ME564 | HW #3 | Solutions

6.2 # 1,2,4: For the vectors given, determine the products or explain why the product is undefined.

$$a = \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} \quad B = \begin{bmatrix} 2 & -3 \\ 0 & 2 \\ 0 & 1 \end{bmatrix} \quad c = \begin{bmatrix} 4 & 6 & 2 \\ 6 & 0 & 3 \\ 2 & 3 & -1 \end{bmatrix} \quad d = \begin{bmatrix} 4 & 3 & 0 \end{bmatrix}$$

**6.2.1** i) $Ba$ ii) $a^T B$ iii) $aB$

i) $Ba = \begin{bmatrix} 2 & -3 \\ 0 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} = (2\times 3) \times (1 \times 3) \Rightarrow \boxed{\text{undefined}}$

ii) $a^T B = \begin{bmatrix} 1 & 4 & 3 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 0 & 2 \\ 0 & 1 \end{bmatrix} = [1(2) + 4(0) + 3(0), \ 1(-3) + 4(2) + 3(1)]$

$$\boxed{a^T B = [2, 8]}$$

iii) $aB = \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 0 & 2 \\ 0 & 1 \end{bmatrix} = (1 \times 3)(2 \times 3) \Rightarrow \boxed{\text{undefined}}$

**6.2.2** i) $C^2$, ii) $C^T C$ iii) $CC^T$

$$C^2 = \begin{bmatrix} 4 & 6 & 2 \\ 6 & 0 & 3 \\ 2 & 3 & -1 \end{bmatrix} \begin{bmatrix} 4 & 6 & 2 \\ 6 & 0 & 3 \\ 2 & 3 & -1 \end{bmatrix}$$

$$C^2 = \begin{bmatrix} 16 + 36 + 4 & 24 + 0 + 6 & 8 + 18 - 2 \\ 24 + 0 + 6 & 36 + 0 + 9 & 12 + 0 - 3 \\ 8 + 18 - 2 & 12 + 0 - 3 & 4 + 9 + 1 \end{bmatrix}$$

$$C^2 = \begin{bmatrix} 56 & 30 & 24 \\ 30 & 45 & 9 \\ 24 & 9 & 14 \end{bmatrix} = C^T C = CC^T$$ because $C$ is a symmetric matrix.

**6.2.4** i) Ca ii) Cd iii) dC

i) $Ca = \begin{bmatrix} 4 & 6 & 2 \\ 6 & 0 & 3 \\ 2 & 3 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 + 6(4) + 2(3) \\ 6(1) + 0 + 3(3) \\ 2(1) + 3(4) - 1(3) \end{bmatrix} = \boxed{\begin{bmatrix} 34 \\ 15 \\ 41 \end{bmatrix}}$

ii) $Cd = \begin{bmatrix} 4 & 6 & 2 \\ 6 & 0 & 3 \\ 2 & 3 & -1 \end{bmatrix} \begin{bmatrix} 4 & 3 & 0 \end{bmatrix} = (3\times3)\ (1\times3)$
$=$ undefined

iii) $dC = \begin{bmatrix} 4 & 3 & 0 \end{bmatrix} \begin{bmatrix} 4 & 6 & 2 \\ 6 & 0 & 3 \\ 2 & 3 & -1 \end{bmatrix}$

$dC = [4(4) + 3(6) + 0,\ 4(6) + 3(0) + 0(3),\ 4(2) + 3(3) + 0]$

$\boxed{dC = [34 \quad 24 \quad 17]}$

**6.3.5** Solve the following system by Gauss elimination or indicate non-existance of solutions.

$$x + y - z = 9$$
$$8y + 6z = -6$$
$$-2x + 4y - 6z = 40$$

$\Rightarrow \begin{bmatrix} 1 & 1 & -1 \\ 0 & 8 & 6 \\ -2 & 4 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ -6 \\ 40 \end{bmatrix}$

