Kinematics and Dynamics
ME 230
Homework #1
Week 2

1.1 Why are the time derivatives of $\mathbf{E}_x$,$\mathbf{E}_y$ and $\mathbf{E}_z$ all equal to zero?

The unit vectors $\mathbf{E}_x$,$\mathbf{E}_y$ and $\mathbf{E}_z$ define the directions of the axes of the Cartesian coordinate system, and are fixed with respect to the origin chosen and with each other. Thus, their time rate of change is zero.

$$\mathbf{E}_x = 1.\mathbf{E}_x + 0.\mathbf{E}_y + 0.\mathbf{E}_z \Rightarrow \frac{d\mathbf{E}_x}{dt} = 0.\mathbf{E}_x + 0.\mathbf{E}_y + 0.\mathbf{E}_z = 0$$
$$\mathbf{E}_y = 0.\mathbf{E}_x + 1.\mathbf{E}_y + 0.\mathbf{E}_z \Rightarrow \frac{d\mathbf{E}_y}{dt} = 0.\mathbf{E}_x + 0.\mathbf{E}_y + 0.\mathbf{E}_z = 0$$
$$\mathbf{E}_z = 0.\mathbf{E}_x + 0.\mathbf{E}_y + 1.\mathbf{E}_z \Rightarrow \frac{d\mathbf{E}_z}{dt} = 0.\mathbf{E}_x + 0.\mathbf{E}_y + 0.\mathbf{E}_z = 0$$

■

1.5 The motion of a particle is such that its position vector is

$$\mathbf{r}(t) = 3t\mathbf{E}_x + 4t\mathbf{E}_y + 10\mathbf{E}_z \text{ (meters).}$$

Show that the path of the particle is a straight line and that the particle moves along this line at a constant speed of 5 meters per second. Using this information, show that the arc-length parameter s is given by

$$s(t) = 5(t - t_0) + s_0.$$

Finally, show that the particle moves 50 meters along its path every 10 seconds.

Given, the position vector of the particle at any instant of time t,

$$\mathbf{r}(t) = 3t\mathbf{E}_x + 4t\mathbf{E}_y + 10\mathbf{E}_z \text{ (meters).} \tag{1}$$

Thus, its instantaneous velocity is given by,

$$\mathbf{v}(t) = \frac{d\mathbf{r}}{dt}(t) = 3\mathbf{E}_x + 4\mathbf{E}_y + 0\mathbf{E}_z \text{ (meters/second)}$$

The speed of the particle is given by,

$$v = \frac{ds}{dt} = \|\mathbf{v}\| = \sqrt{\mathbf{v} \cdot \mathbf{v}} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5 \text{ (meters/second)} \tag{a}$$

And, from the above equation,

$$\frac{ds}{dt} = 5$$

Integrating both sides we have,

$$\Rightarrow \int_{s_0}^{s} ds = \int_{t_0}^{t} 5\, dt$$

$$\Rightarrow s(t) = 5(t - t_0) + s_0 \tag{b}$$

For every 10 seconds, i.e $t - t_0 = 10$, the distance moved by the particle along its path is given by,

$$\Delta s = s - s_0 = 5(t - t_0) = 5 \times 10 = 50 \text{ (meters)} \tag{c}$$

■

1.8 A projectile is launched at time $t_0 = 0$ seconds from a location $\mathbf{r}(t_0) = \mathbf{0}$. The initial velocity of the projectile is $\mathbf{v}(t_0) = v_0 \cos(\alpha)\mathbf{E}_x + v_0 \sin(\alpha)\mathbf{E}_y$. Here, v_0 and α are constants. During its flight, a vertical gravitational force $-mg\mathbf{E}_y$ acts on the projectile. Modeling the projectile as a particle of mass m, show that its path is a parabola:

$$y(x) = -\left(\frac{g}{2v_0^2 \cos^2(\alpha)}\right) x^2 + \tan(\alpha)x.$$

Why is this result not valid when $\alpha = \pm\pi/2$?

