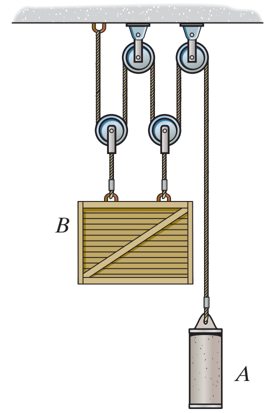
15–30.

The crate B and cylinder A have a mass of 200 kg and 75 kg, respectively. If the system is released from rest, determine the speed of the crate and cylinder when $t = 3$ s. Neglect the mass of the pulleys.



SOLUTION

Free-Body Diagram: The free-body diagrams of cylinder A and crate B are shown in Figs. b and c. $\mathbf{v}_A$ and $\mathbf{v}_B$ must be assumed to be directed downward so that they are consistent with the positive sense of s_A and s_B shown in Fig. a.

Principle of Impulse and Momentum: Referring to Fig. b,

$$(+\downarrow) \qquad m(v_1)_y + \Sigma \int_{t_1}^{t_2} F_y dt = m(v_2)_y$$

$$75(0) + 75(9.81)(3) - T(3) = 75v_A$$

$$v_A = 29.43 - 0.04T \tag{1}$$

From Fig. b,

$$(+\downarrow) \qquad m(v_1)_y + \Sigma \int_{t_1}^{t_2} F_y dt = m(v_2)_y$$

$$200(0) + 2500(9.81)(3) - 4T(3) = 200v_B$$

$$v_B = 29.43 - 0.06T \tag{2}$$

Kinematics: Expressing the length of the cable in terms of s_A and s_B and referring to Fig. a,

$$s_A + 4s_B = l \tag{3}$$

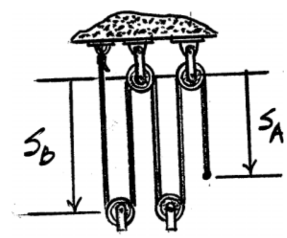
Taking the time derivative,

$$v_A + 4v_B = 0 \tag{4}$$

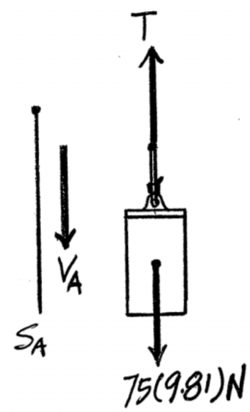
Solving Eqs. (1), (2), and (4) yields

$v_B = -2.102 \text{ m/s} = 2.10 \text{ m/s} \uparrow$ $\qquad$ $v_A = 8.409 \text{ m/s} = 8.41 \text{ m/s} \downarrow$ **Ans.**

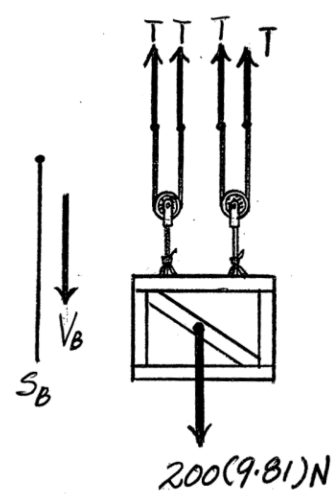
$T = 525.54$ N



(a)



(b)



(c)

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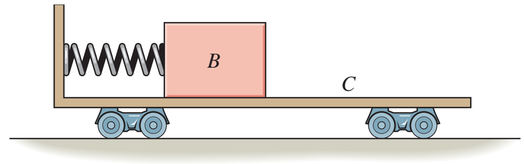
15–42.

The block has a mass of 50 kg and rests on the surface of the cart having a mass of 75 kg. If the spring which is attached to the cart and not the block is compressed 0.2 m and the system is released from rest, determine the speed of the block with respect to the *cart* after the spring becomes undeformed. Neglect the mass of the wheels and the spring in the calculation. Also neglect friction. Take $k = 300$ N/m.

