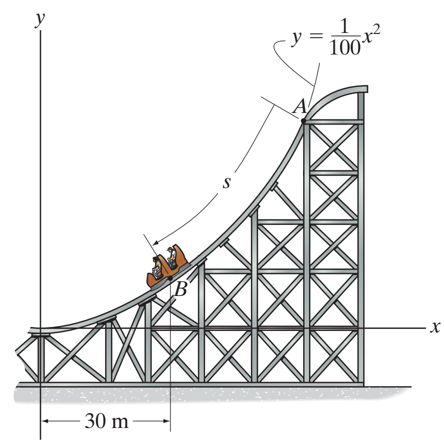
12–121.

When the roller coaster is at B, it has a speed of 25 m/s, which is increasing at $a_t = 3\ \text{m/s}^2$. Determine the magnitude of the acceleration of the roller coaster at this instant and the direction angle it makes with the x axis.



SOLUTION

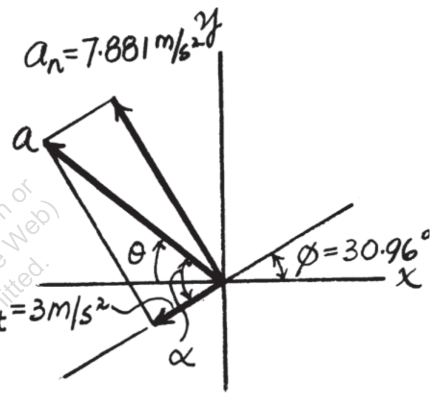
Radius of Curvature:

$$y = \frac{1}{100}x^2$$

$$\frac{dy}{dx} = \frac{1}{50}x$$

$$\frac{d^2y}{dx^2} = \frac{1}{50}$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} = \left.\frac{\left[1 + \left(\frac{1}{50}x\right)^2\right]^{3/2}}{\left|\frac{1}{50}\right|}\right|_{x=30\text{ m}} = 79.30\text{ m}$$



(a)

Acceleration:

$$a_t = \dot{v} = 3\ \text{m/s}^2$$

$$a_n = \frac{v_B^{\,2}}{\rho} = \frac{25^2}{79.30} = 7.881\ \text{m/s}^2$$

The magnitude of the roller coaster's acceleration is

$$a = \sqrt{a_t^{\,2} + a_n^{\,2}} = \sqrt{3^2 + 7.881^2} = 8.43\ \text{m/s}^2 \qquad \textbf{Ans.}$$

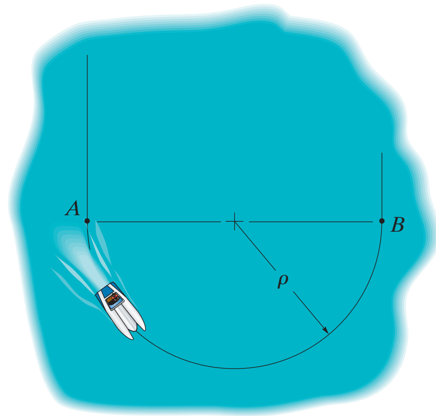
The angle that the tangent at B makes with the x axis is $\phi = \tan^{-1}\left(\left.\frac{dy}{dx}\right|_{x=30\text{ m}}\right) = \tan^{-1}\left[\frac{1}{50}(30)\right] = 30.96°$.

As shown in Fig. a, $\mathbf{a}_n$ is always directed towards the center of curvature of the path. Here, $\alpha = \tan^{-1}\left(\frac{a_n}{a_t}\right) = \tan^{-1}\left(\frac{7.881}{3}\right) = 69.16°$. Thus, the angle θ that the roller coaster's acceleration makes with the x axis is

$$\theta = \alpha - \phi = 38.2° \qquad \textbf{Ans.}$$

12–123.

The speedboat travels at a constant speed of 15 m/s while making a turn on a circular curve from A to B. If it takes 45 s to make the turn, determine the magnitude of the boat's acceleration during the turn.



