ME 564 HW # 3

1 - Use Gram-Schmidt orthogonalization process to obtain an orthonormal basis, determine the dimension of the space
use mathematica (or other computing resource) to check result.

a) $\{v_1, v_2, v_3\}^* = (\{1,0\}, \{1,1\}, \{2,1\})$

Let $\underline{w}_i$ be basis

$$\underline{w}_1 = \underline{v}_1 = \begin{bmatrix}1\\0\end{bmatrix}$$

$$\underline{w}_2 = \underline{v}_2 - \frac{\underline{v}_2 \cdot \underline{w}_1}{\underline{w}_1 \cdot \underline{w}_1}\underline{w}_1 = \begin{bmatrix}1\\1\end{bmatrix} - \frac{(1,1)\cdot(1,0)}{(1,0)\cdot(1,0)}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1\\1\end{bmatrix} - \frac{1\cdot 1 + 1\cdot 0}{1\cdot 1 + 0\cdot 0}\begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}1\\1\end{bmatrix} - \begin{bmatrix}1\\0\end{bmatrix} = \begin{bmatrix}0\\1\end{bmatrix}$$

$$\underline{w}_3 = \underline{v}_3 - \frac{\underline{v}_3 \cdot \underline{w}_1}{\underline{w}_1 \cdot \underline{w}_1}\underline{w}_1 - \frac{\underline{v}_3 \cdot \underline{w}_2}{\underline{w}_2 \cdot \underline{w}_2}\underline{w}_2 = \begin{bmatrix}2\\1\end{bmatrix} - \frac{2\cdot 1 + 1\cdot 0}{1}\begin{bmatrix}1\\0\end{bmatrix} - \frac{2\cdot 0 + 1\cdot 1}{1}\begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}2\\1\end{bmatrix} - \begin{bmatrix}2\\0\end{bmatrix} - \begin{bmatrix}0\\1\end{bmatrix} = \Big[$$

$\Rightarrow$ no contribution from $w_3$

Note: by inspection $\underline{v}_3 = \underline{v}_1 + \underline{v}_2 \Rightarrow$ not lin. indep.

Orthonormal basis are: $\left\{ \hat{w}_1 = \begin{bmatrix}1\\0\end{bmatrix}, \hat{w}_2 = \begin{bmatrix}0\\1\end{bmatrix} \right\}$ and dimension $= 2$

b) $(\underline{v}_1, \underline{v}_2, \underline{v}_3, \underline{v}_4) = (\{1,0,-1\}, \{1,1,1\}, \{3,2,1\}, \{3,4,5\})$

$$\underline{w}_1 = \underline{v}_1 = \begin{bmatrix}1\\0\\-1\end{bmatrix}$$

$$\underline{w}_2 = \underline{v}_2 - \frac{\underline{v}_2 \cdot \underline{w}_1}{\underline{w}_1 \cdot \underline{w}_1}\underline{w}_1 = \begin{bmatrix}1\\1\\1\end{bmatrix} - \frac{1\cdot 1 + 1\cdot 0 + 1\cdot(-1)}{1\cdot 1 + 0\cdot 0 - 1\cdot(-1)}\begin{bmatrix}1\\0\\-1\end{bmatrix} = \begin{bmatrix}1\\1\\1\end{bmatrix} - \cancelto{0}{\frac{0}{2}\begin{bmatrix}1\\0\\-1\end{bmatrix}} = \begin{bmatrix}1\\1\\1\end{bmatrix}$$

$$\underline{w}_3 = \underline{v}_3 - \frac{\underline{v}_3 \cdot \underline{w}_1}{\underline{w}_1 \cdot \underline{w}_1}\underline{w}_1 - \frac{\underline{v}_3 \cdot \underline{w}_2}{\underline{w}_2 \cdot \underline{w}_2}\underline{w}_2 = \begin{bmatrix}3\\2\\1\end{bmatrix} - \frac{3\cdot 1 + 2\cdot 0 + 1\cdot(-1)}{2}\begin{bmatrix}1\\0\\-1\end{bmatrix} - \frac{3+2+1}{1+1+1}\begin{bmatrix}1\\1\\1\end{bmatrix} =$$

