Dichroic Mirrors/Beamsplitters

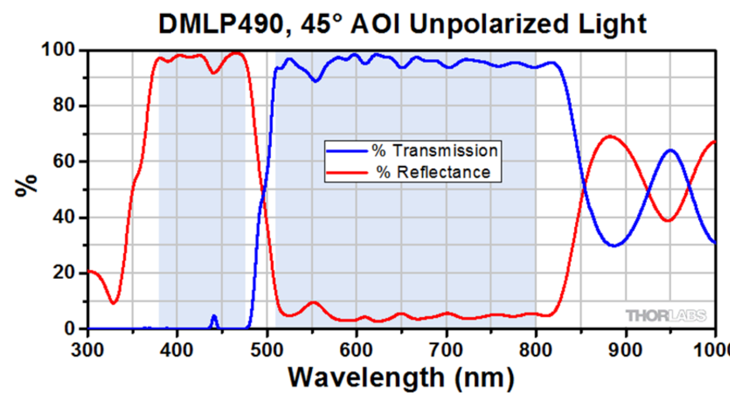


Example of Dichroic Mirrors/Beamsplitters

Thorlab Longpass Dichroic Mirrors/Beamsplitters: 490 nm Cutoff Wavelength

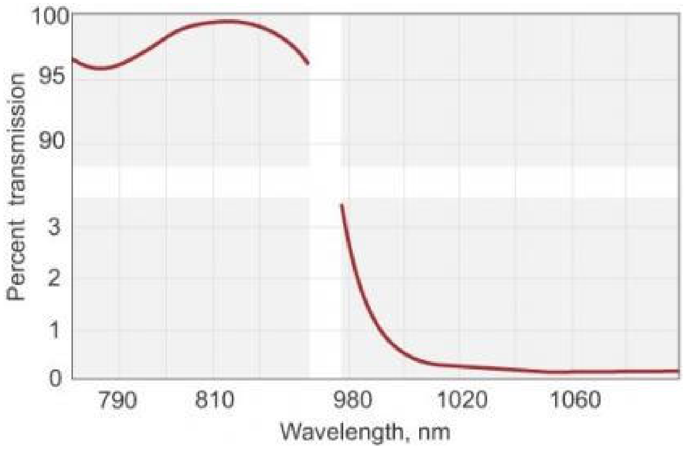
Specification:

Cutoff Wavelength 490 nm (transmission)
Transmission Band (Tabs > 85%, Tavg > 90%)
510 - 800 nm
Reflection Band (Rabs > 90%, Ravg > 95%)
380 - 475 nm



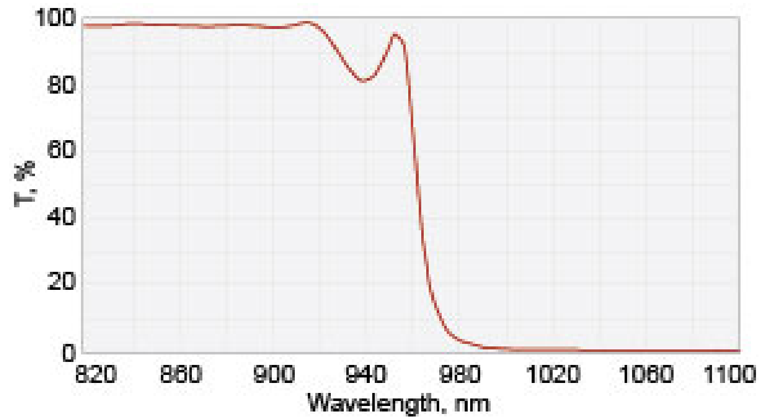
Dichroic Mirrors

Harmonic separators are dichroic beamsplitters used to reflect one wavelength and to transmit the others. Reflectance is higher than 99.5% for the wavelength of interest and transmittance is at least 90% for the rejected wavelengths. The rear surface of harmonic separators is antireflection coated.



031-6800. HR>99.5@1064 nm, HT>95%@808 nm, AOI=0°

- Laser Damage Threshold: >2J/cm2, 8 ns pulse, 1064 nm typical for BK7 substrates; >5J/cm2 8 ns pulse, 1064 nm typical for UVFS substrates
- Back side antireflection coated: R<0.5%
- Parallelism: 30 arcsec



EKSMA OPTICS offers BK7 dichroic mirrors for fast off-the-shelf delivery. Available substrate types of Non-Standard BK7 Dichroic Mirrors are Concave/Convex, Plano/Concave, Plano/Convex and Plano.

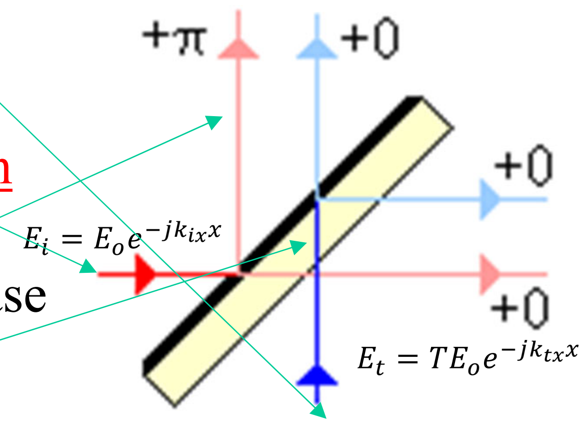
Phase Shifter

A beam splitter that consists of a **glass plate with a reflective dielectric coating on one side gives a phase shift of 0 or π,** depending on the side from which it is incident (see figure). Transmitted waves have no phase shift. Reflected waves entering from the reflective side (red) are phase-shifted by π, whereas reflected waves entering from the glass side (blue) have no phase shift. This is due to the Fresnel equations, according to which reflection causes a phase shift only when light passing through a material of low refractive index is reflected at a material of high refractive index. This is the case in the transition of air to reflector, but not from glass to reflector (given that the refractive index of the reflector is in between that of glass and that of air).

This does not apply to partial reflection by conductive (metallic) coatings, where other phase shifts occur in all paths (reflected and transmitted).

R = -1 => $E_r = e^{j\pi} E_o e^{-jk_{rz}z}$ (phase shifter)

$E_r = RE_o e^{jk_{rx}x}$



$n_{air} < n_{reflector} < n_{glass}$

Difference between reflection, phase shifting, backward travelling wave and phase conjugated wave

Fresnel Equation

$\mu_1, \varepsilon_1, n_1$ $\mu_2, \varepsilon_2, n_2$

$$H^r = (\hat{x}k_{rz} + \hat{z}k_{rx})\frac{R_l E_o}{\omega\mu_1}e^{-jk_{rx}x + jk_{rz}z}$$

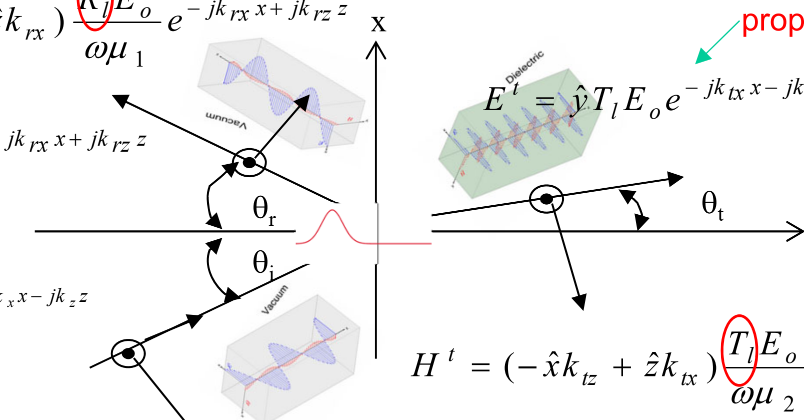
$$E^r = \hat{y}R_l E_o e^{-jk_{rx}x + jk_{rz}z}$$

$$E^i = \hat{y}E_o e^{-jk_x x - jk_z z}$$

$$H^i = (-\hat{x}k_z + \hat{z}k_x)\frac{E_o}{\omega\mu_1}e^{-jk_x x - jk_z z}$$

Negative sign means positive propagating direction

$$E^t = \hat{y}T_l E_o e^{-jk_{tx}x - jk_{tz}z}$$

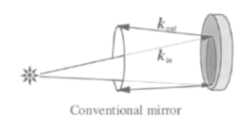


$$H^t = (-\hat{x}k_{tz} + \hat{z}k_{tx})\frac{T_l E_o}{\omega\mu_2}e^{-jk_{tx}x - jk_{tz}z}$$

