Text book Problem
1.3.16: Solve The IVP:

$$\frac{\partial y}{\partial x} = 1 + 4y^2 \qquad y(0) = 0$$

separate integrate $\displaystyle\int \frac{dy}{1+4y^2} = \int dx \Rightarrow$ Tricks for integration Required.

Trick Let $2y = z$ Then $dy = \frac{dz}{2}$

$$\int \frac{dz}{2(1+z^2)} = \int dx \qquad \text{Table has } \int \frac{dx}{a^2+x^2}$$

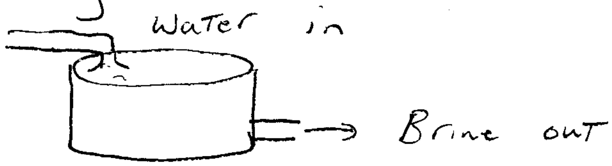
end Trick $\arctan z = 2x + c \Rightarrow 2y = \tan(2x+c)$

$y = \frac{1}{2}\tan(2x+c)$ apply BC.

$0 = \frac{1}{2}\tan(0+c) \qquad c = 0, \pi, 2\pi \ldots$

$$\boxed{y = \frac{1}{2}\tan(2x + n\pi) \quad n \text{ any integer}}$$

Mixing Problem (A Classic!) P 1.4.14




Tank has 2gal/min fresh water pumped in
2 gal/min mixture pumped out

initially has 400 gallons with 100 lb of brine (IC!)

Let $y$ be ~~concentration~~ Mass of Salt at time $t$.

Then clearly, the concentration is $\frac{y}{V} = \frac{y}{400\text{gal}}$

This must be leaving at the outflow rate, and nothing is coming in, so the model is

$$\frac{dy}{dt} = \dot{V}\frac{y}{400\text{gal}} = \frac{2\text{gal}}{\text{min}}\left(\frac{y}{400\text{gal}}\right) = \frac{y}{200} \frac{\text{kg}}{\text{min}}$$

$$\frac{dy}{dt} = \frac{y}{200} \underset{\text{integrate}}{\overset{\text{separate}}{\Rightarrow}} \int\frac{dy}{y} = \int\frac{dt}{200}$$

$$\ln y = \frac{t}{200} + \tilde{c} \qquad \tilde{c} = \ln c$$

$$y = ce^{t/200} \qquad \text{Apply I.C.}$$

$$y(0) = 100 = ce^{0/200} = c \Rightarrow c = 100 \text{ kg}$$

$$y = 100\, e^{t/200}$$

$$y(60) = 100\, e^{60/200} = \boxed{74 \text{ lb}}$$

Exact eqns Problem 1

Find general Solution for

$$(1+2xy^2)\,dx + (1+2x^2y)\,dy = 0$$

exact? $M = 1+2xy^2$

$N = 1+2x^2y$

$$\left.\begin{aligned} \frac{\partial M}{\partial y} &= 4xy \\ \frac{\partial N}{\partial x} &= 4xy \end{aligned}\right\} \text{yep, moving on}$$

integrate WRT $x$

$$u = \int (1+2xy^2)\,dx + f(y) = x + x^2y^2 + f(y)$$

Now differentiate WRT $y$

$$\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left[x + x^2y^2 + f(y)\right] = 2x^2y + \frac{\partial f}{\partial y}$$

Looking at $N$, we see $\frac{\partial f}{\partial y} = 1$

So $f = \int dy = y$

So general Solution is

$$\boxed{C = x + y + x^2y^2}$$

exactifying equations

Solve $0 = 3y^2\cos y\, dx + (2xy\cos y - xy^2\sin y)dy$

$$P = 3y^2\cos y$$
$$Q = 2xy\cos y - xy^2\sin y$$

$$\frac{\partial P}{dy} = 6y\cos y - 3y^2\sin y$$

$$\frac{\partial Q}{dx} = 2y\cos y - y^2\sin y$$

Apply Theorem 1 Page 30

$$F = \exp\int R(x)\,dx$$

where $R(x) = \frac{1}{Q}\left(\frac{\partial P}{\partial y} - \frac{\partial Q}{\partial x}\right)$

$$R(x) = \frac{1}{2xy\cos y - xy^2\sin y}\left(6y\cos y - 3y^2\sin y - \underbrace{(2y\cos y - y^2\sin y)}_{(2y\cos y - y^2\sin y)}\right)$$

$$R(x) = \left(\frac{1}{x}\right)\left(\frac{1}{2y\cancel{\cos y - y^2\sin y}}\right)\left(4y\cancel{\cos y - 2y^2\sin y}\right)$$

$$R(x) = \frac{2}{x} \Rightarrow \int R(x)\,dx = \int \frac{2}{x}dx = 2\ln x$$

$$F = \exp 2\ln(x) = x^2$$

Put that back in and integrate.

Exact eqn example 2 continued.

Now The ODE is

$$0 = 3x^2y^2 \cos y \, dx + (2x^3y\cos y - x^3y^2 \sin y)\, dy$$

$$u = \int 3x^2y^2 \cos y \, dx + f(y)$$

$$= x^3y^2\cos y + f(y)$$

differentiate and compare to the y part $(Q(x,y))$

$$\frac{du}{dy} = 2x^3y\cos y - x^3y^2\sin y + \frac{\partial f}{\partial y}$$

inspection shows $\frac{\partial f}{\partial y} = 0$ so $f = \text{const}$

General Solution is

$$\boxed{C = x^3y^2\cos y}$$