

- Your completed exam must be submitted by **2PM Wednesday March 4**.
- Please put in enough time to do a reasonable job, but please do not spend 6 days on this!
- For all questions, you can work the problems by hand, but **use your computing skills to produce graphics** that give meaning to your results. Also, write a few sentences to interpret your plots and explain why they look reasonable (or not).
- Note that presentation counts. What you hand in should present the essentials of your methods and results while being both **clear and concise**.
- **If you have questions, contact me or Elisabetta, not anyone else.** You are not allowed to discuss this exam with anyone else (including your classmates). If you use any other sources, they must be fully cited and distinguished from your own work.
- Please make sure that your work is independent (which includes making sure that you do not leave your computer files where they are accessible by others.)

1. (30 pts.) Consider two rods of identical material and cross section. One of the rods has length D and has been stored in an ice bath (at 0°). The other rod has length $2D$ and has been stored in 100° boiling water. At time $t = 0$, the two rods are brought into end-to-end contact with the remaining boundaries fully insulated. Solve the heat equation to determine how the temperature evolves over time in the resulting rod of length $3D$.

2. (30 pts.) a) A uniform solid cube has 5 faces held at constant temperature $u = 0$ and the sixth face is held at constant temperature $u = 100$. Determine the steady-state temperature in the cube; i.e. solve the three-dimensional Laplace's equation on $0 < x < 1$, $0 < y < 1$, $0 < z < 1$ with boundary conditions $u(0, y, z) = u(1, y, z) = u(x, 0, z) = u(x, 1, z) = u(x, y, 0) = 0$, and $u(x, y, 1) = 100$. Convey your solution by plotting the temperature distribution along slices through the cube. What is the temperature at the center of the cube? Does that value make sense?

b) If you had a die (singular of dice) with each side held at a temperature corresponding to the number of pips on the face, how would you determine the steady-state temperature distribution? (Do not actually do this, just write a few sentences describing how to do it.) What would the temperature be at the center of the die? (Determine the numerical result.)

3. (40 pts.) A uniform cylinder has radius $\rho = 1$ and height $h = 4$ sits upright atop the origin of the horizontal x-y plane. The cylinder has initial temperature $u = 0$. Starting at time $t = 0$, the top face of the cylinder is held at constant temperature $u(r, \theta, 4) = 100$ while the rest of the surface is held at temperature $u = 0$. Solve the heat equation in cylindrical coordinates to determine how the temperature in the cylinder evolves.

Extra credit: If the preceding questions took you more than about 3 hours, please do not attempt this extra credit question!

Determine the first few mode shapes and frequencies (eigenfunctions and eigenvalues) of an elliptical drum by solving the wave equation with the two-dimensional Laplacian expressed in elliptical polar coordinates. (See, for example, <http://mathworld.wolfram.com/EllipticCylindricalCoordinates.html> for details on elliptic cylindrical coordinates.) Take $a = 1$ and let the boundary correspond to $u = 1$, and find solutions of $y_t = c^2 \nabla^2 y$ with boundary condition $y(1, v, t) = 0$.