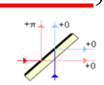
R_l=reflection coefficient
T_l=transmission coefficient

TE = transverse electric, perpendicularly polarized (E perpendicular to plan of incident)

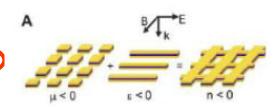
$E_i = E_o e^{-jk_{ix}x - jk_{iz}z}$



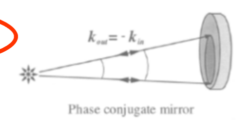
- Plain reflection and transmission: $k_{rx} = k_{ix}$ and **n = positive**, $E_r = RE_o e^{-jk_{rx}x + jk_{iz}z}$, $E_t = TE_o e^{-jk_{tx}x - jk_{iz}z}$
- Phase shifting: **R = -1** => $E_r = e^{j\pi}E_o e^{-jk_{rx}x + jk_{iz}z}$



- Backward travelling wave: **n = negative**, $E_i = E_o e^{-jk_{ix}x - jk_{iz}z}$ and $E_t = TE_o e^{jk_{tx}x + jk_{iz}z}$

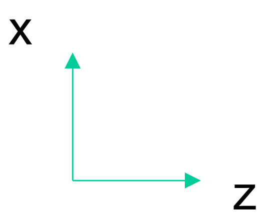


- Phase conjugated wave: $E_i = E_o e^{-jk_{ix}x - jk_{iz}z - \phi}$ => $E_r = E_o e^{jk_{rx}x - jk_{iz}z + \phi}$



Phase Conjugated Mirror

- Phase conjugation is a physical transformation of a wave field where the resulting field has a reversed propagation direction but keeps its amplitudes and phases



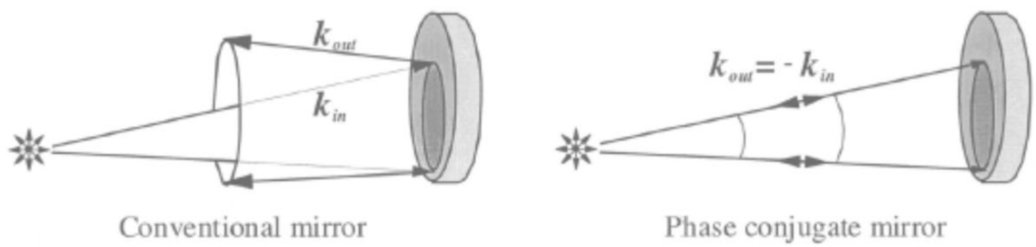


Figure 1: The wave reflects differently from PCM than from an ordinary mirror.[5]

$$E_i = E_o e^{-jk_{ix}x - jk_{iz}z - \phi} \Rightarrow E_r = E_o e^{-jk_{rx}x + jk_{iz}z + \phi}$$

Phase Conjugation

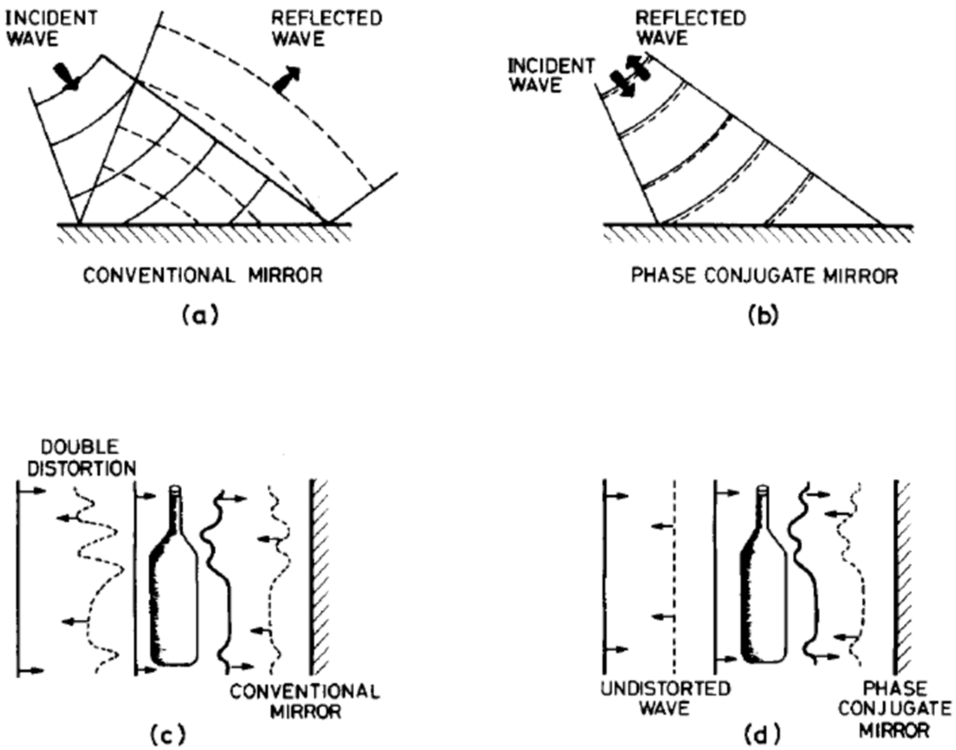


FIG. 1. Reflection from conventional and phase conjugate mirrors.

The secret is that the reflected wave is the complex conjugate of the incident wave so that a wave of the form $E = |\varepsilon(r)|\cos(\omega t - k \cdot r - \varphi(r))$ is reflected as

$$E^* = |\varepsilon(r)|\cos(\omega t + k \cdot r + \varphi(r)) \equiv |\varepsilon(r)|\cos(-\omega t - k \cdot r - \varphi(r)).$$

Thus as well as reversing the direction of light, a phase conjugate mirror also reverses its phase

if a hologram made by the superposition of a reference wave and a coherent wave reflected from a subject is illuminated from the back by a reference wave in the opposite direction (conjugate) to the original reference wave, a single real image is produced which is the complex conjugate of the beam originally reflected from the subject.

This type of real-time holography is more completely and correctly described in non-linear optical terms as four-wave mixing.[6] Within this framework, when three coherent optical beams are mixed together in a non-linear optical medium, it is possible to generate a fourth coherent wave whose wavelength and direction depend upon the relative angles of the beams. Waves E_1, E_2 and E_3 may be regarded as inducing in the medium an electrical polarization which oscillates with a frequency of $\omega_4 = (\omega_1 + \omega_2 - \omega_3)$. This polarization radiates a wave at the same frequency ω_4, with a phase $\varphi_4 = (\varphi_1 + \varphi_2 - \varphi_3)$ and in the direction of the wave vector $k_4 = (k_1 + k_2 - k_3)$.

DEGENERATE FOUR WAVE MIXING

(DFWM)

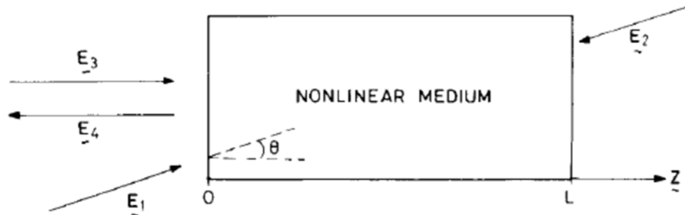


FIG. 2. Degenerate Four Wave Mixing. E_1 and E_2 are pump waves. E_3 is a probe wave and E_4 the generated phase conjugate wave.