$$= \begin{bmatrix}3\\2\\1\end{bmatrix} - \frac{2}{2}\begin{bmatrix}1\\0\\-1\end{bmatrix} - \frac{\cancelto{2}{6}}{\cancel{3}}\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\underline{w}_4 = \underline{v}_4 - \frac{\underline{v}_4 \cdot \underline{w}_1}{\underline{w}_1 \cdot \underline{w}_1}\underline{w}_1 - \frac{\underline{v}_4 \cdot \underline{w}_2}{\underline{w}_2 \cdot \underline{w}_2}\underline{w}_2 - \cancelto{0}{\frac{\underline{v}_4 \cdot \underline{w}_3}{\underline{w}_3 \cdot \underline{w}_3}}\underline{w}_3 =$$

$$= \begin{bmatrix}3\\4\\5\end{bmatrix} - \frac{3-5}{2}\begin{bmatrix}1\\0\\-1\end{bmatrix} - \frac{3+4+5}{3}\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}3\\4\\5\end{bmatrix} + \begin{bmatrix}1\\0\\-1\end{bmatrix} - 4\begin{bmatrix}1\\1\\1\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

orthonormal basis are $\left\{ \hat{w}_1 = \begin{bmatrix}\sqrt{2}/2\\0\\-\sqrt{2}/2\end{bmatrix}, \hat{w}_2 = \begin{bmatrix}\sqrt{3}/3\\\sqrt{3}/3\\\sqrt{3}/3\end{bmatrix} \right\}$

dimension $= 2$

Note that $v_3 = 2v_2 + v_1$ and $v_4 = 4v_2 - v_1$

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<< LinearAlgebra`Orthogonalization`
```

-------Prob 1 a -------

```
{w1, w2, w3} = GramSchmidt[{{0, 1}, {1, 1}, {2, 1}}]
```

```
GramSchmidt::zeromag :
 The magnitude of {0, 0} was zero under the inner product Dot.
   This suggests that the vectors are linearly dependent.
```

```
{{0, 1}, {1, 0}, {0, 0}}
```

-------Prob 1 b -------

```
{u1, u2, u3} =
 GramSchmidt[{{1, 0, -1}, {1, 1, 1}, {3, 2, 1}, {3, 4, 5}}]
```

```
GramSchmidt::zeromag :
 The magnitude of {0, 0, 0} was zero under the inner product
   Dot. This suggests that the vectors are linearly dependent.
```

Set::shape :
Lists {u1, u2, u3} and $\left\{\left\{\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right\}, \left\{\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right\}, \{0, 0, 0\}, \{0, 0, 0\}\right\}$ are not the same shape. More…

$$\left\{\left\{\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}}\right\}, \left\{\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right\}, \{0, 0, 0\}, \{0, 0, 0\}\right\}$$

-------Prob 7.7 .18 -------

```
m = {{2, -5}, {4, 6}}
```

```
{{2, -5}, {4, 6}}
```

```
m.{x, y} == {23, -2}
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$\{2x - 5y, 4x + 6y\} == \{23, -2\}$

```
Solve[%, {x, y}]
```