augmented matrix is then

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 9 \\ 0 & 8 & 6 & -6 \\ -2 & 4 & -6 & 40 \end{array}\right] \xrightarrow{2R_1 + R_3 \to R_3} \begin{bmatrix} 1 & 1 & -1 & 9 \\ 0 & 8 & 6 & -6 \\ 0 & 6 & -8 & 58 \end{bmatrix} \longrightarrow$$

$$\xrightarrow{6R_2 - 8(R_3) \to R_3} \begin{bmatrix} 1 & 1 & -1 & 9 \\ 0 & 8 & 6 & -6 \\ 0 & 0 & 100 & -500 \end{bmatrix} \xrightarrow{R_3/100 \to R_3} \begin{bmatrix} 1 & 1 & -1 & 9 \\ 0 & 8 & 6 & -6 \\ 0 & 0 & 1 & -5 \end{bmatrix} \longrightarrow$$

$$\xrightarrow{R_2 - 6R_3 \to R_2} \begin{bmatrix} 1 & 1 & -1 & 9 \\ 0 & 8 & 0 & 24 \\ 0 & 0 & 1 & -5 \end{bmatrix} \xrightarrow{R_2/8 \to R_2} \begin{bmatrix} 1 & 1 & -1 & 9 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -5 \end{bmatrix} \longrightarrow$$

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$$\left[\begin{array}{cccc} 1 & 1 & -1 & 9 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -5 \end{array}\right] \xrightarrow[R_1' - R_2 \to R_1'']{R_1 + R_3 \to R_1'} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -5 \end{array}\right] \Rightarrow \boxed{\begin{array}{l} x = 1 \\ y = 3 \\ z = -5 \end{array}}$$

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**6.3.9** Use Gauss elimination to solve the system

$$\begin{aligned} 0x + 4y + 3z &= 8 \\ 2x + 0y - z &= 2 \\ 3x + 2y + 0z &= 5 \end{aligned} \Rightarrow \begin{bmatrix} 0 & 4 & 3 \\ 2 & 0 & -1 \\ 3 & 2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 2 \\ 5 \end{bmatrix}$$

Forming the augmented matrix

$$\begin{bmatrix} 0 & 4 & 3 & 8 \\ 2 & 0 & -1 & 2 \\ 3 & 2 & 0 & 5 \end{bmatrix} \xrightarrow[R_3 \to R_1']{R_1 \to R_3'} \begin{bmatrix} 3 & 2 & 0 & 5 \\ 2 & 0 & -1 & 2 \\ 0 & 4 & 3 & 8 \end{bmatrix} \xrightarrow{2R_1 - 3R_2 \to R_2'}$$

$$\begin{bmatrix} 3 & 2 & 0 & 5 \\ 0 & 4 & 3 & 4 \\ 0 & 4 & 3 & 8 \end{bmatrix} \xrightarrow{R_2 - R_3 \to R_3'} \begin{bmatrix} 3 & 2 & 0 & 5 \\ 0 & 4 & 3 & 4 \\ 0 & 0 & 0 & -4 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & 0 & 5 \\ 0 & 4 & 3 & 4 \\ 0 & 0 & 0 & -4 \end{bmatrix} \text{ implies } \underline{0 = -4}$$

$$\boxed{\text{No Solution}}$$

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**6.3.13** Use Gauss Elimination to solve the system

$$\begin{aligned} 0w + 5x + 5y - 10z &= 0 \\ 2w - 3x - 3y + 6z &= 2 \\ 4w + x + y - 2z &= 4 \end{aligned} \Rightarrow \begin{bmatrix} 0 & 5 & 5 & -10 \\ 2 & -3 & -3 & 6 \\ 4 & 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix}$$