For the setup depicted in Fig. 2, all three frequencies are chosen identical (degenerate) and the counter-propagating pump waves are conjugates of each other (so that $k_2 = -k_1$ and $\varphi_2 = -\varphi_1$). Thus the generated wave travels in the direction $k_4 = -k_3$ with a phase $\varphi_4 = \varphi_3$ and so is the conjugate of E_3.

Phase Conjugation

The general concept of phase conjugation is very simple – the phase of the incident light gets conjugated on reflection. To describe this let us define [2] the incident wave in the direction $\hat{e}_z$ with:

$$E_3(x,y,z;t) = E_0(x,y,z)\ e^{i(\Phi(x,y,z)-\omega t)} + c.c.\ , \tag{1}$$

where $\Phi(x,y,z) = kz + \varphi(x,y,z)$ is the dependance of the space component of the wave's phase. The $\varphi(x,y,z)$ describes the distortion of the wave front (its phase) by the medium while passing through it. We can now rewrite this as:

$$\begin{aligned} E_3(x,y,z;t) &= E_0(x,y,z)\ e^{i(kz+\varphi(x,y,z))}\ e^{-i\omega t} + c.c. \\ &= A_3(x,y,z)\ e^{-i\omega t} + c.c.\ , \end{aligned} \tag{2}$$

where we have defined A_3 as the complex amplitude of the incident wave. Now we can write the reflected wave in the same way, while taking into account also that its phase gets conjugated. This means that both components of the propagation vector will change in sign as $\mathbf{k} \rightarrow -\mathbf{k}$ and the same for its phase distortion: $\varphi \rightarrow -\varphi$. Reflected wave is then:

$$\begin{aligned} E_4(x,y,z;t) &= E_0(x,y,z)\ e^{i(-\Phi(x,y,z)-\omega t)} + c.c. \\ &= E_0(x,y,z)\ e^{i(-kz-\varphi(x,y,z))}\ e^{-i\omega t} + c.c. \\ &= A_4(x,y,z)\ e^{-i\omega t} + c.c.\ , \end{aligned} \tag{3}$$

where we have again defined A_4 as the complex amplitude of the wave. Comparing the two expressions we see that $A_4 = A_3^*$. To better understand this, it is worth writting down the real parts of the waves:

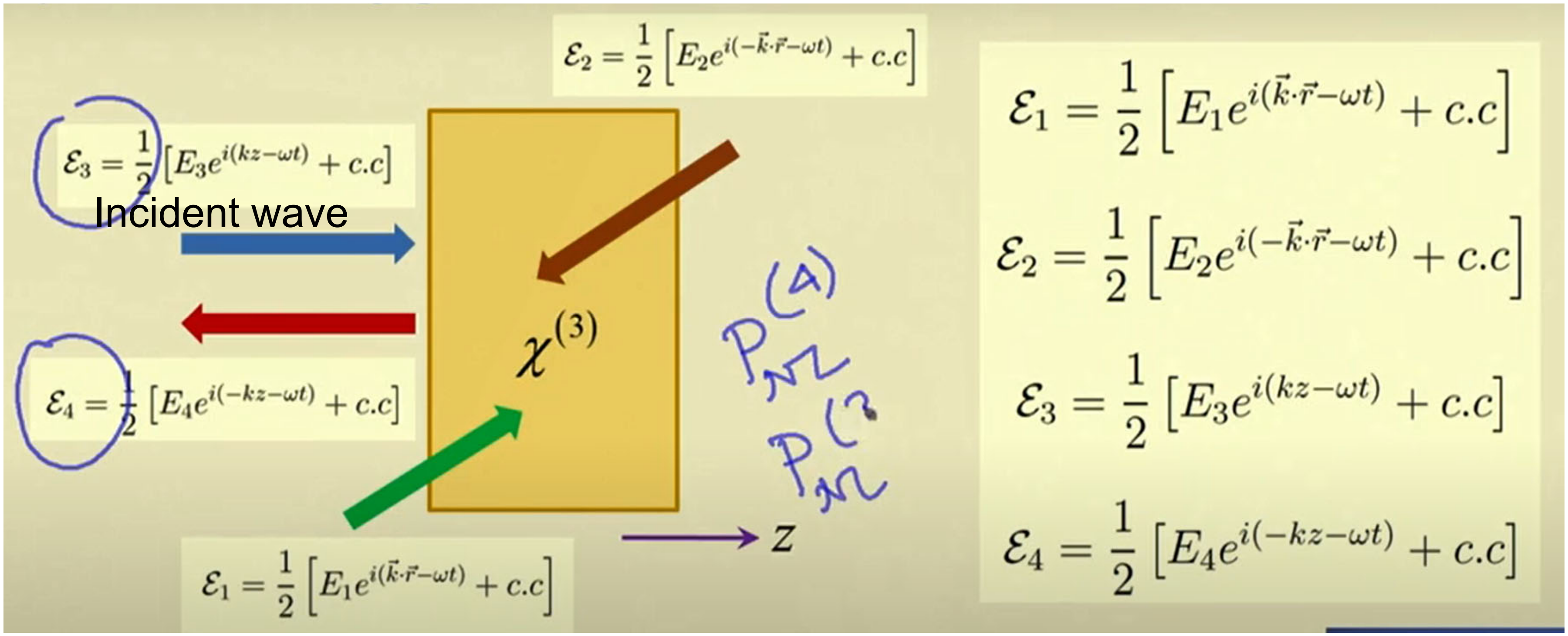
$$\begin{aligned} Re[E_4(x,y,z;t)] &= 2\ E_0 \cos(kz + \varphi(x,y,z) + \omega t) \\ &= 2\ E_0 \cos(-(-kz - \varphi(x,y,z) - \omega t)) \\ &= 2\ E_0 \cos(kz + \varphi(x,y,z) - \omega(-t)))\ . \end{aligned} \tag{4}$$

Thus we see:

$$Re[E_4(x,y,z;t)] = Re[E_3(x,y,z;-t)]\ . \tag{5}$$

From this, we can interpret the reflected wave as the incident wave, reversed as a wave going back in time, exactly following the path it previously took. If we illuminate such mirror with light from a point source, the reflected light will not expand after reflection, but rather focus back to its source.

4 wave mixing



E1 in incident and E2 are pump wave, where E 3 and E4 are wave generated in Z direction.

Nonlinear polarization terms is the source of E 3 and E4

4 wave mixing

Nonlinear polarization effect

$$E_T = (\mathcal{E}_1 + \mathcal{E}_2 + \mathcal{E}_3 + \mathcal{E}_4)$$

Nonlinear polarization

$$P_{NL} = \epsilon_0 \chi^{(3)} E_T^3$$

$$P_{NL} = \epsilon_0 \chi^{(3)} (\mathcal{E}_1 + \mathcal{E}_2 + \mathcal{E}_3 + \mathcal{E}_4)^3$$

$$\boxed{P_{NL}^{(4)} = \frac{1}{8}\epsilon_0 \chi^{(3)} \left[6E_1E_2E_3^* e^{i(-kz-\omega t)} + c.c\right]}$$

$$P_{NL}^{(3)} = \frac{1}{8}\epsilon_0 \chi^{(3)} \left[6E_1E_2E_4^* e^{i(kz-\omega t)} + c.c\right]$$

$$\mathcal{E}_1 = \frac{1}{2}\left[E_1 e^{i(\vec{k}\cdot\vec{r}-\omega t)} + c.c\right]$$

$$\mathcal{E}_2 = \frac{1}{2}\left[E_2 e^{i(-\vec{k}\cdot\vec{r}-\omega t)} + c.c\right]$$

$$\mathcal{E}_3 = \frac{1}{2}\left[E_3 e^{i(kz-\omega t)} + c.c\right]$$

$$\boxed{\mathcal{E}_4 = \frac{1}{2}\left[E_4 e^{i(-kz-\omega t)} + c.c\right]}$$