$\{\{x \to 4, y \to -3\}\}$

-------Prob 7.7 .19 -------

```
m = {{0, 3, 4}, {4, 2, -1}, {1, -1, 5}}
```

```
{{0, 3, 4}, {4, 2, -1}, {1, -1, 5}}

m.{x, y, z} == {14.8, -6.3, 13.5}

{3 y + 4 z, 4 x + 2 y - z, x - y + 5 z} == {14.8, -6.3, 13.5}

Solve[%, {x, y, z}]

{{x → -1.2, y → 0.8, z → 3.1}}
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**7.2.1** $\quad \underset{=}{A}\,\underset{\sim}{a}$ , $\underset{=}{A}\,\underset{\sim}{b}$ , $\underset{=}{A}\,\underset{\sim}{b}^T$ , $\underset{=}{A}\,\underset{=}{B}$

**Sol$^n$**

$$\underset{=}{A}\,\underset{\sim}{a} = \begin{bmatrix} 6 & -2 & -2 \\ 10 & -3 & 1 \\ -10 & 5 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 1 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 30-2-4 \\ 50-3+2 \\ -50+5+2 \end{bmatrix} = \begin{bmatrix} 24 \\ 49 \\ -43 \end{bmatrix} \qquad \text{Ans}$$

$\underset{=}{A}\,\underset{\sim}{b}$ is undefined since $\underset{=}{A}$ is a $3\times 3$ matrix $(m\times n = 3\times 3)$ but $\underset{\sim}{b}$ is a $1\times 3$ matrix (or $r\times p = 1\times 3$) and that $n \neq r$. Thus, matrix $\underset{=}{A}\,\underset{\sim}{b}$ is undefined (p. 279 in text). $\qquad$ Ans

$$\underset{=}{A}\,\underset{\sim}{b}^T = \begin{bmatrix} 6 & -2 & -2 \\ 10 & -3 & 1 \\ -10 & 5 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 0 \\ 8 \end{bmatrix} = \begin{bmatrix} 18-0-16 \\ 30-0+8 \\ -30+0+8 \end{bmatrix} = \begin{bmatrix} 2 \\ 38 \\ -22 \end{bmatrix} \qquad \text{Ans}$$

$$\underset{=}{A}\,\underset{=}{B} = \begin{bmatrix} 6 & -2 & -2 \\ 10 & -3 & 1 \\ -10 & 5 & 1 \end{bmatrix} \begin{bmatrix} 9 & 4 & -4 \\ 4 & 7 & 0 \\ -4 & 0 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} 54-8+8 & 24-14-0 & -24-0-22 \\ 90-12-4 & 40-21+0 & -40-0+11 \\ -90+20-4 & -40+35+0 & 40+0+11 \end{bmatrix}$$

$$= \begin{bmatrix} 54 & 10 & -46 \\ 74 & 19 & -29 \\ -74 & -5 & 51 \end{bmatrix} \qquad \text{Ans}$$

$\boxed{\begin{matrix} 7.2 \\ \#21 \end{matrix}}$

$U_1+U_2$ is upper triangular. eg. $\begin{bmatrix}1&1\\0&1\end{bmatrix}+\begin{bmatrix}1&2\\0&1\end{bmatrix}=\begin{bmatrix}2&3\\0&2\end{bmatrix}$

$U_1\cdot U_2$ is upper triangular. eg. $\begin{bmatrix}1&1&1\\0&1&1\\0&0&1\end{bmatrix}\cdot\begin{bmatrix}1&2&3\\0&1&2\\0&0&1\end{bmatrix}=\begin{bmatrix}1&3&6\\0&1&3\\0&0&1\end{bmatrix}$

$U_1^2$ is upper triangular eg. $\begin{bmatrix}1&1\\0&1\end{bmatrix}\begin{bmatrix}1&2\\0&1\end{bmatrix}=\begin{bmatrix}1&3\\0&1\end{bmatrix}$

$U_1+L_1$ is not triangular eg. $\begin{bmatrix}1&1\\0&1\end{bmatrix}+\begin{bmatrix}1&0\\1&1\end{bmatrix}=\begin{bmatrix}2&1\\1&2\end{bmatrix}$