Form the augmented matrix:

$$\begin{bmatrix} 0 & 5 & 5 & -10 & 0 \\ 2 & -3 & -3 & 6 & 2 \\ 4 & 1 & 1 & -2 & 4 \end{bmatrix} \xrightarrow[R_3 \to R_1']{R_1 \to R_3'} \left[\begin{array}{cccc|c} 4 & 1 & 1 & -2 & 4 \\ 2 & -3 & -3 & 6 & 2 \\ 0 & 5 & 5 & -10 & 0 \end{array}\right] \longrightarrow$$

$$\xrightarrow{R_1 - 2R_2 \to R_2'} \begin{bmatrix} 4 & 1 & 1 & -2 & 4 \\ 0 & 7 & 7 & -14 & 0 \\ 0 & 5 & 5 & -10 & 0 \end{bmatrix}$$

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6.3.13 continued

$$\begin{bmatrix} 4 & 1 & 1 & -2 & 4 \\ 0 & 7 & 7 & -14 & 0 \\ 0 & 5 & 5 & -10 & 0 \end{bmatrix} \xrightarrow[R_2/7 \to R_2']{7R_3 - 5R_2 \to R_3'} \begin{bmatrix} 4 & 1 & 1 & -2 & 4 \\ 0 & 1 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\xrightarrow{R_1 - R_2 \to R_1'} \begin{bmatrix} 4 & 0 & 0 & 0 & 4 \\ 0 & 1 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \to$$

This Gives The Following Set of Solutions

$$\boxed{W = 1 \;;\; x + y - 2z = 0}$$

## 6.4.7

Are The Vectors Linearly dependant or independant?

$[1\ 9\ 9\ 8] \quad [2\ 0\ 0\ 3] \quad [2\ 0\ 0\ 8]$

Test w/ Theorem (4) P 336

$$\begin{bmatrix} 1 & 9 & 9 & 8 \\ 2 & 0 & 0 & 3 \\ 2 & 0 & 0 & 8 \end{bmatrix} \xrightarrow[R_3 - R_2 \to R_3']{2R_1 - R_2 \to R_2'} \begin{bmatrix} 1 & 9 & 9 & 8 \\ 0 & 18 & 18 & 13 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

Rank = 3, 3 vectors $\therefore \Rightarrow$ Linearly independant.

## 6.4.12

Find The Rank of

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \xrightarrow[R_1 \to R_3']{R_3 \to R_1'} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \xrightarrow{R_2 - R_2 \to R_2'} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 1 & 1 \end{bmatrix}$$

$$\xrightarrow{R_3 - R_2 \to R_3} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & +2 \end{bmatrix} \to \text{Rank } 3, \ (3 \text{ independant Vectors.})$$

## 6.4.13

Determine the Rank by the method in the text or by inspection

$$\text{Rank}\begin{bmatrix} 3 & -1 & 5 \\ 2 & -4 & 6 \\ 10 & 0 & 14 \end{bmatrix} \quad \text{Test by Theorem 1}$$

$$\begin{bmatrix} 3 & -1 & 5 \\ 2 & -4 & 6 \\ 10 & 0 & 14 \end{bmatrix} \xrightarrow[10(R_1) - 3(R_3) \to R_3']{2(R_1) - 3(R_2) \to R_2'} \begin{bmatrix} 3 & -1 & 5 \\ 0 & 10 & -8 \\ 0 & -10 & 8 \end{bmatrix} \xrightarrow{R_2 + R_3 \to R_3'}$$

$$\begin{bmatrix} 3 & -1 & 5 \\ 0 & 10 & -8 \\ 0 & 0 & 0 \end{bmatrix} \longrightarrow \boxed{2 \text{ independant vectors} \therefore \text{Rank } 2}$$

## 6.6 #19

Solve by Cramers Rule, and check with Gauss elimination

$$\begin{aligned} 3x + 7y + 8z &= -13 \\ 2x \quad\quad + 9z &= -5 \\ -4x + y - 26z &= 2 \end{aligned} \Longrightarrow \begin{bmatrix} 3 & 7 & 8 \\ 2 & 0 & 9 \\ -4 & 1 & -26 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -13 \\ -5 \\ 2 \end{bmatrix}$$

$$\underline{A}\,\underline{x} = \underline{b}$$

Det A will be a useful quantity, so I'll calculate that first.