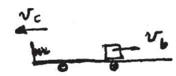


SOLUTION

$T_1 + V_1 = T_2 + V_2$

$(0 + 0) + \frac{1}{2}(300)(0.2)^2 = \frac{1}{2}(50)(v_b)^2 + \frac{1}{2}(75)(v_c)^2$

$12 = \; = 50\, v_b^2 + 75\, v_c^2$



$(\overset{+}{\rightarrow})$ $\quad \Sigma m v_1 = \Sigma m v_2$

$0 + 0 = 50\, v_b - 75\, v_c$

$v_b = 1.5 v_c$

$v_c = 0.253 \text{ m/s} \leftarrow$

$v_b = 0.379 \text{ m/s} \rightarrow$

$\mathbf{v}_b = \mathbf{v}_c + \mathbf{v}_{b/c}$

$(\overset{+}{\rightarrow})$ $\quad 0.379 = -0.253 + \mathbf{v}_{b/c}$

$v_{b/c} = 0.632 \text{ m/s} \rightarrow$ **Ans.**

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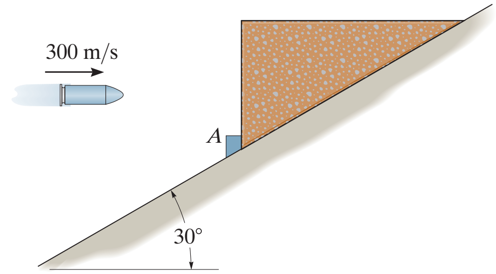
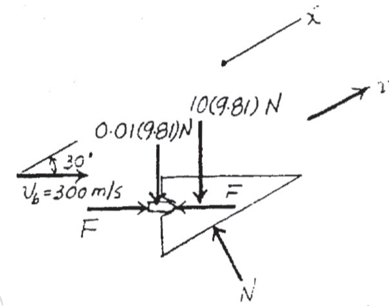
15–57.

The 10-kg block is held at rest on the smooth inclined plane by the stop block at A. If the 10-g bullet is traveling at 300 m/s when it becomes embedded in the 10-kg block, determine the distance the block will slide up along the plane before momentarily stopping.



SOLUTION

Conservation of Linear Momentum: If we consider the block and the bullet as a system, then from the FBD, the *impulsive* force F caused by the impact is *internal* to the system. Therefore, it will cancel out. Also, the weight of the bullet and the block are *nonimpulsive forces*. As the result, linear momentum is conserved along the x' axis.



$$m_b(v_b)_{x'} = (m_b + m_B)\,v_{x'}$$

$$0.01(300\cos 30°) = (0.01 + 10)\,v$$

$$v = 0.2595 \text{ m/s}$$

Conservation of Energy: The datum is set at the blocks initial position. When the block and the embedded bullet is at their highest point they are h *above* the datum. Their gravitational potential energy is $(10 + 0.01)(9.81)h = 98.1981h$. Applying Eq. 14–21, we have

$$T_1 + V_1 = T_2 + V_2$$

$$0 + \frac{1}{2}(10 + 0.01)\left(0.2595^2\right) = 0 + 98.1981h$$

$$h = 0.003433 \text{ m} = 3.43 \text{ mm}$$

$$d = 3.43 / \sin 30° = 6.87 \text{ mm}$$ **Ans.**

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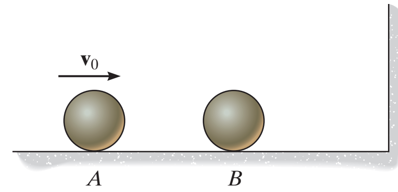
15–70.

Two smooth spheres A and B each have a mass m. If A is given a velocity of v_0, while sphere B is at rest, determine the velocity of B just after it strikes the wall. The coefficient of restitution for any collision is e.



SOLUTION

Impact: The first impact occurs when sphere A strikes sphere B. When this occurs, the linear momentum of the system is conserved along the x axis (line of impact). Referring to Fig. a,

$$(\overset{+}{\rightarrow}) \qquad m_A v_A + m_B v_B = m_A (v_A)_1 + m_B (v_B)_1$$