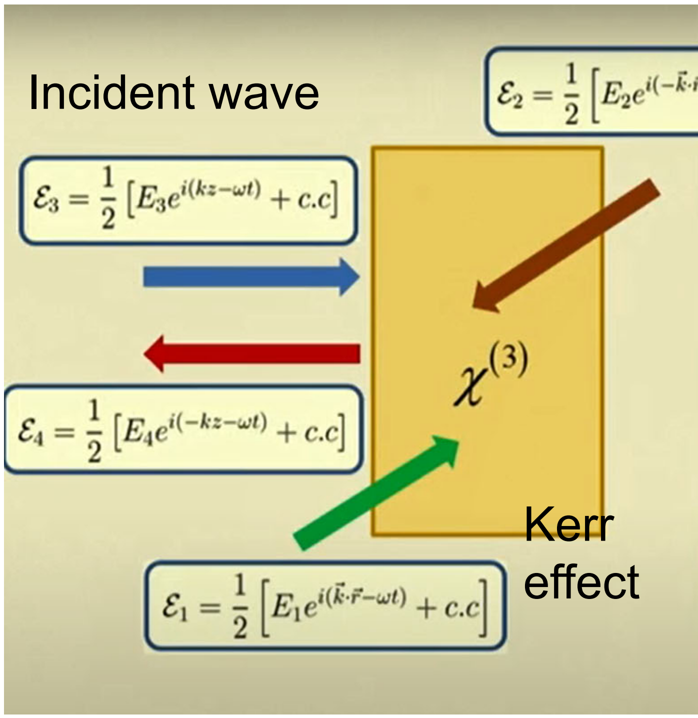
Complex conjugate of E_3 Therefore it's important to find out the Nonlinear polarization terms of E_3 and E_4

Nonlinear Maxwell equations

This is the nonlinear Maxwell equation, the source term is that last terms which we derived earlier. If E3 is the incident wave, we have two other waves going opposite direction is generated behave as a pump and mixing E1 E2 and E3 which give rise to another term E4

Gradient of E_4

Source term derived earlier



$$\boxed{\nabla^2\mathcal{E}_4 - \mu_0\epsilon(\omega)\frac{\partial^2\mathcal{E}_4}{\partial t^2} = \mu_0\frac{\partial^2 P_{NL}^{(4)}}{\partial t^2}}$$

$$\nabla^2\mathcal{E}_4 = \frac{1}{2}\left[\frac{\partial^2 E_4}{\partial z^2} - 2ik\frac{\partial E_4}{\partial z} - k^2 E_4\right]e^{-i(kz+\omega t)} + c.c$$

$$\mu_0\epsilon(\omega)\frac{\partial^2\mathcal{E}_4}{\partial t^2} = -\frac{1}{2}\mu_0\epsilon(\omega)\omega^2 E_4 e^{-i(kz+\omega t)} + c.c$$

$$\mu_0\frac{\partial^2 P_{NL}^{(4)}}{\partial t^2} = -\frac{3}{4}\mu_0\epsilon_0\chi^{(3)}\omega^2 E_1 E_2 E_3^* e^{-i(kz+\omega t)} + c.c$$

$$-2ik\frac{dE_4}{dz} = -\frac{3}{2}\mu_0\epsilon_0\chi^{(3)}\omega^2 E_1 E_2 E_3^*$$

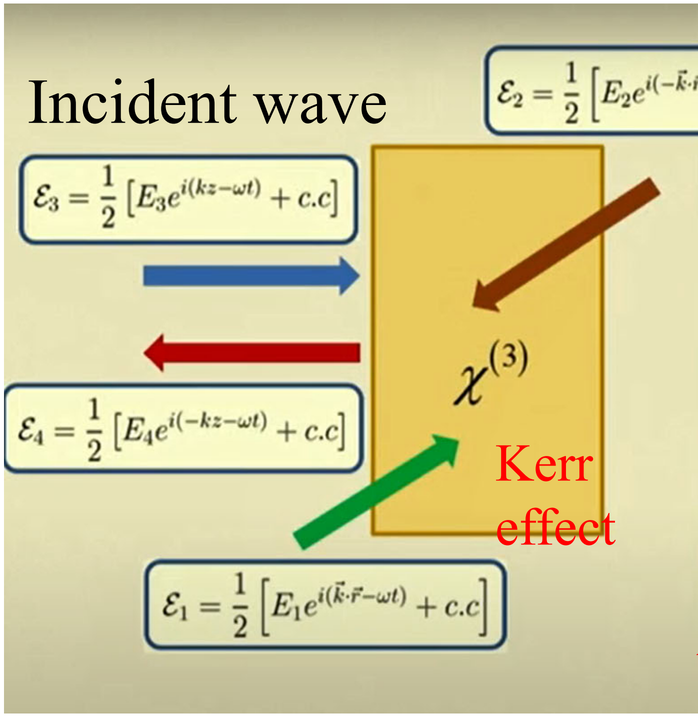
$$P_{NL}^{(4)} = \frac{1}{8}\epsilon_0\chi^{(3)}\left[6E_1E_2E_3^* e^{i(-kz-\omega t)} + c.c\right]$$

Nonlinear Maxwell equations

This is the nonlinear Maxwell equation, the source term is that last terms which we derived earlier. If E3 is the incident wave, we have two other waves going opposite direction is generated behave as a pump and mixing E1 E2 and E3 which give rise to another term E4

Source term derived earlier

Gradient of E_4



$$\mathcal{E}_2 = \frac{1}{2}\left[E_2 e^{i(-\vec{k}\cdot\vec{r}-\omega t)} + c.c\right]$$

$$\mathcal{E}_3 = \frac{1}{2}\left[E_3 e^{i(kz-\omega t)} + c.c\right]$$

$$\mathcal{E}_4 = \frac{1}{2}\left[E_4 e^{i(-kz-\omega t)} + c.c\right]$$

$$\mathcal{E}_1 = \frac{1}{2}\left[E_1 e^{i(\vec{k}\cdot\vec{r}-\omega t)} + c.c\right]$$

$$\boxed{\nabla^2\mathcal{E}_4 - \cancel{\mu_0\epsilon(\omega)\frac{\partial^2\mathcal{E}_4}{\partial t^2}} = \mu_0\frac{\partial^2 P_{NL}^{(4)}}{\partial t^2}}$$

$$\nabla^2\mathcal{E}_4 = \frac{1}{2}\left[\cancel{\frac{\partial^2 E_4}{\partial z^2}} - 2ik\frac{\partial E_4}{\partial z} - \cancel{k^2 E_4}\right]e^{-i(kz+\omega t)} + c.c$$

First term cancelled out due to slow varying approximation. These two terms will cancelled out

$$\mu_0\epsilon(\omega)\frac{\partial^2\mathcal{E}_4}{\partial t^2} = -\frac{1}{2}\mu_0\epsilon(\omega)\omega^2 E_4 e^{-i(kz+\omega t)} + c.c$$

$$\mu_0\frac{\partial^2 P_{NL}^{(4)}}{\partial t^2} = -\frac{3}{4}\mu_0\epsilon_0\chi^{(3)}\omega^2 E_1 E_2 E_3^* e^{-i(kz+\omega t)} + c.c$$

Simplify to

$$-2ik\frac{dE_4}{dz} = -\frac{3}{2}\mu_0\epsilon_0\chi^{(3)}\omega^2 E_1 E_2 E_3^*$$

$$P_{NL}^{(4)} = \frac{1}{8}\epsilon_0\chi^{(3)}\left[6E_1E_2E_3^* e^{i(-kz-\omega t)} + c.c\right]$$

Further simplify the last equation from previous page

$$\boxed{\frac{dE_4}{dz} = -i\frac{3}{4}\frac{\chi^{(3)}\omega^2}{kc^2}E_1E_2E_3^*}$$

$$\frac{dE_4}{dz} = -i\frac{3}{4}\frac{\chi^{(3)}\omega}{nc}E_1E_2E_3^*$$

Further simply by sub n for k etc.