$$\text{Det}[A] = \text{Det}\begin{vmatrix} 3 & 7 & 8 \\ 2 & 0 & 9 \\ -4 & 1 & -26 \end{vmatrix} \Rightarrow \text{using the 2nd row}$$

$$\begin{vmatrix} 3 & 7 & 8 \\ 2 & 0 & 9 \\ -4 & 1 & -26 \end{vmatrix} = -2\begin{vmatrix} 7 & 8 \\ 1 & -26 \end{vmatrix} + 0 - 9\begin{vmatrix} 3 & 7 \\ -4 & 1 \end{vmatrix}$$

$$|A| = -2\left((7)(-26) - (1)(8)\right) - 9\left[3(1) - 7(-4)\right]$$

$$|A| = 101$$

Now get x, y, and z by replacing the appropriate column, (cramer's Rule)

$$X = \frac{\begin{vmatrix} -13 & 7 & 8 \\ -5 & 0 & 9 \\ 2 & 1 & -26 \end{vmatrix}}{\begin{vmatrix} 3 & 7 & 8 \\ 2 & 0 & 9 \\ -4 & 1 & -26 \end{vmatrix}} = \frac{-7\begin{vmatrix} -5 & 9 \\ 2 & -26 \end{vmatrix} + 0 - 1\begin{vmatrix} -13 & 7 \\ 2 & -26 \end{vmatrix}}{101}$$

$$X = \frac{-7\left[(-5)(-26) - (9)(2)\right] - 1\left[(-13)(-26) - (2)(7)\right]}{101}$$

$$\boxed{X = \frac{-707}{101} = -7}$$

$$y = \frac{\begin{vmatrix} 3 & -13 & 8 \\ 2 & -5 & 9 \\ 4 & 2 & -26 \end{vmatrix}}{\begin{vmatrix} 3 & 7 & 8 \\ 2 & 0 & 9 \\ 4 & 1 & -26 \end{vmatrix}} = \frac{3\begin{vmatrix} -5 & 9 \\ 2 & -26 \end{vmatrix} - 2\begin{vmatrix} -13 & 8 \\ 2 & -26 \end{vmatrix} + 4\begin{vmatrix} -13 & 8 \\ -5 & 9 \end{vmatrix}}{101}$$

$$y = \frac{3\left[-5(-26) - 9(2)\right] - 2\left[(-13)(-26) - 8(2)\right] + 4\left[(-13)(9) - (-5)(8)\right]}{101}$$

$$\boxed{y = 0}$$

$$z = \frac{\begin{vmatrix} 3 & 7 & -13 \\ 2 & 0 & -5 \\ 4 & 1 & 2 \end{vmatrix}}{101} = \frac{-7\begin{vmatrix} 2 & -5 \\ 4 & 2 \end{vmatrix} - 1\begin{vmatrix} 3 & -13 \\ 2 & -5 \end{vmatrix}}{101}$$

$$z = \frac{-7\left[2(2) - (-5)(4)\right] - 1\left[3(-5) - (-13)(2)\right]}{101}$$

$$\boxed{z = \frac{101}{101} = 1}$$

**6.7.1** Compute the inverse by elimination, and compare to the transpose of the cofactor matrix.

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

Augment with the 3 vectors of the identity matrix

$$\left[\begin{array}{ccc|ccc} 2 & 0 & -1 & 1 & 0 & 0 \\ 5 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 3 & 0 & 0 & 1 \end{array}\right] \xrightarrow{2R_2 - 5R_1 \to R_2'}$$