$$m v_0 + 0 = m(v_A)_1 + m(v_B)_1$$

$$(v_A)_1 + (v_B)_1 = v_0 \qquad \textbf{(1)}$$

$$(\overset{+}{\rightarrow}) \qquad e = \frac{(v_B)_1 - (v_A)_1}{v_A - v_B}$$

$$e = \frac{(v_B)_1 - (v_A)_1}{v_0 - 0}$$

$$(v_B)_1 - (v_A)_1 = e v_0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

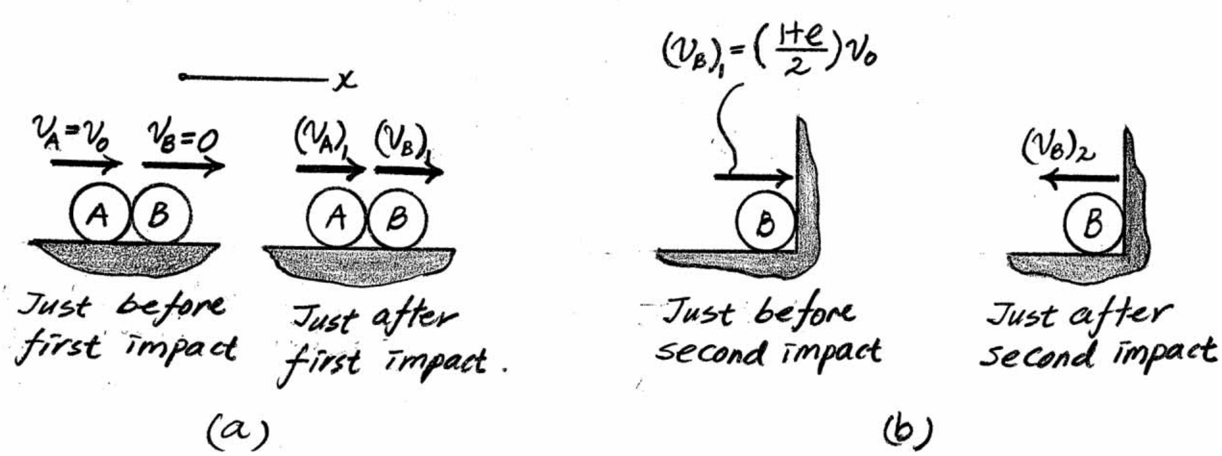
$$(v_B)_1 = \left(\frac{1+e}{2}\right) v_0 \rightarrow \qquad\qquad (v_A)_1 = \left(\frac{1-e}{2}\right) v_0 \rightarrow$$

The second impact occurs when sphere B strikes the wall, Fig. b. Since the wall does not move during the impact, the coefficient of restitution can be written as

$$(\overset{+}{\rightarrow}) \qquad e = \frac{0 - \left[-(v_B)_2\right]}{(v_B)_1 - 0}$$

$$e = \frac{0 + (v_B)_2}{\left[\dfrac{1+e}{2}\right] v_0 - 0}$$

$$(v_B)_2 = \frac{e(1+e)}{2} v_0 \qquad \textbf{Ans.}$$



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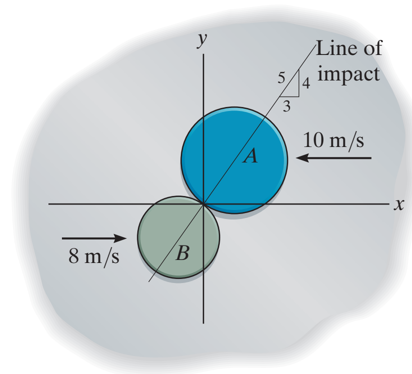
15–93.

Disks A and B have a mass of 15 kg and 10 kg, respectively. If they are sliding on a smooth horizontal plane with the velocities shown, determine their speeds just after impact. The coefficient of restitution between them is $e = 0.8$.



SOLUTION

Conservation of Linear Momentum: By referring to the impulse and momentum of the system of disks shown in Fig. a, notice that the linear momentum of the system is conserved along the n axis (line of impact). Thus,

$$+\nearrow \quad m_A\left(v_A\right)_n + m_B\left(v_B\right)_n = m_A\left(v_A'\right)_n + m_B\left(v_B'\right)_n$$

$$15(10)\left(\frac{3}{5}\right) - 10(8)\left(\frac{3}{5}\right) = 15v_A' \cos\phi_A + 10v_B' \cos\phi_B$$

$$15v_A' \cos\phi_A + 10v_B' \cos\phi_B = 42 \qquad \textbf{(1)}$$

Also, we notice that the linear momentum of disks A and B are conserved along the t axis (tangent to? plane of impact). Thus,