Coupling coefficient.

$$\boxed{\kappa = \frac{3}{4}\frac{\chi^{(3)}\omega}{nc}E_1E_2}$$

E1 and E2 are amplitude of this pump and if it's strong and no depletion of the pump then E 1 E2 becomes constant

$$\boxed{\frac{dE_4}{dz} = -i\kappa E_3^*}$$

$$\boxed{\frac{dE_3}{dz} = i\kappa E_4^*}$$

$$\boxed{\frac{d^2E_4}{dz^2}} = -i\kappa\boxed{\frac{dE_3^*}{dz}} = \boxed{-i\kappa(-i\kappa^*)E_4}$$

$$\boxed{\frac{d^2E_4}{dz^2} = -|\kappa|^2E_4}$$

Second order differentiation

Solution is sinusoidal

$$\boxed{E_4(z) = a\cos(|\kappa|z) + b\sin(|\kappa|z)}$$

Once we find E4 we can find E3

$$E_3^* = \frac{1}{(-i\kappa)}\frac{dE_4}{dz} = \frac{i}{\kappa}\frac{dE_4}{dz}$$

$$\boxed{E_3^*(z) = \frac{i}{\kappa}|\kappa|[-a\sin(|\kappa|z) + b\cos(|\kappa|z)}$$

Exact the same way to derive E_3 as E_4

Trigonometric Functions in Terms of Exponential Functions

$$\sin x = \frac{e^{ix} - e^{-ix}}{2i} \qquad \csc x = \frac{2i}{e^{ix} - e^{-ix}}$$

$$\cos x = \frac{e^{ix} + e^{-ix}}{2} \qquad \sec x = \frac{2}{e^{ix} + e^{-ix}}$$

$$\tan x = \frac{e^{ix} - e^{-ix}}{i\left(e^{ix} + e^{-ix}\right)} \qquad \cot x = \frac{i\left(e^{ix} + e^{-ix}\right)}{e^{ix} - e^{-ix}}$$

Exponential Function vs. Trigonometric and Hyperbolic Functions

$$\boxed{e^{ix} = \cos x + i \sin x} \qquad e^{x} = \cosh x + \sinh x$$

Hyperbolic Functions in Terms of Exponential Functions

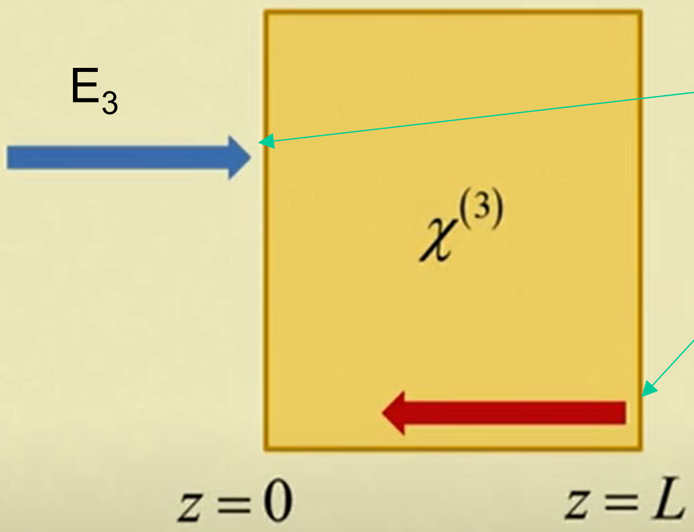
$$\sinh x = \frac{e^{x} - e^{-x}}{2} \qquad \operatorname{csch} x = \frac{2}{e^{x} - e^{-x}}$$

$$\cosh x = \frac{e^{x} + e^{-x}}{2} \qquad \operatorname{sech} x = \frac{2}{e^{x} + e^{-x}}$$

$$\tanh x = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}} \qquad \coth x = \frac{e^{x} + e^{-x}}{e^{x} - e^{-x}}$$

Fins *a and b* coefficients in the solution

Boundary condition



$$\boxed{E_4(L) = 0} \qquad \boxed{E_3(0) = E_{30}}$$

$$E_4(L) = \boxed{a\cos(|\kappa|L) + b\sin(|\kappa|L) = 0}$$

If $E_4(L) = 0$ then

$$\boxed{a = -b\tan(|\kappa|L)}$$

$$\boxed{E_3^*(0) = \frac{i}{\kappa}|\kappa|b = E_{30}^*}$$

$$E_3^* = \frac{1}{(-i\kappa)}\frac{dE_4}{dz} = \frac{i}{\kappa}\boxed{\frac{dE_4}{dz}} \qquad \boxed{\frac{dE_4}{dz}} = -i\kappa\boxed{E_3^*}$$

and $$E_3^*(z) = \frac{i}{\kappa}|\kappa|[-a\overset{0}{\sin(|\kappa|z)} + b\overset{1}{\cos(|\kappa|z)}]$$

From last page, a and b were derived now we want to find E_4 and R"

$$b = -i\frac{\kappa}{|\kappa|}E_{30}^*$$

$$a = i\frac{\kappa}{|\kappa|}E_{30}^*\tan(|\kappa|L)$$

$$E_4(z) = a\cos(|\kappa|z) + b\sin(|\kappa|z)$$

$$E_4(z) = i\frac{\kappa}{|\kappa|}E_{30}^*\left[\tan(|\kappa|L)\cos(|\kappa|z) - \sin(|\kappa|z)\right]$$

This represents what E_4 is at Z=0: $$E_4(0) = i\frac{\kappa}{|\kappa|}E_{30}^*\tan(|\kappa|L)$$

Reflection coefficient at Z=0

$$R = \left|\frac{E_4(0)}{E_3(0)}\right|^2 = \tan^2(|\kappa|L)$$

$$E_3^*(0) = \frac{i}{\kappa}|\kappa|b = E_{30}^*$$

From nonlinear Maxwell equation and wave equation and nonlinear polarization equation, we derive the E4 and then E3* and also coupling coefficient (in terms of E1 and E2) and also solution for E4 and E3*.

Then we use the E4 as E3* and B.C. to find the a and b coefficient and E3 and E4. Then substitute Z=0 on both E3 and E 4 to find reflecting coefficient R at Z=0