$U_1L_1$ is not triangular eg. $\begin{bmatrix}1&1&1\\0&1&1\\0&0&1\end{bmatrix}\begin{bmatrix}1&0&0\\2&1&0\\3&2&1\end{bmatrix}=\begin{bmatrix}6&3&1\\5&3&1\\3&2&1\end{bmatrix}$

$L_1+L_2$ is lower triangular eg. $\begin{bmatrix}1&0\\1&1\end{bmatrix}+\begin{bmatrix}1&0\\2&1\end{bmatrix}=\begin{bmatrix}2&0\\3&2\end{bmatrix}$

$L_1L_2$ is lower triangular eg. $\begin{bmatrix}1&0&0\\1&1&0\\1&1&1\end{bmatrix}\begin{bmatrix}1&0&0\\2&1&0\\3&2&1\end{bmatrix}=\begin{bmatrix}1&0&0\\3&1&0\\6&3&1\end{bmatrix}$

$L_1^2$ is lower triangular eg. $\begin{bmatrix}1&0&0\\1&1&0\\1&1&1\end{bmatrix}\begin{bmatrix}1&0&0\\1&1&0\\1&1&1\end{bmatrix}=\begin{bmatrix}1&0&0\\2&1&0\\3&2&1\end{bmatrix}$

Use transposition: make $U_1=L_1^T$ $U_2=L_2^T$

$$\left.\begin{aligned} L_1+L_2 &= (U_1+U_2)^T \\ L_1\cdot L_2 &= U_1^T\cdot U_2^T = (U_2U_1)^T \\ L_1^2 &= (U_1^2)^T \end{aligned}\right\}\ \text{So they are all triangular}$$

Solve the following system or indicate the nonexistence of solutions (Show the details of your work.)

7.3.5

$$\begin{aligned} 0.8x + 1.2y - 0.6z &= -7.8 \\ 2.6x \qquad\quad + 1.7z &= 15.3 \\ 4.0x - 7.3y - 1.5z &= 1.1 \end{aligned}$$

Augmented Matrix $\underline{\underline{A}}$, let $R_i$ represents $i^{th}$ row

$$\left[\begin{array}{ccc|c} 0.8 & 1.2 & -0.6 & -7.8 \\ 2.6 & 0 & 1.7 & 15.3 \\ 4.0 & -7.3 & -1.5 & 1.1 \end{array}\right] \begin{array}{l} R_1 \\ \frac{0.8}{2.6}R_2 - R_1 \\ \frac{0.8}{4.0}R_3 - R_1 \end{array} \longrightarrow \left[\begin{array}{ccc|c} 0.8 & 1.2 & -0.6 & -7.8 \\ 0 & -1.2 & 1.12 & 12.51 \\ 0 & -2.66 & 0.3 & 8.02 \end{array}\right]$$

$$\begin{array}{r} R_1 \\ R_2 \\ \xrightarrow{\frac{-1.2}{-2.66}R_3 - R_2} \end{array} \left[\begin{array}{ccc|c} 0.8 & 1.2 & -0.6 & -7.8 \\ 0 & -1.2 & 1.12 & 12.51 \\ 0 & 0 & -0.99 & -8.89 \end{array}\right]$$

Back substitution

$$\left.\begin{aligned} -0.99z &= -8.89 \rightarrow z = 9 \\ -1.2y + 1.12z &= 12.51 \rightarrow y = \frac{12.51 - 1.12(9)}{-1.2} = -2 \\ 0.8x + 1.2y - 0.6z &= -7.8 \rightarrow x = \frac{-7.8 + 0.6(9) - 1.2(-2)}{0.8} = 0 \end{aligned}\right\} \underline{\underline{\text{Ans}}}$$

7.3.9

$$\begin{aligned} 4y + 4z &= 24 \\ 3x - 11y - 2z &= -6 \\ 6x - 17y + z &= 18 \end{aligned}$$

Sol$^n$ Gauss Elimination

Augmented Matrix $\underset{=}{A}$, let $R_i \equiv i^{th}$ Row

$$\left[\begin{array}{ccc|c} 0 & 4 & 4 & 24 \\ 3 & -11 & -2 & -6 \\ 6 & -17 & 1 & 18 \end{array}\right] \xrightarrow{\substack{\text{interchange} \\ R_1 \,\&\, R_2}} \begin{array}{c} \\ \end{array}\left[\begin{array}{ccc|c} 3 & -11 & -2 & -6 \\ 0 & 4 & 4 & 24 \\ 6 & -17 & 1 & 18 \end{array}\right]\begin{array}{l} R_1 \leftarrow \text{now} \\ R_2 \\ R_3 \end{array}$$

