ME 564 HW #2 Solution Key

Problem 1.6.7 Find The general Solution To The Differential equation

$$y' + ky = e^{-kx}$$

Refer To eqn (4) Page 34 of Kreyszig

$r(x) = e^{-kx}$ $\quad P(x) = k \quad$ $h = \int P\,dx = \int k\,dx = kx$

$$y(x) = e^{-kx}\left[\int e^{kx} \cdot e^{-kx}\,dx + c\right]$$

$$y(x) = e^{-kx}\left[e^{kx-kx}\,dx + c\right]$$

$$\boxed{y(x) = e^{-kx}\left[x + c\right]}$$

Problem 1.6.9 Find The general Solution To The Differential eqn.

$$xy' = 2y + x^3 e^x$$

Rearranging into $y' + P(x)y = r(x)$ form

$$y' - \frac{2}{x}y = x^2 e^x \qquad r(x) = x^2 e^x \qquad P(x) = \frac{-2}{x}$$

$$h = \int P(x)\,dx = \int \frac{-2}{x}\,dx = -2\ln x = \ln x^{-2}$$

Again, using eqn (4)

$$y(x) = e^{-\ln x^{-2}}\left[\int e^{\ln x^{-2}} x^2 e^x\,dx + c\right]$$

$$y(x) = x^2\left[\int \cancel{x^{-2}}\,\cancel{x^2}\,e^x\,dx + c\right]$$

$$\boxed{y(x) = x^2\left[e^x + c\right]}$$

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Problem 1.6.20

Solve The initial value Problem

$$y' + 6x^2 y = e^{-2x^3}/x^2 \; ; \quad y(1) = 0$$

$$P(x) = 6x^2 \qquad r(x) = \frac{e^{-2x^3}}{x^2} \qquad h = \int P(x)\,dx = \int 6x^2\,dx = 2x^3$$

by eqn (4) Page 34

$$y = e^{-2x^3}\left[\int \cancel{e^{2x^3}} \cdot \frac{\cancel{e^{-2x^3}}}{x^2}\,dx + c\right]$$

$$y = e^{-2x^3}\left[\int \frac{1}{x^2}\,dx + c\right]$$

$$\boxed{y = e^{-2x^3}\left[\frac{-1}{x} + c\right]}$$ ⇐ General Solution:

Now Apply initial Condition

$$y(1) = 0 = e^{-2(1)^3}\left[\frac{-1}{1} + c\right]$$

$$c = 1$$

$$\boxed{y = e^{-2x^3}\left[1 - \frac{1}{x}\right]}$$

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Problem 1.6.31 Reduction of Nonlinear DE's
Reduce The Following To Linear Form and Solve

$$y' + 2y = y^2$$

Following The Reduction of Form along

Let $u(x) = [y(x)]^{1-2} = y(x)^{-1}$

by The chain Rule $du = \frac{du}{dy}\,dy = \frac{d(y^{-1})}{dy}\,dy$

$$du = (-1)\,y^{-2}\,dy$$

Rewriting The Given equation, solving for y

$$y' = y^2 - 2y$$

Substitute into The differential

$$u' = -y^{-2}(y^2 - 2y) = -1 + 2y^{-1}$$

and $u = y^{-1}$ So, substitution gives

$$u' = 2u - 1 \quad \text{or} \quad u' - 2u = -1 \Rightarrow \text{Now Solve}$$

$$p(x) = -2 \qquad r(x) = -1$$

$$h = \int p\,dx = -2x$$

$$u = e^{2x}\left[\int e^{-2x}(-1)\,dx + c\right] = e^{2x}\left[\frac{e^{-2x}}{2} + c\right]$$

Finally, multiplying Through

$$u = \left[\frac{1 + 2ce^{2x}}{2}\right]$$

but $u = \frac{1}{y}$ so $y = \frac{1}{u}$

$$\boxed{y = \left[\frac{2}{1 + 2ce^{2x}}\right]}$$

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Problem 6.8.1

is The Set a vector space or Not? if so, determine dimension and basis.

