Again on this assignment, you should employ computing resources and include plots to communicate your results.

1. Section 12.5 # 33. Solve for a general temperature distribution on the right edge, i.e. \( u(a, y) = f(y) \), and also find the solution for the particular case \( f(y) = 20 \).

2. Find the steady state temperature in a solid hemisphere 
   \( (0 \leq r \leq b, 0 \leq \varphi \leq \pi / 2, 0 \leq \vartheta \leq 2\pi) \) if the planar base is held at zero temperature while the hemispherical portion is kept at constant temperature 100.

3. Find the steady-state temperature distribution in a spherical shell \( (a \leq r \leq b) \) if the inner surface is held at zero temperature while the outer surface has temperature distribution \( u(b, \varphi) = \varphi(\pi - \varphi) \).

4. Section 12.8 # 11, 13

5. Section 12.9 # 6, 15

6. Determine the vibration of a semi-circular membrane \( (0 \leq r \leq b, 0 \leq \vartheta \leq \pi) \) with initial conditions \( u(r, \vartheta, 0) = \vartheta(\pi - \vartheta)\sin\left(\frac{\vartheta}{a}\right) \), \( u_r(r, \vartheta, 0) = 0 \). Determine mode shapes, nodal lines and natural frequencies for at least the first 5 modes. Create an animation of the membrane in motion and turning a graphic showing some frames from the animation (e.g. using GraphicsArray).