$$+\nwarrow \quad m_A\left(v_A\right)_t = m_A\left(v_A'\right)_t$$

$$15(10)\left(\frac{4}{5}\right) = 15v_A' \sin\phi_A$$

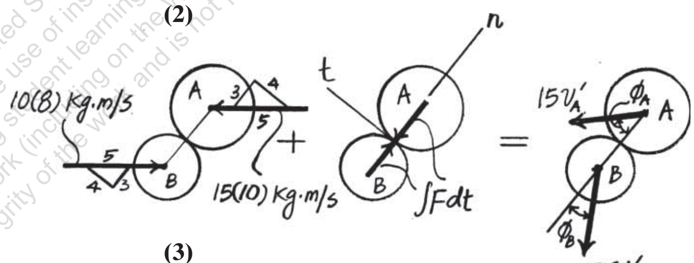
$$v_A' \sin\phi_A = 8 \qquad \textbf{(2)}$$

and

$$+\nwarrow \quad m_B\left(v_B\right)_t = m_B\left(v_B'\right)_t$$

$$10(8)\left(\frac{4}{5}\right) = 10\,v_B' \sin\phi_B$$

$$v_B' \sin\phi_B = 6.4 \qquad \textbf{(3)}$$



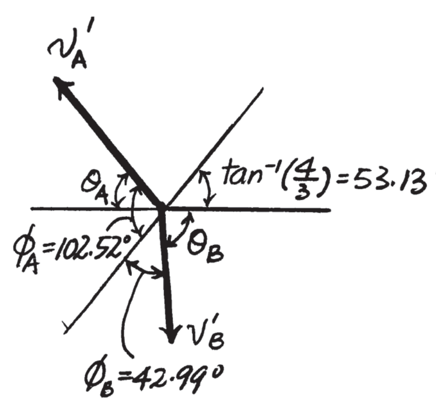
(a)

Coefficient of Restitution: The coefficient of restitution equation written along the n axis (line of impact) gives

$$+\nearrow \quad e = \frac{(v_B')_n - (v_A')_n}{(v_A)_n - (v_B)_n}$$

$$0.8 = \frac{v_B' \cos\phi_B - v_A' \cos\phi_A}{10\left(\frac{3}{5}\right) - \left[-8\left(\frac{3}{5}\right)\right]}$$

$$v_B' \cos\phi_B - v_A' \cos\phi_A = 8.64 \qquad \textbf{(4)}$$



(b)

Solving Eqs. (1), (2), (3), and (4), yeilds

$$v_A' = 8.19 \text{ m/s} \qquad \textbf{Ans.}$$

$$\phi_A = 102.52^\circ$$

$$v_B' = 9.38 \text{ m/s} \qquad \textbf{Ans.}$$

$$\phi_B = 42.99^\circ$$

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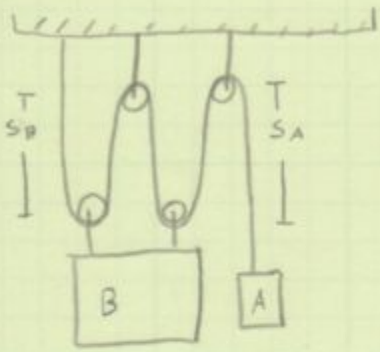
Hibbeler, 15-30



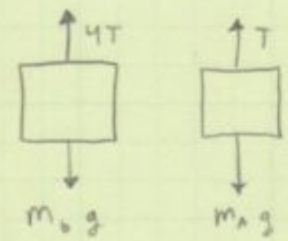
Given m_A, m_B, determine V_A, V_B @ $t = 3$ sec

Relating V_A, V_B via kinematics,

$$4S_B + S_A + C = l$$

$$4V_B = -V_A$$

Free body diagrams



Using impulse-momentum,

$$\sum m_i (\underline{v}_i)_1 + \sum \int_{t_1}^{t_2} \underline{F}_i \, dt = \sum m_i (\underline{v}_i)_2$$

$$\underbrace{\sum m_i (\underline{v}_i)_1}_{0}$$

For mass A,

$$\Sigma F = T - m_A g \, , \quad \sum \int_{t_1=0}^{t=3} [T - m_A g] \, dt = [T - m_A g] \, t$$