At any time t, the acceleration of the particle is given by,

$$\mathbf{a}(t) = \frac{d\mathbf{v}}{dt}(t)$$

Now, during the flight, the net force acting on the particle is given by,

$$\mathbf{F}_net(t) = m\mathbf{a}(t) = -mg\mathbf{E}_y$$

$$\Rightarrow \mathbf{a}(t) = \frac{d\mathbf{v}}{dt}(t) = -g\mathbf{E}_y$$

Integrating both sides we have,

$$\Rightarrow \mathbf{v}(t) - \mathbf{v}(t_0) = -g(t - t_0)\mathbf{E}_y$$

$$\Rightarrow \mathbf{v}(t) = v_0 \cos(\alpha)\mathbf{E}_x + (v_0 \sin(\alpha) - gt)\mathbf{E}_y$$

Integrating both sides to get the position vector at any time t during the flight we have,

$$\mathbf{r}(t) - \mathbf{r}(t_0) = v_0 \cos(\alpha)(t - t_0)\mathbf{E}_x + \left(v_0 \sin(\alpha)(t - t_0) - \frac{g(t^2 - t_0^2)}{2}\right)\mathbf{E}_y$$

$$\Rightarrow \mathbf{r}(t) = v_0 \cos(\alpha)t\mathbf{E}_x + \left(v_0 \sin(\alpha)t - \frac{gt^2}{2}\right)\mathbf{E}_y$$

Now, let

$$x = v_0 \cos(\alpha)t$$

This implies,

$$t = \frac{x}{v_0 \cos(\alpha)} \tag{2}$$

Now,

$$y = v_0 \sin(\alpha)t - \frac{gt^2}{2}$$

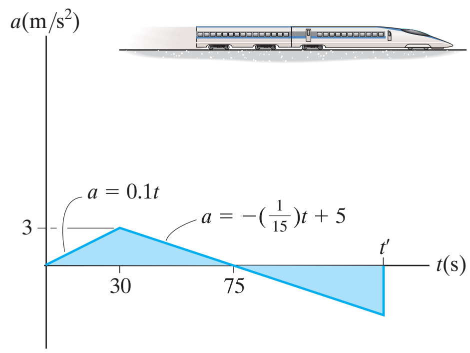
$$\Rightarrow y(x) = v_0 \sin(\alpha)\frac{x}{v_0 \cos(\alpha)} - \frac{g}{2}\frac{x^2}{v_0^2 \cos^2(\alpha)}$$

$$\Rightarrow \boxed{y(x) = -\left(\frac{g}{2v_0^2 \cos^2(\alpha)}\right) x^2 + \tan(\alpha)x}.$$

This result is not valid for $\alpha = \pm\pi/2$ because from (2), the time parameter is not defined which means that such an α is not possible during this flight trajectory. ∎

12–51.

The a–t graph of the bullet train is shown. If the train starts from rest, determine the elapsed time t' before it again comes to rest. What is the total distance traveled during this time interval? Construct the v–t and s–t graphs.



SOLUTION

$v-t$ Graph: For the time interval $0 \leq t < 30$ s, the initial condition is $v = 0$ when $t = 0$ s.

$$(\overset{+}{\rightarrow}) \qquad dv = a\,dt$$

$$\int_0^v dv = \int_0^t 0.1t\,dt$$

$$v = \left(0.05t^2\right) \text{ m/s}$$

When $t = 30$ s,

$$v\Big|_{t=30\text{ s}} = 0.05\left(30^2\right) = 45 \text{ m/s}$$

or the time interval $30 \text{ s} < t \leq t'$, the initial condition is $v = 45$ m/s at $t = 30$ s.

$$(\overset{+}{\rightarrow}) \qquad dv = a\,dt$$

$$\int_{45\text{ m/s}}^v dv = \int_{30\text{ s}}^t \left(-\frac{1}{15}t + 5\right)dt$$

$$v = \left(-\frac{1}{30}t^2 + 5t - 75\right) \text{ m/s}$$

Thus, when $v = 0$,

$$0 = -\frac{1}{30}t'^2 + 5t' - 75$$

Choosing the root $t' > 75$ s,

$$t' = 133.09 \text{ s} = 133 \text{ s}$$ **Ans.**

Also, the change in velocity is equal to the area under the a–t graph. Thus,

$$\Delta v = \int a\,dt$$

$$0 = \frac{1}{2}(3)(75) + \frac{1}{2}\left[\left(-\frac{1}{15}t' + 5\right)(t' - 75)\right]$$

$$0 = -\frac{1}{30}t'^2 + 5t' - 75$$

This equation is the same as the one obtained previously.