1) All vectors satisfying $V_1 - 3V_2 + 2V_3 = 0$ in $\mathbb{R}^3$

One way to check for a vector space is to see if its Rules are met. here's a good way. If I pick two sets of vectors $V = [V_1\ V_2\ V_3]$ and $W = [W_1\ W_2\ W_3]$, ~~then~~ either ~~like~~ both satisfying ~~eqn~~ The eqn Given; Then an arbitrary Linear Combination such as $U = aV + bW$ should also satisfy The equation.

Lets Look:

choose $\underline{V} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$ Such That $V_1 - 3V_2 + 2V_3 = 0$

and $\underline{W} = \begin{bmatrix} W_1 \\ W_2 \\ W_3 \end{bmatrix}$ Such That $W_1 - 3W_2 + 2W_3 = 0$

Let $\underline{U} = a\underline{V} + b\underline{W} = \begin{bmatrix} aV_1 + bW_1 \\ aV_2 + bW_2 \\ aV_3 + bW_3 \end{bmatrix}$

Apply it to The equation we ~~...~~ are wanting To satisfy.

$$U_1 - 3U_2 + 2U_3 = [a_1V_1 + b_1W_1] - 3[a_2V_2 + bW_2] + \cancel{[aV_3 + bW]}$$
$$+ 2[aV_3 + bW_3]$$

~~Rearranging Σ $a_1V_1 - 3a_2V_2$~~

Collecting V/W:

Problem 6.8.1 continued.

$$u_1 - 3u_2 + 2u_3 = aV_1 + bW_1 - 3aV_2 - 3bW_2 + 2aV_3 + 2bW_3$$

Rearrange, collecting V's and W's, and factoring out constants

$$u_1 - 3u_2 + 2u_3 = a\underbrace{\left[V_1 - 3V_2 + 2V_3\right]} + b\underbrace{\left[W_1 - 3W_2 + 2W_3\right]}$$

chosen to be 0

$$u_1 - 3u_2 + 2u_3 = a(0) + b(0) = 0$$

So it is a vector space.

Now, how do we find the dimension?

We have 3 dimensions, V_1, V_2 and V_3 constrained by one equation $V_1 = 3V_2 - 2V_3$, so Dimension = 2 for this set

We can use $V_1 = 3V_2 - 2V_3$ to determine a basis for the space.

Since there are two dimensions in the space, we can fully define it with the set $\underline{B} = [B_1 \; B_2]$

Note you basically make up the basis set, using the contraints given by the space, I'll use the technique the book used for the answers in the back

if we let $V_2 = 1$ and $V_3 = 0$, then $V_1 = 3V_2 = 3$

So $\underline{B_1} = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$

if we let $V_2 = 0$, $V_3 = 1$ then $V_1 = -2V_3 = -2$

So $\underline{B_2} = \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix}$ Giving us a basis of

$$\underline{\underline{B}} = \begin{bmatrix} B_1 & B_2 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

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Problem 6.8.2 | Is The Given Set A Vector Space

All ordered Quadruples of Non-Negative Numbers.

A vector within a valid vector space must remain in that space under scalar multiplication.

These sets fail this test; watch

$$(-1)\begin{bmatrix}1\\2\\3\\4\end{bmatrix} = \begin{bmatrix}-1\\-2\\-3\\-4\end{bmatrix} \neq \text{Non-Negative}$$

$\therefore$ Not in The Set in question

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6.8.14 Find The Euclidean Norm of The vector $\begin{bmatrix}2\\0\\3\\0\\8\end{bmatrix}$

$$\left\|\begin{bmatrix}2\\0\\3\\0\\8\end{bmatrix}\right\| = \sqrt{[2\ 0\ 3\ 0\ 8]\cdot\begin{bmatrix}2\\0\\3\\0\\8\end{bmatrix}}$$

$$= \sqrt{2\cdot2+0\cdot0+3\cdot3+0\cdot0+8\cdot8}$$

$$= \sqrt{4+9+64}$$

$$= \boxed{\sqrt{77}}$$

6.8.17 Find The Euclidean Norm of The vector $\begin{bmatrix}3\\2\\-2\\4\\0\end{bmatrix}$

$$\left\|\begin{bmatrix}3\\2\\-2\\4\\0\end{bmatrix}\right\| = \sqrt{[3\ 2\ -2\ 4\ 0]\cdot\begin{bmatrix}3\\2\\-2\\4\\0\end{bmatrix}}$$

$$= \sqrt{3^2+2^2+(-2)^2+4^2+0^2}$$

$$= \sqrt{9+4+4+16}$$

$$= \sqrt{33}$$

6.8.19 Show that the vectors in Problems 14 and 19 are orthogonal

Use the dot product.