$$\kappa = \frac{3}{4}\frac{\chi^{(3)}\omega}{nc}E_1E_2$$

Reflection coefficient

$$R = \left|\frac{E_4(0)}{E_3(0)}\right|^2 = \tan^2(|\kappa|L)$$

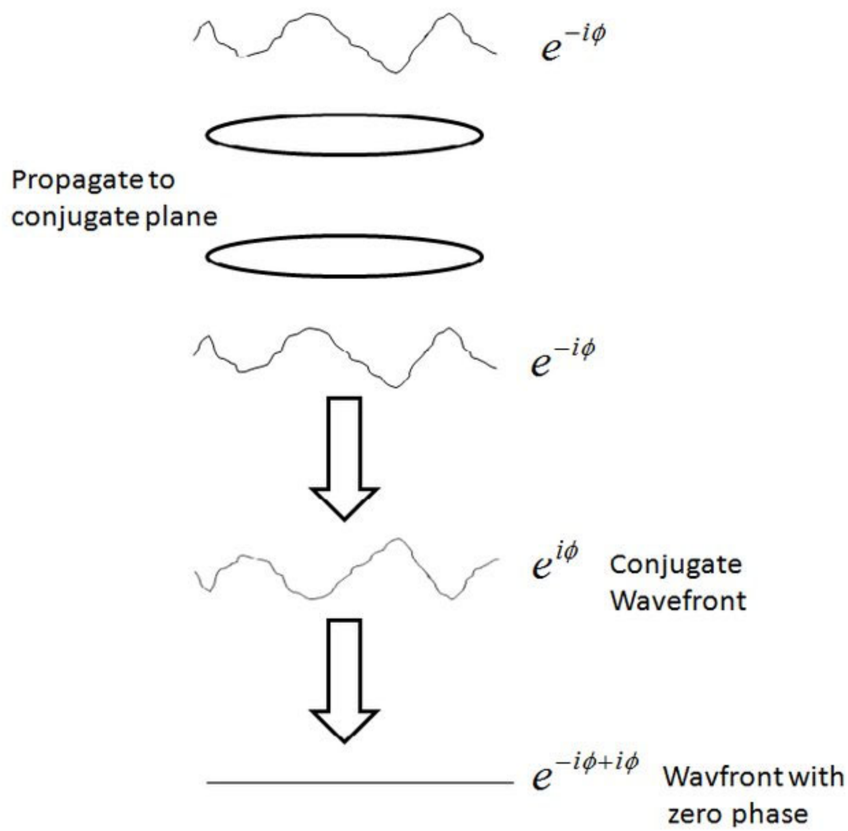
E1 and E2 are amplitude of this pump and if it's strong and no depletion of the pump then E 1 E2 becomes constant

$\boxed{\text{If } |\kappa|L > \pi/4 \text{ then } R > 1,}$ that means we have *amplification*. It is called the amplifying mirror. $\boxed{\text{If } |\kappa|L = \pi/4, \text{ then } R \to \infty,}$ that means we have reflected signal without any input signal, it is called *oscillation*.

Key point is you can have a reflected wave just opposite phase. So this is really a phase conjugated kind of wave just using this 4wave mixer. We can have the conjugate wave or this conjugate wave to remove the distortion.

3 conditions: one is reflected, one is oscillated and one is amplified:

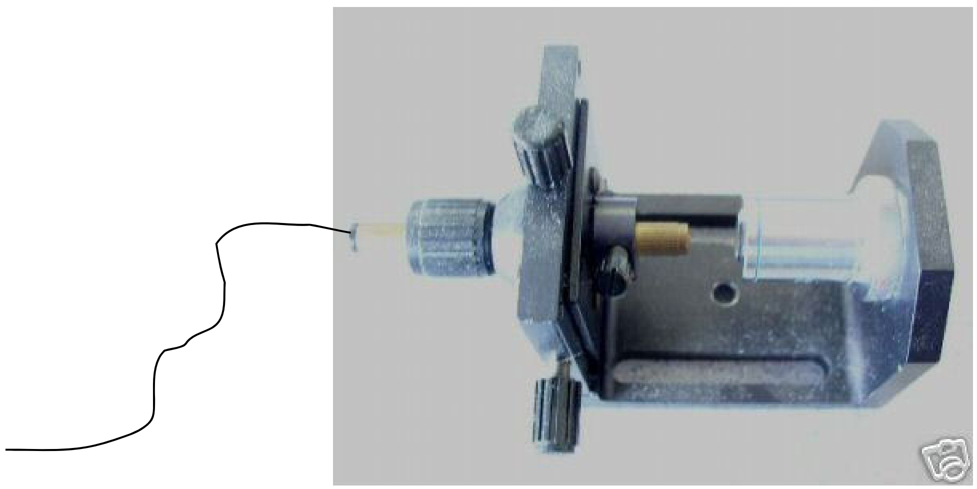
- Amplification is possible due to the pumping. This pumping basically gives rise to the fact that some sort of amplification maybe possible
- If kl = pi/4 then R tends to infinity, we have some sort of wave without any kind of input, so it's nothing but the oscillation