$$[T - m_A g] t = m_A V_A \quad (I)$$

For mass B,

$$\Sigma F = 4T - m_b g \, , \quad \sum \int_0^t [4T - m_b g] \, dt = [4T - m_b g] \, t$$

$$[4t - m_b g] t = m_b V_B \quad (II)$$

Equating tension from (I), (II),

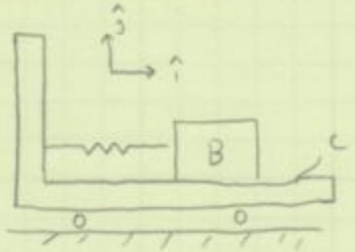
$$T = \frac{m_A V_A}{t} + m_A g \quad , \quad T = \frac{m_B V_B}{4t} + \frac{m_b g}{4}$$

$$\frac{m_A V_A}{t} + m_A g = \frac{m_B V_B}{4t} + \frac{m_b g}{4} \qquad \text{Subing in } -4V_B = V_A$$

$$\left[\frac{-4m_A}{t} - \frac{m_B}{4t} \right] V_B = \frac{m_b g}{4} - m_A g$$

$$\boxed{V_B = \frac{\left[\frac{m_b}{4} - m_A \right] g}{\frac{1}{t} \left[-4m_A - \frac{m_b}{4} \right]} = 2.10 \text{ m/sec upwards} \qquad V_A = -4V_B}$$

Hibbeler, 15.42



Given m_B, m_c, x_o, k, determine

$V_{B/c}$ (velocity of block as seen from cart)

- Notice, block B moves to the right, cart C moves to the left

Conservation of energy,

$$\underbrace{T_1}_{0} + V_1 + \underbrace{\Sigma U_{1-2}}_{0} = T_2 + V_2$$

$$\frac{1}{2} k x^2 = \frac{1}{2}\left[m_b v_b^2 + m_c v_c^2\right]$$

$$k x^2 = m_b v_b^2 + m_c v_c^2 \quad (I)$$

Conservation of momentum,

$$\underbrace{\Sigma(m_i v_i)_1}_{0} = \Sigma(m_i v_i)_2$$

$$0 = m_b V_B - m_c V_c \quad , \quad m_b V_B = m_c V_c \, , \quad \frac{m_b V_B}{m_c} = V_c \quad (II)$$

Substituting (II) into (I),

$$k x^2 = m_b v_b^2 + m_c \left[\frac{m_b V_B}{m_c}\right]^2$$

$$k x^2 = m_b V_B^2 + \frac{m_b^2 V_B^2}{m_c}$$

$$k x^2 = \left[m_b + \frac{m_b^2}{m_c}\right] V_B^2$$

$$\boxed{\sqrt{\frac{k x^2}{\left[m_b + \frac{m_b^2}{m_c}\right]}} = V_B = 0.38 \text{ m/sec } \hat{i}}$$

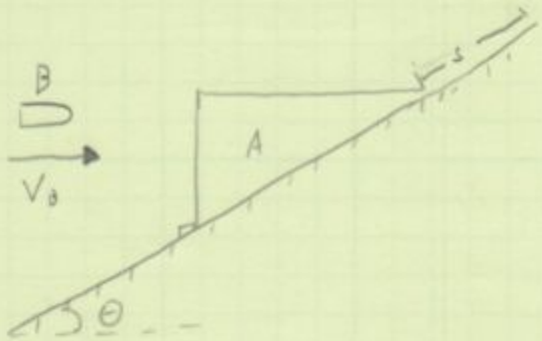
$$\boxed{V_c = \frac{m_b}{m_c}\sqrt{\frac{k x^2}{\left[m_b + \frac{m_b^2}{m_c}\right]}} = -0.25 \text{ m/sec } \hat{i}}$$

$$V_B = V_c + V_{B/c}$$

$$V_{B/c} = V_B - V_c$$

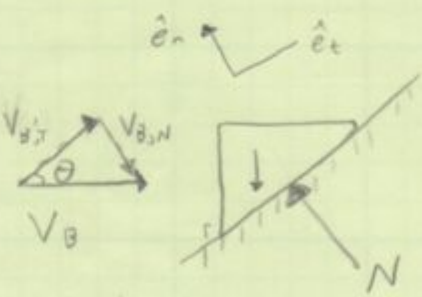
$$\boxed{V_{B/c} = 0.63 \text{ m/sec } \hat{i}}$$