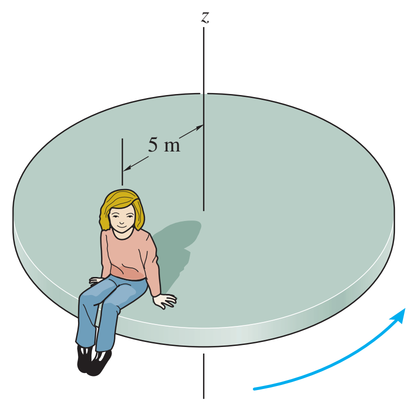
SOLUTION

Acceleration: During the turn, the boat travels $s = vt = 15(45) = 675$ m. Thus, the radius of the circular path is $\rho = \frac{s}{\pi} = \frac{675}{\pi}$ m. Since the boat has a constant speed, $a_t = 0$. Thus,

$$a = a_n = \frac{v^2}{\rho} = \frac{15^2}{\left(\frac{675}{\pi}\right)} = 1.05 \text{ m/s}^2$$ **Ans.**

***13–52.**

A girl, having a mass of 15 kg, sits motionless relative to the surface of a horizontal platform at a distance of $r = 5$ m from the platform's center. If the angular motion of the platform is *slowly* increased so that the girl's tangential component of acceleration can be neglected, determine the maximum speed which the girl will have before she begins to slip off the platform. The coefficient of static friction between the girl and the platform is $\mu = 0.2$.



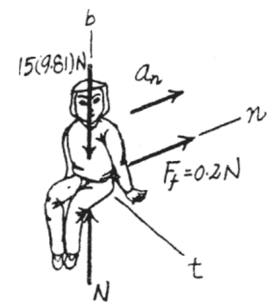
SOLUTION

Equation of Motion: Since the girl is on the verge of slipping, $F_f = \mu_s N = 0.2N$. Applying Eq. 13–8, we have

$$\Sigma F_b = 0; \qquad N - 15(9.81) = 0 \qquad N = 147.15\text{ N}$$

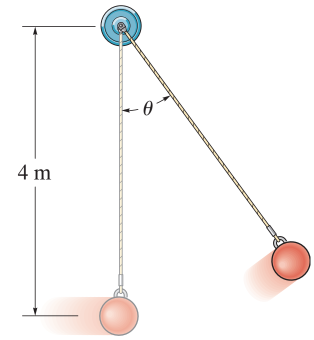
$$\Sigma F_n = ma_n; \qquad 0.2(147.15) = 15\left(\frac{v^2}{5}\right)$$

$$v = 3.13\text{ m/s} \qquad \textbf{Ans.}$$



***13–72.**

The ball has a mass of 30 kg and a speed $v = 4$ m/s at the instant it is at its lowest point, $\theta = 0°$. Determine the tension in the cord and the rate at which the ball's speed is decreasing at the instant $\theta = 20°$. Neglect the size of the ball.



SOLUTION

$+\nwarrow\Sigma F_n = ma_n;\qquad T - 30(9.81)\cos\theta = 30\left(\frac{v^2}{4}\right)$

$+\nearrow\Sigma F_t = ma_t;\qquad -30(9.81)\sin\theta = 30a_t$

$a_t = -9.81\sin\theta$

$a_t\,ds = v\,dv$ Since $ds = 4\,d\theta$, then

$$-9.81\int_0^\theta \sin\theta\,(4\,d\theta) = \int_4^v v\,dv$$

$$9.81(4)\cos\theta\Big|_0^\theta = \frac{1}{2}(v)^2 - \frac{1}{2}(4)^2$$

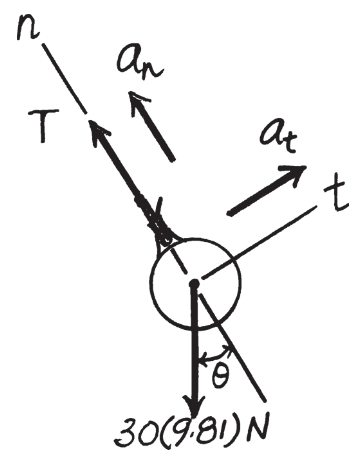
$$39.24(\cos\theta - 1) + 8 = \frac{1}{2}v^2$$