$$\left[\begin{array}{ccc|ccc} 2 & 0 & -1 & 1 & 0 & 0 \\ 0 & 2 & 5 & -5 & 2 & 0 \\ 0 & 1 & 3 & 0 & 0 & 1 \end{array}\right] \xrightarrow{2R_3 - R_2 \to R_3'}$$

$$\left[\begin{array}{ccc|ccc} 2 & 0 & -1 & 1 & 0 & 0 \\ 0 & 2 & 5 & -5 & 2 & 0 \\ 0 & 0 & 1 & 5 & -2 & 2 \end{array}\right] \xrightarrow[R_3 + R_1 \to R_1']{R_2 - 5R_3 \to R_2'}$$

$$\left[\begin{array}{ccc|ccc} 2 & 0 & 0 & 6 & -2 & 2 \\ 0 & 2 & 0 & -30 & 12 & -10 \\ 0 & 0 & 1 & 5 & -2 & 2 \end{array}\right] \xrightarrow[R_2/2 \to R_2']{R_1/2 \to R_1'}$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -1 & 1 \\ 0 & 1 & 0 & -15 & 6 & -5 \\ 0 & 0 & 1 & 5 & -2 & 2 \end{array}\right]$$

$$A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

Now compare to the cofactor matrix

See Next Page ...

6.7.1 continued: Comparing the inverse of the given matrix, to the transpose of the cofactor matrix:

$$C = \begin{bmatrix} \begin{vmatrix} 1 & 0 \\ 1 & 3 \end{vmatrix} & -\begin{vmatrix} 5 & 0 \\ 0 & 3 \end{vmatrix} & \begin{vmatrix} 5 & 1 \\ 0 & 1 \end{vmatrix} \\ -\begin{vmatrix} 0 & -1 \\ 1 & 3 \end{vmatrix} & \begin{vmatrix} 2 & -1 \\ 0 & 3 \end{vmatrix} & -\begin{vmatrix} 2 & 0 \\ 0 & 1 \end{vmatrix} \\ \begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix} & -\begin{vmatrix} 2 & -1 \\ 5 & 0 \end{vmatrix} & \begin{vmatrix} 2 & 0 \\ 5 & 1 \end{vmatrix} \end{bmatrix}$$

$$C = \begin{bmatrix} 3 & -15 & 5 \\ -1 & 6 & -2 \\ 1 & -5 & 2 \end{bmatrix} \therefore C^T = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$C^T = A^{-1}$; Neato!

**6.7.3** Compute the inverse by elimination, Compare to the Transpose of the Cofactor matrix.

$$A = \begin{bmatrix} 3 & -1 & 5 \\ 2 & 6 & 4 \\ 5 & 5 & 9 \end{bmatrix}$$ Augmenting $[A \,\vdots\, I]$ and Solving

$$\left[\begin{array}{ccc|ccc} 3 & -1 & 5 & 1 & 0 & 0 \\ 2 & 6 & 4 & 0 & 1 & 0 \\ 5 & 5 & 9 & 0 & 0 & 1 \end{array}\right] \xrightarrow[3R_3 - 5R_1 \to R_3']{2R_1 - 3R_2 \to R_2'}$$

$$\left[\begin{array}{ccc|ccc} 3 & -1 & 5 & 1 & 0 & 0 \\ 0 & -20 & -2 & 2 & -3 & 0 \\ 0 & 20 & -2 & -5 & 0 & 3 \end{array}\right] \xrightarrow{R_2 + R_3 \to R_3}$$

$$\left[\begin{array}{ccc|ccc} 3 & -1 & 5 & 1 & 0 & 0 \\ 0 & -20 & -2 & 2 & -3 & 0 \\ 0 & 0 & 0 & -3 & -3 & 3 \end{array}\right]$$