$$\xrightarrow{} \begin{array}{c} R_1 \\ R_2 \\ \frac{R_3}{2} - R_1 \end{array} \left[\begin{array}{cccc} 3 & -11 & -2 & -6 \\ 0 & 4 & 4 & 24 \\ 0 & 2.5 & 2.5 & 15 \end{array}\right] \xrightarrow{} \begin{array}{c} R_1 \\ R_2 \\ \frac{4}{2.5}R_3 - R_2 \end{array} \left[\begin{array}{ccc|c} 3 & -11 & -2 & -6 \\ 0 & 4 & 4 & 24 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Back substitution

$$4y + 4z = 24 \qquad y = \frac{24 - 4z}{4} = 6 - z$$

$$3x - 11y - 2z = -6 \qquad x = \frac{-6 + 2z + 11y}{3} = \frac{-6 + 2z + 11(6 - z)}{3}$$

$$= 20 - 3z$$

$\therefore$ $x = 20 - 3t$, $y = 6 - t$, $z = t$ ( arbitrary unknown) Ans

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7.3.16 is on the last page

Find the rank and a basis for the row space and for the column space

7.4.7 $\begin{bmatrix} 8 & 0 & 4 \\ 0 & 2 & 0 \\ 4 & 0 & 2 \\ 0 & 4 & 0 \end{bmatrix} = \underline{\underline{A}}$

Sol$^n$ Row-reduce $\underline{\underline{A}}$

$$\left[\begin{matrix} 8 & 0 & 4 \\ 0 & 2 & 0 \\ 4 & 0 & 2 \\ 0 & 4 & 0 \end{matrix}\right] \begin{matrix} R_1 \\ R_2 \\ 2R_3 - R_1 \\ \frac{R_4}{2} - R_2 \end{matrix} \longrightarrow \begin{bmatrix} 8 & 0 & 4 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Row-reduce $\underline{\underline{A}}^T$

$$\left[\begin{matrix} 8 & 0 & 4 & 0 \\ 0 & 2 & 0 & 4 \\ 4 & 0 & 2 & 0 \end{matrix}\right] \begin{matrix} R_1 \\ R_2 \\ 2R_3 - R_1 \end{matrix} \longrightarrow \begin{bmatrix} 8 & 0 & 4 & 0 \\ 0 & 2 & 0 & 4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\therefore$ The rank of $\underline{\underline{A}}$ is 2 and basis are

$[8\ 0\ 4]$ and $[0\ 2\ 0]$ for the row space and

$[8\ 0\ 4\ 0]^T$ and $[0\ 2\ 0\ 4]^T$ for the column space. $\Big\}$ Ans

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7.4.12 $\begin{bmatrix} 0 & 0 & -7 & 1 \\ 0 & 0 & 5 & 0 \\ -7 & 5 & 0 & 2 \\ 1 & 0 & 2 & 0 \end{bmatrix} = \underline{\underline{A}}$

Sol$^n$ Row-reduce $\underline{\underline{A}}$

$$\left[\begin{matrix} 0 & 0 & -7 & 1 \\ 0 & 0 & 5 & 0 \\ -7 & 5 & 0 & 2 \\ 1 & 0 & 2 & 0 \end{matrix}\right] \begin{matrix} \text{interchange} \\ R_1 \,\&\, R_3 \\ R_2 \,\&\, R_4 \end{matrix} \longrightarrow \left[\begin{matrix} -7 & 5 & 0 & 2 \\ 1 & 0 & 2 & 0 \\ 0 & 0 & -7 & 1 \\ 0 & 0 & 5 & 0 \end{matrix}\right] \begin{matrix} R_1 \\ 7R_2 + R_1 \\ R_3 \\ \frac{7}{5}R_4 + R_3 \end{matrix} \longrightarrow \begin{bmatrix} -7 & 5 & 0 & 2 \\ 0 & 5 & 14 & 2 \\ 0 & 0 & -7 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Row-reduce $A^T$