At $\theta = 20°$

$v = 3.357$ m/s

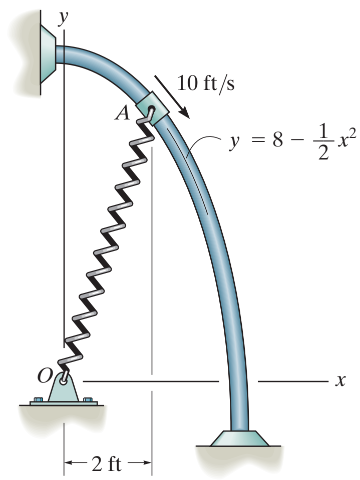
$a_t = -3.36\text{ m/s}^2 = 3.36\text{ m/s}^2 \swarrow$ **Ans.**

$T = 361$ N **Ans.**



***13–84.**

The 5-lb collar slides on the smooth rod, so that when it is at A it has a speed of 10 ft/s. If the spring to which it is attached has an unstretched length of 3 ft and a stiffness of $k = 10$ lb/ft, determine the normal force on the collar and the acceleration of the collar at this instant.



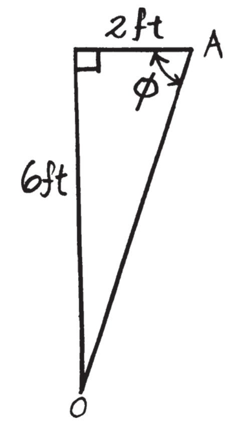
SOLUTION

$$y = 8 - \frac{1}{2}x^2$$

$$-\frac{dy}{dx} = \tan\theta = x\Big|_{x=2} = 2 \qquad \theta = 63.435^\circ$$

$$\frac{d^2y}{dx^2} = -1$$

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}}{\left|\frac{d^2y}{dx^2}\right|} = \frac{(1 + (-2)^2)^{\frac{3}{2}}}{|-1|} = 11.18 \text{ ft}$$



$$y = 8 - \frac{1}{2}(2)^2 = 6$$

$$OA = \sqrt{(2)^2 + (6)^2} = 6.3246$$

$$F_s = kx = 10(6.3246 - 3) = 33.246 \text{ lb}$$

$$\tan\phi = \frac{6}{2}; \ \phi = 71.565^\circ$$

$+\swarrow\Sigma F_n = ma_n;$ $\quad 5\cos 63.435^\circ - N + 33.246\cos 45.0^\circ = \left(\frac{5}{32.2}\right)\left(\frac{(10)^2}{11.18}\right)$

$N = 24.4$ lb **Ans.**

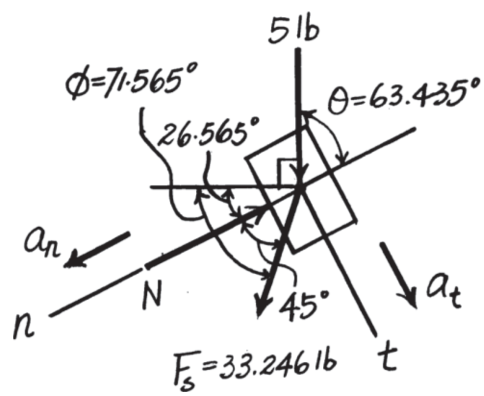
$+\searrow\Sigma F_t = ma_t;$ $\quad 5\sin 63.435^\circ + 33.246\sin 45.0^\circ = \left(\frac{5}{32.2}\right)a_t$

$$a_t = 180.2 \text{ ft/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{(10)^2}{11.18} = 8.9443 \text{ ft/s}^2$$

$$a = \sqrt{(180.2)^2 + (8.9443)^2}$$

$a = 180$ ft/s^2 **Ans.**



HW#3 ME 230, Spring 2014
O'Reilly: 3.6, 3.9, 4.1, 4.3
Hibbeler: 12-121, 12-123, 13-52, 13-72, 13-84