$$[\,2\;0\;3\;0\;8\,]\cdot\begin{bmatrix}3\\2\\-2\\4\\0\end{bmatrix} = 2\cdot3 + 0\cdot2 + 3(-2) + (0)4 + 8(0)$$

$$= 6 + 0 - 6 + 0 + 0 = 0 \checkmark$$

Orthogonal

6.8.21 Find all vectors V that are orthogonal to the vector

$$\underset{\sim}{a} = \begin{bmatrix}1\\2\\0\end{bmatrix}$$

Select vectors such that $\underline{a}\cdot\underline{V} = 0$

$$[1\;\;2\;\;0]\begin{bmatrix}V_1\\V_2\\V_3\end{bmatrix} = V_1 + 2V_2 = 0$$

So all vectors $\bar{V}$ where $V_1 = -2V_2$ with arbitrary V_3, are orthogonal. hence the answer in the back

$$\begin{bmatrix}V_1\\-\tfrac{1}{2}V_1\\V_3\end{bmatrix} \quad V_1, V_3 \text{ arbitrary.}$$

For the following set of vectors, compute an orthogonal basis and determine the dimension of the vector space they span

$$\begin{bmatrix}1\\1\\1\end{bmatrix} \begin{bmatrix}1\\3\\5\end{bmatrix} \begin{bmatrix}1\\0\\0\end{bmatrix} \begin{bmatrix}2\\3\\5\end{bmatrix}$$

Let W_i be the ith basis vector:

I'll choose $W_1 = \begin{bmatrix}1\\1\\1\end{bmatrix}$

Then I find W_2 by subtracting the projection ~~Rede~~ onto W_1 by the next vector

$$W_2 = \begin{bmatrix}1\\3\\5\end{bmatrix} - \frac{\begin{bmatrix}1 & 3 & 5\end{bmatrix}\cdot\begin{bmatrix}1\\1\\1\end{bmatrix}}{\begin{bmatrix}1 & 1 & 1\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix}} \begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$\left[W_2 = V_2 - \frac{V_2^T \cdot W_1}{W_1^T W_1} W_1 \right]$$

* This is what I'm doing

$$W_2 = \begin{bmatrix}1\\3\\5\end{bmatrix} - \frac{9}{3}\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}-2\\0\\2\end{bmatrix}$$

Now repeat this idea with the next vector

$$W_3 = V_3 - \frac{V_3 \cdot W_2}{W_2 \cdot W_2} W_2 - \frac{V_3 \cdot W_1}{W_1 \cdot W_1} W_1$$

$$W_3 = \begin{bmatrix}1\\0\\0\end{bmatrix} - \frac{\begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}-2\\0\\2\end{bmatrix}}{\begin{bmatrix}-2 & 0 & 2\end{bmatrix}\begin{bmatrix}-2\\0\\2\end{bmatrix}} \begin{bmatrix}-2\\0\\2\end{bmatrix} - \frac{\begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix}}{\begin{bmatrix}1 & 1 & 1\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix}} \begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$W_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + \frac{+2}{8}\begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\left(\frac{1}{3}\right) = \begin{bmatrix} 1/6 \\ -1/3 \\ +1/6 \end{bmatrix}$$

$$W_3 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$ (Multiplying by 3 does not change direction, and thereby orthogonality)

So ~~the~~ a basis of vectors are

$$W_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad W_2 = \begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix} \quad W_3 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

Other vectors could be found by doing this with the vectors in a different order.

Check:

$$W_1 \cdot W_2 = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix} = -2+2=0 \checkmark$$

$$W_1 \cdot W_3 = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} = 1-2+1=0 \checkmark$$

$$W_3 \cdot W_2 = \begin{bmatrix} 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix} = -2+2=0 \checkmark$$

The vectors have 3 components, and 3 orthogonal basis vectors, so it must have dimension = $\underline{\underline{3}}$