Self-pumped phase conjugate mirror

Light Coupling System

Fiber Direct Focusing



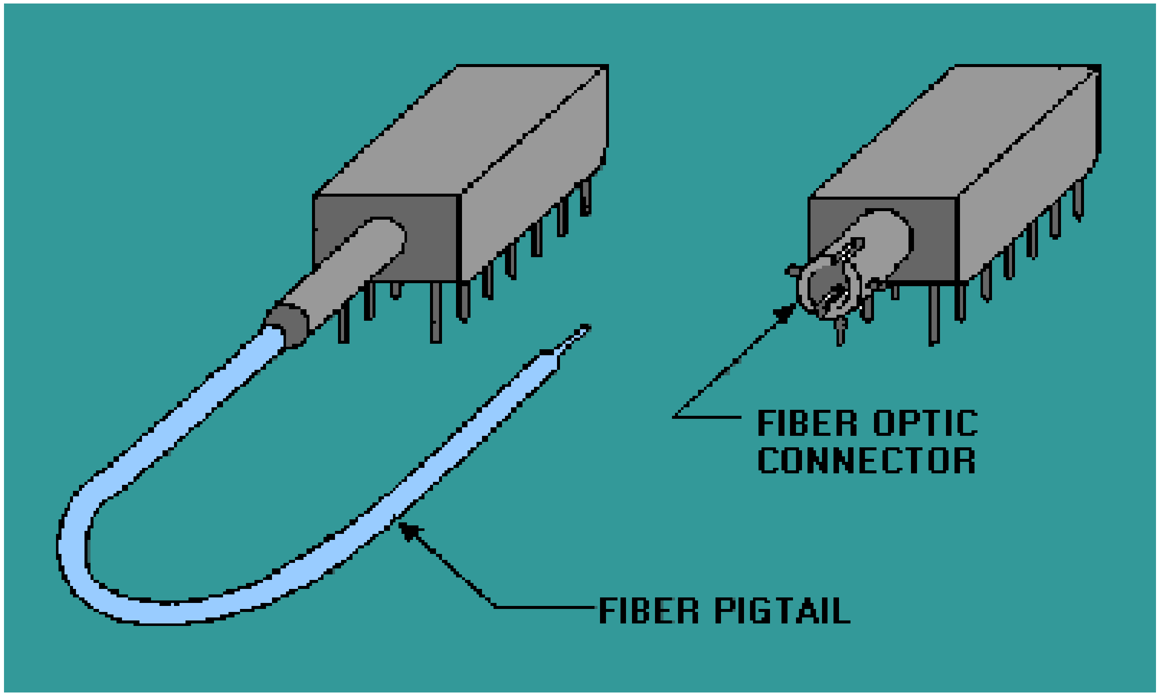
fiber

X-Y stage

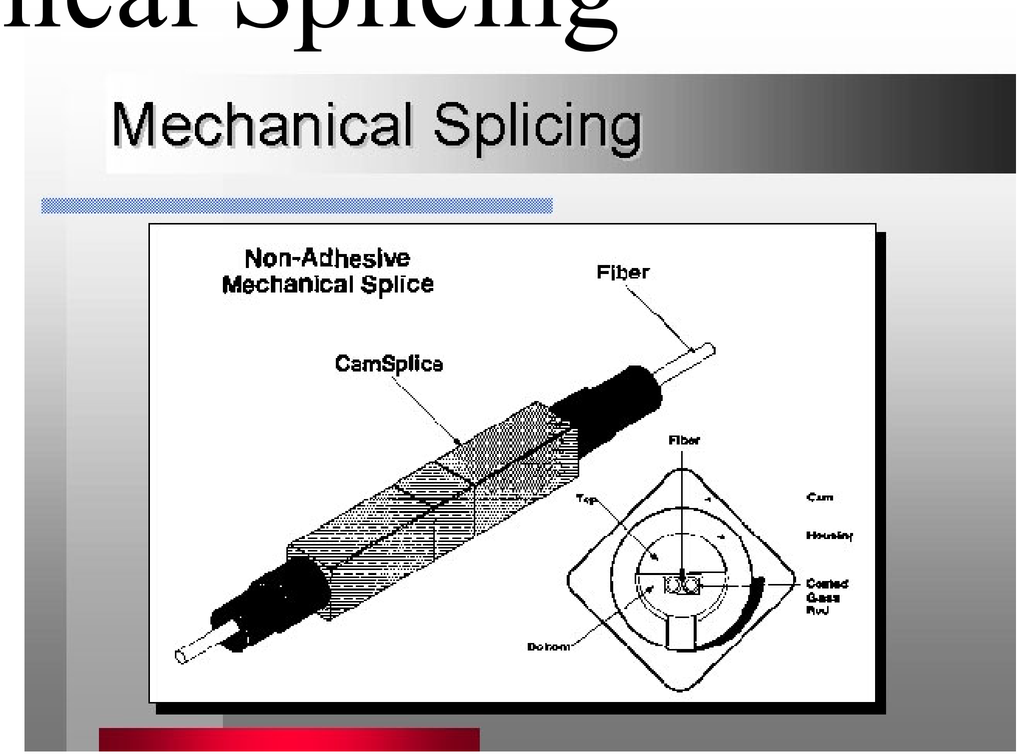
lens

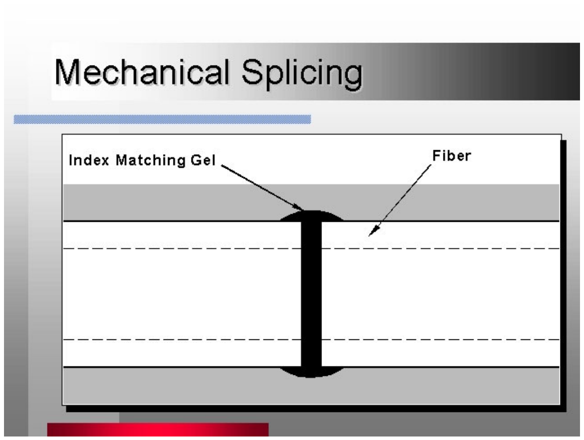
Bare fiber coupling

Pigtailed and connectorized fiber optic devices



Mechanical Splicing



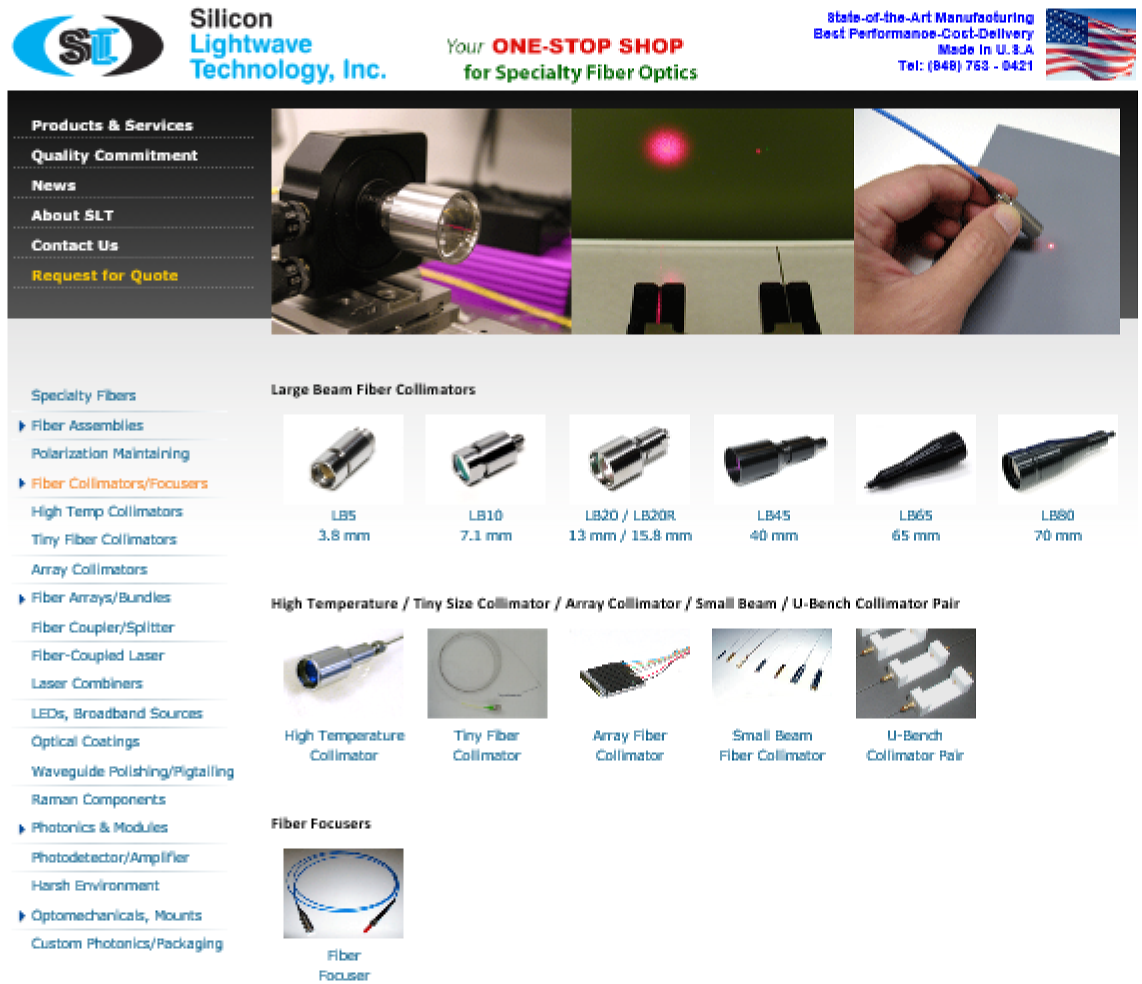


Bare Fiber to Fiber Connection

Mechanical coupler

SMA Fiber Optic Coupler

Pigtail Grin Lens



Tapered Mode Size Converters

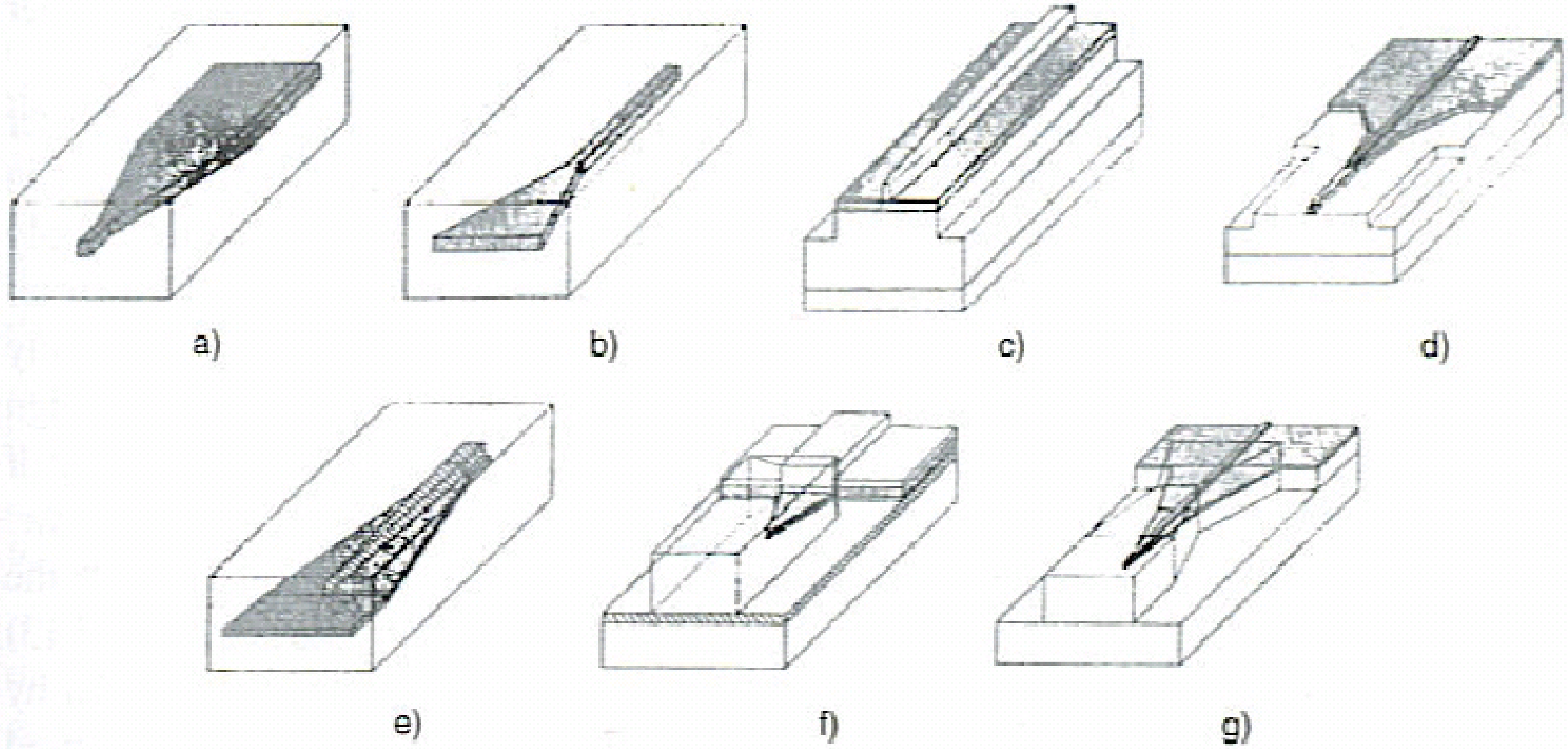
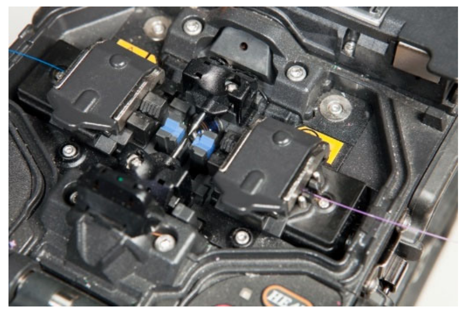
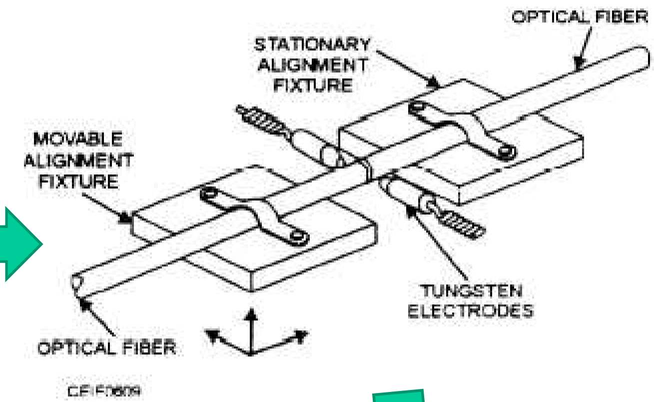


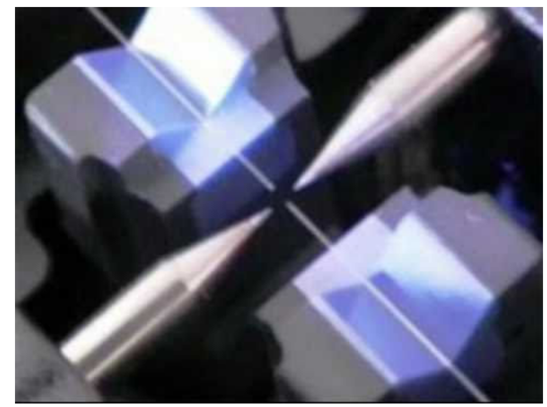