No Solution

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**6.7.5** Same instructions as #1, 3

$$A = \begin{bmatrix} 4 & -1 & -5 \\ 15 & 1 & -5 \\ 5 & 4 & 9 \end{bmatrix}$$ Augmenting and Reducing

$$\left[\begin{array}{ccc|ccc} 4 & -1 & -5 & 1 & 0 & 0 \\ 15 & 1 & -5 & 0 & 1 & 0 \\ 5 & 4 & 9 & 0 & 0 & 1 \end{array}\right] \xrightarrow[4(R_3) - 5(R_1) \to R_3']{4(R_2) - 15(R_1) \to R_2'}$$

$$\left[\begin{array}{ccc|ccc} 4 & -1 & -5 & 1 & 0 & 0 \\ 0 & 19 & 55 & -15 & 4 & 0 \\ 0 & 21 & 61 & -5 & 0 & 4 \end{array}\right] \xrightarrow{19R_3 - 21R_2 \to R_3'}$$

$$\left[\begin{array}{ccc|ccc} 4 & -1 & -5 & 1 & 0 & 0 \\ 0 & 19 & 55 & -15 & 4 & 0 \\ 0 & 0 & 4 & 220 & \cancel{\phantom{0}} & 76 \end{array}\right] \xrightarrow{R_3/4 \to R_3'}$$

$-84$

$$\begin{bmatrix} 4 & -1 & -5 & 1 & 0 & 0 \\ 0 & 19 & 55 & -15 & 4 & 0 \\ 0 & 0 & 1 & 55 & -21 & 19 \end{bmatrix} \xrightarrow[R_1 + 5R_3 \rightarrow R_1']{R_2 - 55R_3 \rightarrow R_2'}$$

$$\begin{bmatrix} 4 & -1 & 0 & 270 & -105 & 95 \\ 0 & 19 & 0 & -3040 & 1159 & -1045 \\ 0 & 0 & 1 & 55 & -21 & 19 \end{bmatrix} \xrightarrow[R_1 + R_2' \rightarrow R_1']{R_2/19 \rightarrow R_2'}$$

$$\begin{bmatrix} 4 & 0 & 0 & 116 & -44 & 40 \\ 0 & 1 & 0 & -160 & 61 & -55 \\ 0 & 0 & 1 & 55 & -21 & 19 \end{bmatrix} \xrightarrow{R_1/4 \rightarrow R_1'}$$

$$\begin{bmatrix} 1 & 0 & 0 & 29 & -11 & 10 \\ 0 & 1 & 0 & -160 & 61 & -55 \\ 0 & 0 & 1 & 55 & -21 & 19 \end{bmatrix} \Rightarrow A^{-1} = \begin{bmatrix} 29 & -11 & 10 \\ -160 & 61 & -55 \\ 55 & -21 & 19 \end{bmatrix}$$

Now Compare to the Cofactor matrix.

$$C = \begin{bmatrix} \begin{vmatrix} 1 & -5 \\ 4 & 9 \end{vmatrix} & -\begin{vmatrix} 15 & -5 \\ 5 & 9 \end{vmatrix} & \begin{vmatrix} 15 & 1 \\ 5 & 4 \end{vmatrix} \\ -\begin{vmatrix} -1 & -5 \\ 4 & 9 \end{vmatrix} & \begin{vmatrix} 4 & -5 \\ 5 & 9 \end{vmatrix} & -\begin{vmatrix} 4 & -1 \\ 5 & 4 \end{vmatrix} \\ \begin{vmatrix} -1 & -5 \\ 1 & -5 \end{vmatrix} & -\begin{vmatrix} 4 & -5 \\ 15 & -5 \end{vmatrix} & \begin{vmatrix} 4 & -1 \\ 15 & 1 \end{vmatrix} \end{bmatrix}$$

$$C = \begin{bmatrix} 29 & -160 & 55 \\ -11 & 61 & -21 \\ 10 & -55 & 19 \end{bmatrix} = (A^{-1})^T$$

$$C^T = \begin{bmatrix} 29 & -11 & 10 \\ -160 & 61 & -55 \\ 55 & -21 & 19 \end{bmatrix} = A^{-1}$$