1. Demonstrate that the following vectors are linearly dependent: \[
\begin{bmatrix}
1 \\
1 \\
1
\end{bmatrix}, \begin{bmatrix}
1 \\
2 \\
3
\end{bmatrix}, \begin{bmatrix}
2 \\
1 \\
0
\end{bmatrix}
\]
Obtain an orthonormal basis for the vector space they span and determine its dimension. (15)

2. Determine all solutions of the following linear systems using Gaussian elimination. (20)

\[
A = \begin{bmatrix}
1 & 1 & 2 \\
1 & 2 & 1 \\
1 & 3 & 0
\end{bmatrix}, \quad B = \begin{bmatrix}
1 & 1 & 2 \\
1 & 2 & 1 \\
1 & 3 & 1
\end{bmatrix}
\]
(Note that the matrices differ only in their lower right entry.)

\[
a) \quad A\bar{x} = \begin{bmatrix} 9 \\ 8 \\ 7 \end{bmatrix}, \quad b) \quad A\bar{x} = \begin{bmatrix} 9 \\ 8 \\ 10 \end{bmatrix}, \quad c) \quad B\bar{x} = \begin{bmatrix} 9 \\ 8 \\ 10 \end{bmatrix}
\]

3. Compute the determinants of the matrices \(A\) and \(B\) from Problem 2. (10)

4. Compute the eigenvalues and eigenvectors of matrix \(A\) from Problem 2. (15)

5. a) Using Cramer’s rule, solve for \(x_1\) as a product of determinants. (5)

\[
\begin{bmatrix}
a_{11} & a_{12} & a_{13} & a_{14} \\
a_{21} & a_{22} & a_{23} & a_{24} \\
a_{31} & a_{32} & a_{33} & a_{34} \\
a_{41} & a_{42} & a_{43} & a_{44}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{bmatrix}
= \begin{bmatrix}
1 \\
0 \\
0 \\
0
\end{bmatrix}
\]

b) Use the Laplace expansion to re-express \(x_1\) with the numerator given by a 3x3 determinant. (5)

6. For each differential equation, select an appropriate solution method (separability, exactness or integrating factor), explain why the method is applicable to the equation and find the solution that satisfies the given initial condition. (10 pts. ea.)

a) \(\frac{dy}{dx} - 2xy = 0; \quad y(0) = 3\)

b) \((x^3 + 1)\frac{dy}{dx} + 3x^2y = 2x; \quad y(2) = 1\)

c) \(x\frac{dy}{dx} + 2y = x + 2; \quad y(1) = 1\)