The slope of the v–$t$graph is zero when $t = 75$ s, which is the instant $a = \dfrac{dv}{dt} = 0$. Thus,

$$v\Big|_{t=75\text{ s}} = -\frac{1}{30}\left(75^2\right) + 5(75) - 75 = 112.5 \text{ m/s}$$

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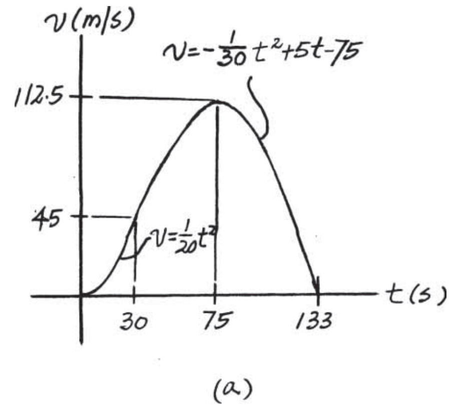
12–51. continued

The v–t graph is shown in Fig. *a*.



$s-t$ Graph: Using the result of v, the equation of the s–t graph can be obtained by integrating the kinematic equation $ds = vdt$. For the time interval $0 \leq t < 30$ s, the initial condition $s = 0$ at $t = 0$ s will be used as the integration limit. Thus,

$$(\overset{+}{\rightarrow}) \qquad ds = vdt$$

$$\int_0^s ds = \int_0^t 0.05t^2\, dt$$

$$s = \left(\frac{1}{60}t^3\right) \text{ m}$$

When $t = 30$ s,

$$s\big|_{t=30\text{ s}} = \frac{1}{60}\left(30^3\right) = 450 \text{ m}$$

For the time interval $30\text{ s} < t \leq t' = 133.09$ s, the initial condition is $s = 450$ m when $t = 30$ s.

$$(\overset{+}{\rightarrow}) \qquad ds = vdt$$

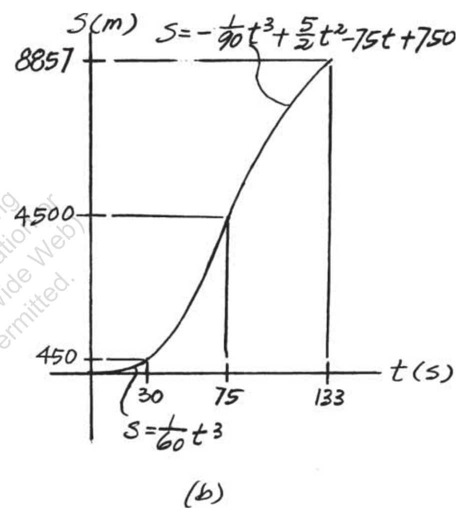
$$\int_{450\text{ m}}^s ds = \int_{30\text{ s}}^t \left(-\frac{1}{30}t^2 + 5t - 75\right)dt$$

$$s = \left(-\frac{1}{90}t^3 + \frac{5}{2}t^2 - 75t + 750\right) \text{ m}$$

When $t = 75$ s and $t' = 133.09$ s,

$$s\big|_{t=75\text{ s}} = -\frac{1}{90}\left(75^3\right) + \frac{5}{2}\left(75^2\right) - 75(75) + 750 = 4500 \text{ m}$$

$$s\big|_{t=133.09\text{ s}} = -\frac{1}{90}\left(133.09^3\right) + \frac{5}{2}\left(133.09^2\right) - 75(133.09) + 750 = 8857 \text{ m} \qquad \textbf{Ans.}$$



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The s–t graph is shown in Fig. *b*.

When $t = 30$ s,

$v = 45$ m/s and $s = 450$ m.

When $t = 75$ s,

$v = v_{max} = 112.5$ m/s and $s = 4500$ m.

When $t = 133$ s,

$v = 0$ and $s = 8857$ m.

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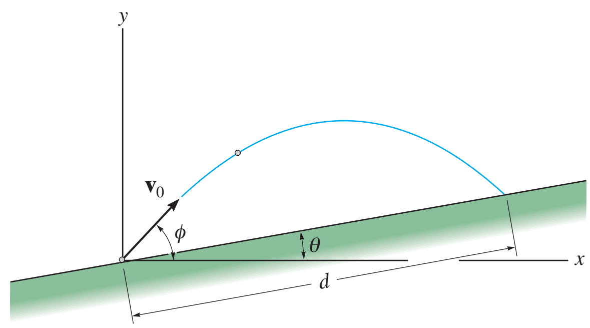
***12–96.**

A projectile is given a velocity $\mathbf{v}_0$. Determine the angle ϕ at which it should be launched so that d is a maximum. The acceleration due to gravity is g.