O'Reilly 3.6

$r = r(s(t))$ show that $\vec{v} = v\hat{e}_t$, $\vec{a} = \dot{v}\hat{e}_t + Kv^2\hat{e}_n$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dr}{ds}\cdot\frac{ds}{dt} \quad \text{where } \frac{ds}{dt} = v \quad \frac{dr}{ds} = \hat{e}_t$$

$$\boxed{\vec{v} = v\hat{e}_t}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv}{dt}\hat{e}_t + v\frac{d\hat{e}_t}{dt} = \dot{v}\hat{e}_t + v\frac{d\hat{e}_t}{ds}\frac{ds}{dt} \quad \text{where } \frac{d\hat{e}_t}{dt} = K\hat{e}_n, \frac{ds}{dt} = v$$

$$\boxed{\therefore\ \vec{a} = \dot{v}\hat{e}_t + v^2K\hat{e}_n}$$

O'Reilly 3.9

$\vec{r} = R\hat{e}_r + \alpha R\theta\hat{E}_z$ $\quad \vec{F} = -mg\hat{E}_z$ $\quad$ show that $s(\theta) = R\sqrt{1+\alpha^2}(\theta-\theta_0) + s(\theta_0)$

$\ddot{s} = R\sqrt{1+\alpha^2}\,\ddot{\theta}$

Using $\vec{F} = m\vec{a}$ + results from Section 3.4, show that

$$\vec{F}\cdot\hat{e}_t = \frac{-mg\alpha}{\sqrt{1+\alpha^2}} = mR\sqrt{1+\alpha^2}\,\ddot{\theta}$$
$$\vec{F}\cdot\hat{e}_n = N_n = mR\dot{\theta}^2$$
$$\vec{F}\cdot\hat{e}_b = N_b - \frac{mg}{\sqrt{1+\alpha^2}}$$

where $N_n\hat{e}_n + N_b\hat{e}_b$ is the normal force exerted on the particle.

$$\frac{d\vec{r}}{dt} = \dot{R}\hat{e}_r + R\dot{\hat{e}}_r + \alpha\dot{R}\theta\hat{E}_z + \alpha R\dot{\theta}\hat{E}_z$$
$$= \dot{R}\hat{e}_r + R\dot{\theta}\hat{e}_\theta + \alpha(\dot{R}\theta + R\dot{\theta})\hat{E}_z$$

If $\dot{R} = 0$: $\displaystyle \frac{d\vec{r}}{dt} = R\frac{d\theta}{dt}\hat{e}_\theta + \alpha R\frac{d\theta}{dt}\hat{E}_z$

$$\frac{d\vec{r}}{dt} = R\frac{d\theta}{dt}\left(\hat{e}_\theta + \alpha\hat{E}_z\right)$$

$$\frac{ds}{dt} = \left|\frac{d\vec{r}}{dt}\right| = \sqrt{R^2\left(\frac{d\theta}{dt}\right)^2(1^2+\alpha^2)}$$

$$\frac{ds}{dt} = R\frac{d\theta}{dt}\sqrt{1+\alpha^2} \quad \text{with } R = \text{const.} \rightarrow \boxed{\ddot{s} = R\ddot{\theta}\sqrt{1+\alpha^2}}$$

$$ds = R\sqrt{1+\alpha^2}\,d\theta$$

$$s(\theta) - s_0(\theta) = R\sqrt{1+\alpha^2}(\theta-\theta_0)$$

$$\boxed{s(\theta) = R\sqrt{1+\alpha^2}(\theta-\theta_0) + s(\theta_0)}$$

O'Reilly 3.9 (continued)

From Section 3.4: $\vec{F} \cdot \hat{e}_t = m \frac{d^2 s}{dt^2} = \underline{m R \sqrt{1+\alpha^2}\, \ddot{\theta}} = -mg \left(\frac{\alpha}{\sqrt{1+\alpha^2}} \right)$

$$\vec{F} \cdot \hat{e}_n = mK \left(\frac{ds}{dt} \right)^2 = mK \left(R^2 \dot{\theta}^2 (1+\alpha^2) \right)$$

$K = \frac{1}{R(1+\alpha^2)}$ for a circular helix (O'Reilly Section 3.3)

$$\vec{F} \cdot \hat{e}_n = \frac{m R^2 \dot{\theta}^2 (1+\alpha^2)}{R(1+\alpha^2)} = \underline{m R \dot{\theta}^2}$$

(O'Reilly Section 3.3) $\hat{e}_b = \frac{-\alpha \hat{e}_\theta + \hat{E}_z}{\sqrt{1+\alpha^2}}$ given $\vec{F} = -mg \hat{E}_z + N_b \hat{e}_b$

$$\text{so } \underline{\vec{F} \cdot \hat{e}_b = \frac{-mg}{\sqrt{1+\alpha^2}} + N_b}$$

