Part 0: Experimental Facts

What is dynamics?
Dynamics is the study of mechanical motion.

Where can we begin with dynamics?
We must be attentive to the data of our experience.
We must begin with experimental facts.

Chapter 0: Space, Time, + Motion

Space:
- is 3-dimensional
- has no origin

Time:
- is 1-dimensional

0.1 Galileo's Principle of Relativity (Arnold)

There exist coordinate systems (called inertial) possessing the following two properties:

1. All the laws of nature at all moments of time are the same in all inertial coordinate sysems.
2. All coordinate systems in uniform rectilinear motion with respect to an inertial one are themselves inertial.

0.2 Newton's Principle of Determinacy (Arnold)

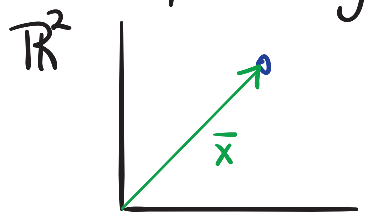
The initial state of a mechanical system
(the totality of positions and velocities of its points at some moment of time)
uniquely determines all of its [future] motion.

0.3 Spacetime is affine (Euclidean Space) (Arnold)

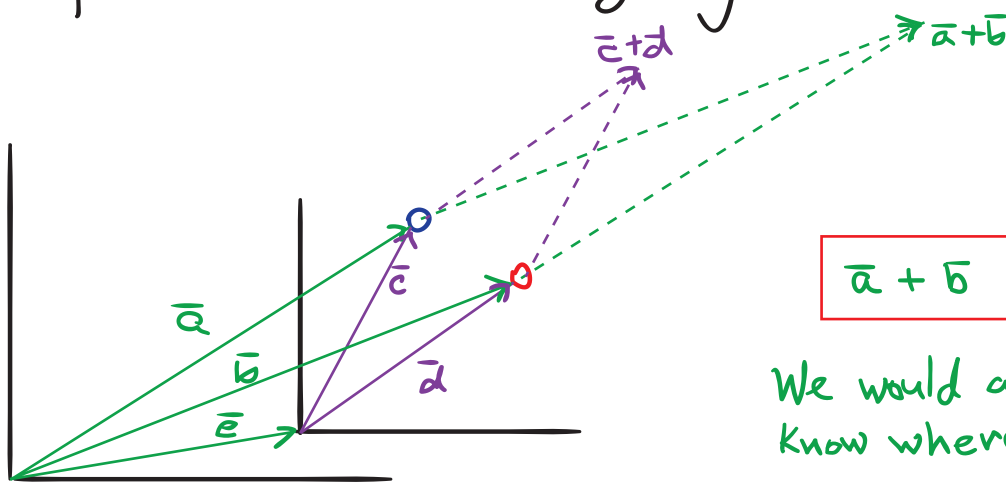
We often represent the locations of objects in spacetime with n-tuples of real numbers, e.g. (1 meter up, 2.3 meters to the right).
2-tuple, i.e. double

The set of all n-tuples of real numbers $\mathbb{R}^n$ is a vector space.
We are used to representing points in space as vectors.

E.g.



But spacetime doesn't behave globally like a vector space.

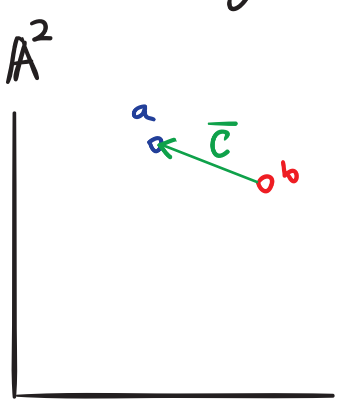


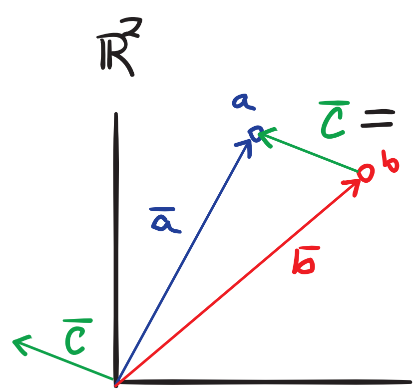
$$\boxed{\bar{a}+\bar{b} \neq \bar{c}+\bar{d}}$$

We would all have to know where the origin is!

Spacetime is actually affine. Affine spaces are like vector spaces, but they have no origin.

We will remember that spacetime is affine, but when we solve problems we will assign an origin and work with a vector space.





$\bar{c} = \bar{a} - \bar{b}$

Finally, we still need to define distances. The Euclidean norm defines the length of a vector $\bar{x}$: (in vector-not affine-space)

$$\boxed{\|\bar{x}\| = \sqrt{\bar{x}\cdot\bar{x}}} \; .$$

The Euclidean metric or "distance function" is (in vector space)

$$\boxed{d(\bar{x},\bar{y}) = \|\bar{x}-\bar{y}\|} \; .$$

An affine space with a Euclidean metric is a Euclidean Space $\mathbb{E}^n$.

Our model of spacetime is a Eulidean Space $\mathbb{E}^4$ (3 spatial, 1 time).

We will often consider only subspaces of $\mathbb{E}^4$!

0.4 Newton's equation of Motion

All future motions of a system are uniquely determined by their initial positions $\bar{r}(t_0)\in\mathbb{R}^n$ and velocities $\bar{v}(t_0)=\dot{r}(t_0)\in\mathbb{R}^n$.

In particular, the acceleration $\bar{a}=\dot{\bar{v}}=\ddot{\bar{r}}$ is determined:

$$\boxed{\bar{a} = \bar{D}(\bar{r},\bar{v},t)} \; .$$

Assuming that the laws of physics remain constant (Galilean relativity), for a closed system,

$$\boxed{\bar{a} = \bar{D}(\bar{r},\bar{v})} \; .$$

Determining $\bar{D}$ for any particular system is an experimental endeavor.

Certain common characteristics will arise, however. For instance, a spring often behaves in a certain way (influences $\bar{a}$ in a certain recognizable manne that depends of $\bar{r}$ + $\bar{v}$).