SOLUTION

$$(\overset{+}{\rightarrow}) \qquad s_x = s_0 + v_0 t$$

$$d\cos\theta = 0 + v_0(\cos\phi)t$$

$$(+\uparrow) \qquad s_y = s_0 + v_0 t + \frac{1}{2}a_c t^2$$

$$d\sin\theta = 0 + v_0(\sin\phi)t + \frac{1}{2}(-g)t^2$$

Thus,

$$d\sin\theta = v_0\sin\phi\left(\frac{d\cos\theta}{v_0\cos\phi}\right) - \frac{1}{2}g\left(\frac{d\cos\theta}{v_0\cos\phi}\right)^2$$

$$\sin\theta = \cos\theta\tan\phi - \frac{gd\cos^2\theta}{2v_0^2\cos^2\phi}$$

$$d = (\cos\theta\tan\phi - \sin\theta)\frac{2v_0^2\cos^2\phi}{g\cos^2\theta}$$

$$d = \frac{v_0^2}{g\cos\theta}\left(\sin 2\phi - 2\tan\theta\cos^2\phi\right)$$

Require:

$$\frac{d(d)}{d\phi} = \frac{v_0^2}{g\cos\theta}\left[\cos 2\phi(2) - 2\tan\theta(2\cos\phi)(-\sin\phi)\right] = 0$$

$$\cos 2\phi + \tan\theta\sin 2\phi = 0$$

$$\frac{\sin 2\phi}{\cos 2\phi}\tan\theta + 1 = 0$$

$$\tan 2\phi = -\text{ctn}\,\theta$$

$$\phi = \frac{1}{2}\tan^{-1}(-\text{ctn}\,\theta)$$ **Ans.**

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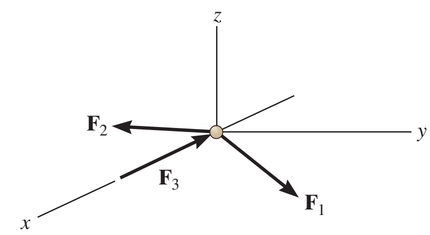
13–1.

The 6-lb particle is subjected to the action of its weight and forces $\mathbf{F}_1 = \{2\mathbf{i} + 6\mathbf{j} - 2t\mathbf{k}\}$ lb, $\mathbf{F}_2 = \{t^2\mathbf{i} - 4t\mathbf{j} - 1\mathbf{k}\}$ lb, and $\mathbf{F}_3 = \{-2t\mathbf{i}\}$ lb, where t is in seconds. Determine the distance the ball is from the origin 2 s after being released from rest.



SOLUTION

$$\Sigma \mathbf{F} = m\mathbf{a}; \quad (2\mathbf{i} + 6\mathbf{j} - 2t\mathbf{k}) + (t^2\mathbf{i} - 4t\mathbf{j} - 1\mathbf{k}) - 2t\mathbf{i} - 6\mathbf{k} = \left(\frac{6}{32.2}\right)(\mathbf{a}_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k})$$

Equating components:

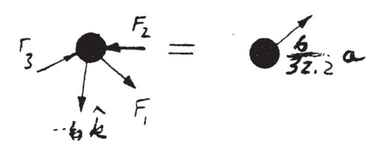
$$\left(\frac{6}{32.2}\right)a_x = t^2 - 2t + 2 \quad \left(\frac{6}{32.2}\right)a_y = -4t + 6 \quad \left(\frac{6}{32.2}\right)a_z = -2t - 7$$

Since $dv = a\,dt$, integrating from $\nu = 0, t = 0$, yields

$$\left(\frac{6}{32.2}\right)v_x = \frac{t^3}{3} - t^2 + 2t \quad \left(\frac{6}{32.2}\right)v_y = -2t^2 + 6t \quad \left(\frac{6}{32.2}\right)v_z = -t^2 - 7t$$

Since $ds = v\,dt$, integrating from $s = 0, t = 0$ yields

$$\left(\frac{6}{32.2}\right)s_x = \frac{t^4}{12} - \frac{t^3}{3} + t^2 \quad \left(\frac{6}{32.2}\right)s_y = -\frac{2t^3}{3} + 3t^2 \quad \left(\frac{6}{32.2}\right)s_z = -\frac{t^3}{3} - \frac{7t^2}{2}$$



When $t = 2$ s then, $s_x = 14.31$ ft, $s_y = 35.78$ ft $s_z = -89.44$ ft

Thus,

$$s = \sqrt{(14.31)^2 + (35.78)^2 + (-89.44)^2} = 97.4 \text{ ft}$$ **Ans.**

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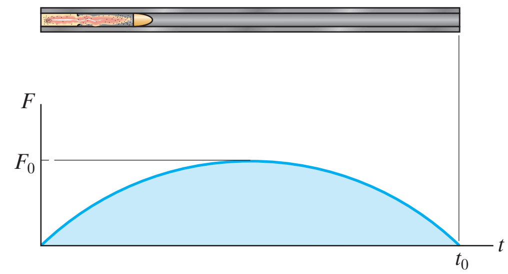
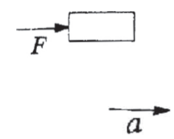
13–13.