O'Reilly 3.9 (continued)

From Section 3.4: $\vec{F}\cdot\hat{e}_t = m\frac{d^2s}{dt^2} = \underline{mR\sqrt{1+\alpha^2}\,\ddot{\theta}} = -mg\left(\frac{\alpha}{\sqrt{1+\alpha^2}}\right)$

$\vec{F}\cdot\hat{e}_n = mK\left(\frac{ds}{dt}\right)^2 = mK\left(R^2\dot{\theta}^2(1+\alpha^2)\right)$

$K = \frac{1}{R(1+\alpha^2)}$ for a circular helix (O'Reilly Section 3.3)

$\vec{F}\cdot\hat{e}_n = \frac{mR^2\dot{\theta}^2(1+\alpha^2)}{R(1+\alpha^2)} = \underline{mR\dot{\theta}^2}$

(O'Reilly Section 3.3) $\hat{e}_b = \frac{-\alpha\hat{e}_\theta + \hat{E}_z}{\sqrt{1+\alpha^2}}$ given $\vec{F} = -mg\hat{E}_z + N_b\hat{e}_b$

so $\underline{\vec{F}\cdot\hat{e}_b = \frac{-mg}{\sqrt{1+\alpha^2}} + N_b}$

O'Reilly 4.1

$\vec{r} = R\hat{e}_r$ show that $\vec{F}_f = -\mu_d\|\vec{N}\|\frac{\dot{\theta}}{|\dot{\theta}|}\hat{e}_\theta$, $N = N_r\hat{e}_r + N_z\hat{e}_z$

In general for a particle on a space curve:

$\vec{F}_f = -\mu_d\|\vec{N}\|\frac{\vec{v}_{rel}}{\|\vec{v}_{rel}\|}$ (O'Reilly Section 4.2.2)

$\frac{d\vec{r}}{dt} = \cancel{\dot{R}}^{\,0}\hat{e}_r + R\dot{\theta}\hat{e}_\theta = R\dot{\theta}\hat{e}_\theta$ $\quad \vec{v}_{rel} = \frac{d\vec{r}}{dt} - 0 = R\dot{\theta}\hat{e}_\theta$

$\vec{F}_f = -\mu_d\|\vec{N}\|\frac{R\dot{\theta}\hat{e}_\theta}{|R\dot{\theta}|} = \underline{-\mu_d\|\vec{N}\|\frac{\dot{\theta}\hat{e}_\theta}{|\dot{\theta}|}}$

O'Reilly 4.3

$\vec{r} = R\hat{e}_r$ show that $\vec{F}_f = F_f\hat{e}_\theta$ $\vec{N} = N_r\hat{e}_r + N_z\hat{E}_z$, $|F_f| \leq \mu_s\sqrt{N_r^2+N_z^2}$

equivalent to: $-\mu_s\sqrt{N_r^2+N_z^2} \leq F_f \leq \mu_s\sqrt{N_r^2+N_z^2}$

In general, for a particle on a space curve:

$\vec{F}_f = F_f\hat{e}_t$ for a particle with $\vec{r} = R\hat{e}_r$, $\hat{e}_\theta = \hat{e}_t$,

so $\vec{F}_f = F_f\hat{e}_\theta$.

$\vec{N} = N\hat{n} \rightarrow \vec{N} = N_r\hat{e}_r + N_z\hat{E}_z$ for a particle sitting in the $r-\theta$ plane.

$|F_f| \leq \mu_s\|\vec{N}\|$ where $\|\vec{N}\|^2 = \vec{N}\cdot\vec{N}$

$|F_f| \leq \mu_s\sqrt{N_r^2+N_z^2}$ $\quad \|\vec{N}\|^2 = N_r^2 + N_z^2$

$\|\vec{N}\| = \pm\sqrt{N_r^2+N_z^2}$

equivalent to

$-\mu_s\sqrt{N_r^2+N_z^2} \leq F_f \leq \mu_s\sqrt{N_r^2+N_z^2}$

* Note: the "$\pm$" in front of the square root gives $\|\vec{N}\|$ to be positive or negative, giving the two equivalent statements for F_f.