Fig. 7.12a–g. Lateral taper designs. **a** Lateral down-tapered buried waveguide. **b** Lateral up-tapered buried waveguide. **c** Single lateral taper transition from a ridge waveguide to a fiber-matched waveguide. **d** Multisection taper transition from a ridge waveguide to a fiber-matched waveguide. **e** Dual lateral overlapping buried waveguide taper. **f** Dual lateral overlapping ridge waveguide taper. **g** Nested waveguide taper transition from a ridge waveguide to a fiber-matched waveguide [7.25] ©1997 IEEE

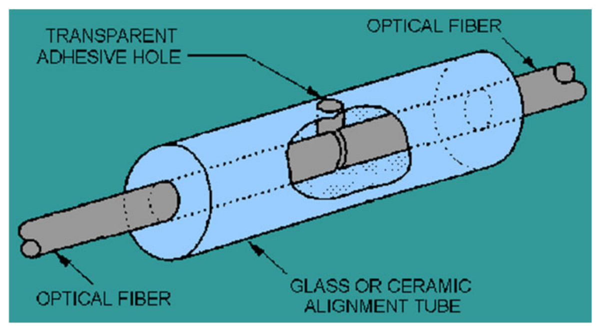
W.Wang

Fusion Splicer

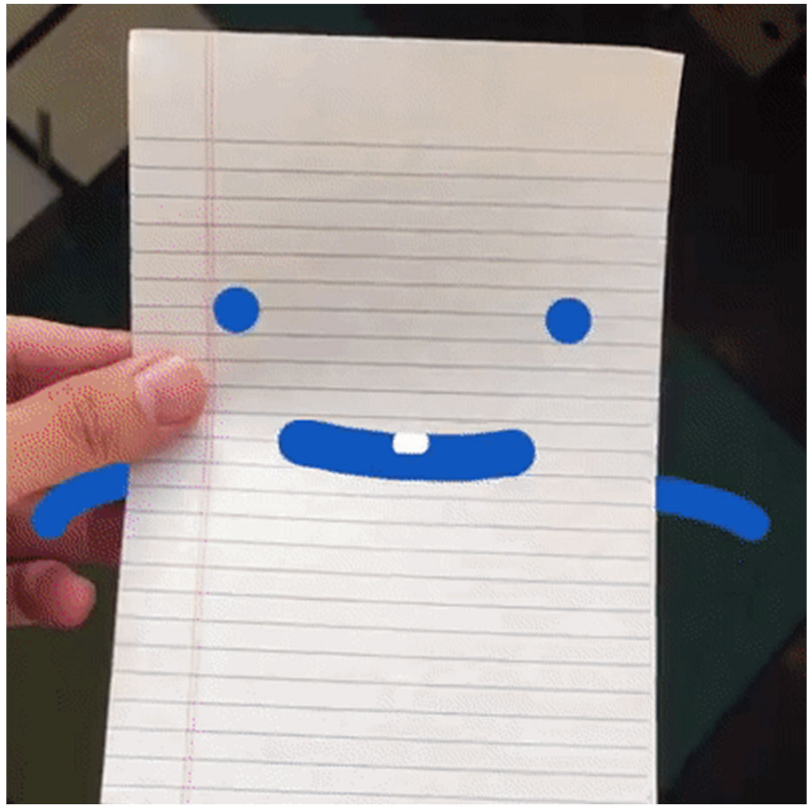








- Connecting two bare fibers together
- With loss < 0.3dB



Hasta Luego