$$\begin{bmatrix} 0 & 0 & -7 & 1 \\ 0 & 0 & 5 & 0 \\ -7 & 5 & 0 & 2 \\ 1 & 0 & 2 & 0 \end{bmatrix} = A \quad \rightarrow \quad A^T = A$$

$\therefore$ Row-reduce $A^T$ will give the same result as that of $A$.

$\therefore$ The rank is 4 and basis are

$\begin{bmatrix} -7 & 5 & 0 & 2 \end{bmatrix}$, $\begin{bmatrix} 0 & 5 & 14 & 2 \end{bmatrix}$, $\begin{bmatrix} 0 & 0 & -7 & 1 \end{bmatrix}$ and $\begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$ for the row space and

$\begin{bmatrix} -7 & 5 & 0 & 2 \end{bmatrix}^T$, $\begin{bmatrix} 0 & 5 & 14 & 2 \end{bmatrix}^T$, $\begin{bmatrix} 0 & 0 & -7 & 1 \end{bmatrix}^T$ and $\begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T$ for the column space.

Ans

**7.4.23**

Rank A = Rank B does not imply Rank $(A^2)$ = Rank $(B^2)$

$$A=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow A^2=\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow \text{Rank } A = \text{Rank }(A^2) = 2$$

$$B=\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \Rightarrow B^2=\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}=\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \text{ Rank } B = 2 = \text{Rank } A \text{ while Rank }(B^2) = 1 \neq \text{Rank }(A^2) = 2$$

$\boxed{7.3 \ \#16}$

$$\left[\begin{array}{cccc|c} -2 & -17 & 4 & 3 & 0 \\ 7 & 0 & 3 & -2 & 0 \\ 0 & 2 & 8 & -6 & -20 \\ 5 & -13 & -1 & 5 & 16 \end{array}\right] \xrightarrow[\substack{2+\frac{7}{2}\times 1 \\ 4+\frac{5}{2}\times 1}]{\substack{1 \\ 3/2}} \left[\begin{array}{cccc|c} -2 & -17 & 4 & 3 & 0 \\ 0 & 1 & 4 & -3 & -10 \\ 0 & -\frac{7\times 17}{2} & \frac{34}{2} & \frac{17}{2} & 0 \\ 0 & -\frac{111}{2} & 9 & \frac{25}{2} & 16 \end{array}\right]$$

$$\xrightarrow[\substack{3\times\frac{2}{17} \\ 4}]{\substack{1 \\ 2}} \left[\begin{array}{cccc|c} -2 & -17 & 4 & 3 & 0 \\ 0 & 1 & 4 & -3 & -10 \\ 0 & -7 & 2 & 1 & 0 \\ 0 & -\frac{111}{2} & 9 & \frac{25}{2} & 16 \end{array}\right] \xrightarrow[\substack{3+7\times 2 \\ 4+\frac{111}{2}\times 2}]{\substack{1 \\ 2}} \left[\begin{array}{cccc|c} -2 & -17 & 4 & 3 & 0 \\ 0 & 1 & 4 & -3 & -10 \\ 0 & 0 & 30 & -20 & -70 \\ 0 & 0 & 231 & -154 & -539 \end{array}\right]$$

$$\xrightarrow[\substack{3/10 \\ 4/7}]{\substack{1 \\ 2}} \left[\begin{array}{cccc|c} -2 & -17 & 4 & 3 & 0 \\ 0 & 1 & 4 & -3 & -10 \\ 0 & 0 & 3 & -2 & -7 \\ 0 & 0 & 33 & -22 & -77 \end{array}\right] \xrightarrow[\substack{3 \\ 4-11\times 3}]{\substack{1 \\ 2}} \left[\begin{array}{cccc|c} -2 & -17 & 4 & 3 & 0 \\ 0 & 1 & 4 & -3 & -10 \\ 0 & 0 & 3 & -2 & -7 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$\therefore$ So there are $\infty$ solutions.