The bullet of mass m is given a velocity due to gas pressure caused by the burning of powder within the chamber of the gun. Assuming this pressure creates a force of $F = F_0 \sin(\pi t/t_0)$ on the bullet, determine the velocity of the bullet at any instant it is in the barrel. What is the bullet's maximum velocity? Also, determine the position of the bullet in the barrel as a function of time.



SOLUTION

$$\xrightarrow{+} \Sigma F_x = ma_x; \qquad F_0 \sin\left(\frac{\pi t}{t_0}\right) = ma$$



$$a = \frac{dv}{dt} = \left(\frac{F_0}{m}\right)\sin\left(\frac{\pi t}{t_0}\right)$$

$$\int_0^v dv = \int_0^t \left(\frac{F_0}{m}\right)\sin\left(\frac{\pi t}{t_0}\right)dt \qquad v = -\left(\frac{F_0 t_0}{\pi m}\right)\cos\left(\frac{\pi t}{t_0}\right)\Big]_0^t$$

$$v = \left(\frac{F_0 t_0}{\pi m}\right)\left(1 - \cos\left(\frac{\pi t}{t_0}\right)\right)$$ **Ans.**

v_{max} occurs when $\cos\left(\frac{\pi t}{t_0}\right) = -1$, or $t = t_0$.

$$v_{max} = \frac{2F_0 t_0}{\pi m}$$ **Ans.**

$$\int_0^s ds = \int_0^t \left(\frac{F_0 t_0}{\pi m}\right)\left(1 - \cos\left(\frac{\pi t}{t_0}\right)\right)dt$$

$$s = \left(\frac{F_0 t_0}{\pi m}\right)\left[\text{t} - \frac{t_0}{\pi}\sin\left(\frac{\pi t}{t_0}\right)\right]_0^t$$

$$s = \left(\frac{F_0 t_0}{\pi m}\right)\left(t - \frac{t_0}{\pi}\sin\left(\frac{\pi t}{t_0}\right)\right)$$ **Ans.**

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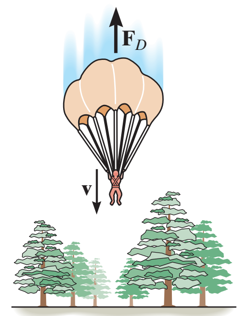
***13–48.**

A parachutist having a mass m opens his parachute from an at-rest position at a very high altitude. If the atmospheric drag resistance is $F_D = kv^2$, where k is a constant, determine his velocity when he has fallen for a time t. What is his velocity when he lands on the ground? This velocity is referred to as the *terminal velocity*, which is found by letting the time of fall $t \rightarrow \infty$.



SOLUTION

$$+\downarrow \Sigma F_z = m\,a_z; \qquad mg - kv^2 = m\frac{dv}{dt}$$

$$m\int_0^v \frac{m\,dv}{(mg - kv^2)} = \int_0^t dt$$

$$\frac{m}{k}\int_0^v \frac{dv}{\frac{mg}{k} - v^2} = t$$

$$\frac{m}{k}\left(\frac{1}{2\sqrt{\frac{mg}{k}}}\right)\ln\left[\frac{\sqrt{\frac{mg}{k}} + v}{\sqrt{\frac{mg}{k}} - v}\right]_0^v = t$$

$$\frac{k}{m}t\left(2\sqrt{\frac{mg}{k}}\right) = \ln\frac{\sqrt{\frac{mg}{k}} + v}{\sqrt{\frac{mg}{k}} - v}$$

$$e^{2t\sqrt{\frac{mg}{k}}} = \frac{\sqrt{\frac{mg}{k}} + v}{\sqrt{\frac{mg}{k}} - v}$$

$$\sqrt{\frac{mg}{k}}\,e^{2t\sqrt{\frac{mg}{k}}} - v\,e^{2t\sqrt{\frac{mg}{k}}} = \sqrt{\frac{mg}{k}} + v$$

$$v = \sqrt{\frac{mg}{k}}\left[\frac{e^{2t\sqrt{\frac{mg}{k}}} - 1}{e^{2t\sqrt{\frac{mg}{k}}} + 1}\right] \qquad \textbf{Ans.}$$

When $t \rightarrow \infty \qquad v_t = \sqrt{\frac{mg}{k}}$ **Ans.**

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