Hibbeler, 15-57



Given m_A, m_B, V_B, determine the distance s block A slides along the plane

- What direction is momentum conserved in? Lets use N-T coordinates



• Momentum is (initially) conserved in the $\hat{e}_t$ direction, but not the $\hat{e}_n$ direction

$$\cos\theta = \frac{V_{B,T}}{V_B} \; ; \; V_{B,T} = V_B \cos\theta$$

Using Impulse-momentum,

$$m_B V_{B,T} + \underbrace{\Sigma \int F\,dt}_{\to 0} = (m_B + m_A) V_{AB,T}$$

$$V_{AB,T} = \frac{m_B V_B \cos\theta}{[m_B + m_A]}$$

Now using conservation of energy,

$$\sin\theta = \frac{y}{s} \;,\; s\sin\theta = y$$

$$T_1 = V_2$$

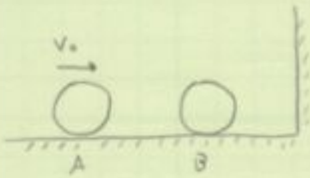
$$V_2 = (m_A + m_B) g y = (m_A + m_B) g s \sin\theta$$

$$T_1 = \frac{1}{2}(m_A + m_B) V_{AB,T}^2$$

$$\frac{1}{2}[\cancel{m_A + m_B}]\left[\frac{m_B V_B \cos\theta}{m_A + m_B}\right]^2 = [\cancel{m_A + m_B}]\, g\, s \sin\theta$$

$$\boxed{\frac{1}{2g\sin\theta}\left[\frac{m_B V_B \cos\theta}{m_A + m_B}\right]^2 = s = 6.86\text{ mm}}$$

Hibbeler 15.70



Given $m_A = m_B = m$, V_0, e, determine:

V_B after it strikes the wall

Define 3 times,

0 - Right before A hits B

1 - Right after A hits B; right before B hits wall

2 - Right after B hits wall

Using conservation of momentum,

$m V_{A,0} = m V_{A,1} + m V_{B,1} \Rightarrow V_{A,0} = V_{A,1} + V_{B,1}$; $V_{A,0} - V_{B,1} = V_{A,1}$

$$e = \frac{V_{B,1} - V_{A,1}}{V_{A,0} - \underbrace{V_{B,0}}_{0}}$$

$V_{B,1} = e V_{A,0} + V_{A,1}$

$V_{B,1} = e V_{A,0} + [V_{A,0} - V_{B,1}]$

$$\boxed{V_{B,1} = \frac{[e+1] V_{A,0}}{2}}$$ ← velocity of B after being struck by A

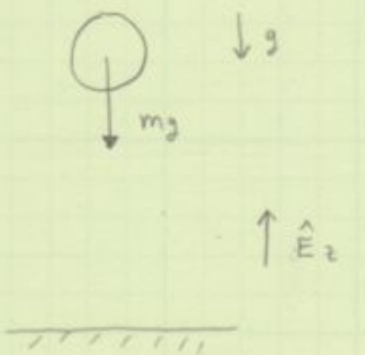
Determining $V_{B,2}$,

$$e = \frac{V_{B,2}}{V_{B,1}}$$

$e V_{B,1} = V_{B,2}$

$$\boxed{\therefore \; V_{B,2} = \frac{e[e+1] V_{A,0}}{2}}$$

O'Reilly 6.2



Show that for a particle of mass m under the influence of gravity, $\underset{\sim}{G} \cdot \hat{E}_z$ is not conserved

Writing down the linear impulse-momentum equation,

$$\underset{\sim}{G}(t_1) - \underset{\sim}{G}(t_0) = \int_{t_0}^{t_1} \underline{F}\, dt$$

Linear momentum is conserved only if

$$\int_{t_0}^{t_1} \underline{F}\, dt = 0$$

The force of gravity is $\underset{\sim}{F} = -mg\,\hat{E}_z$

Then,

$$-\int_{t_0}^{t_1} mg\,\hat{E}_z\, dt = -mg\,[t_1 - t_0]\,\hat{E}_z \neq 0$$

Thus,

$\underset{\sim}{G}(t_1) \neq \underset{\sim}{G}(t_0)$ and linear momentum is NOT conserved