Part 1: Dynamics of a single particle

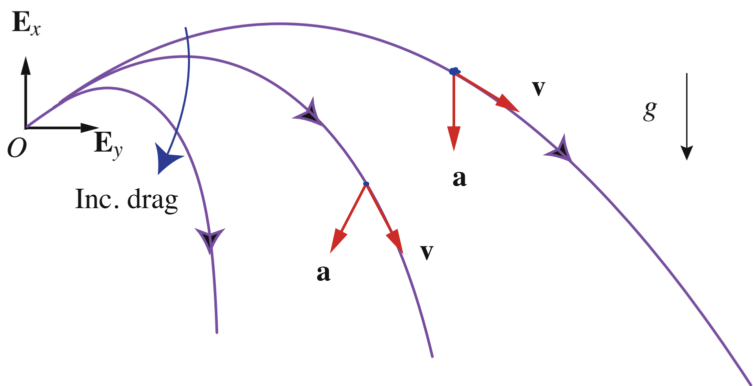
What is a particle?

Chapter 1: Elementary particle dynamics

Topics:
- particle kinematics
- particle dynamics
- Euler's first law (Newton's second law)

1.1 An example

A projectile particle:



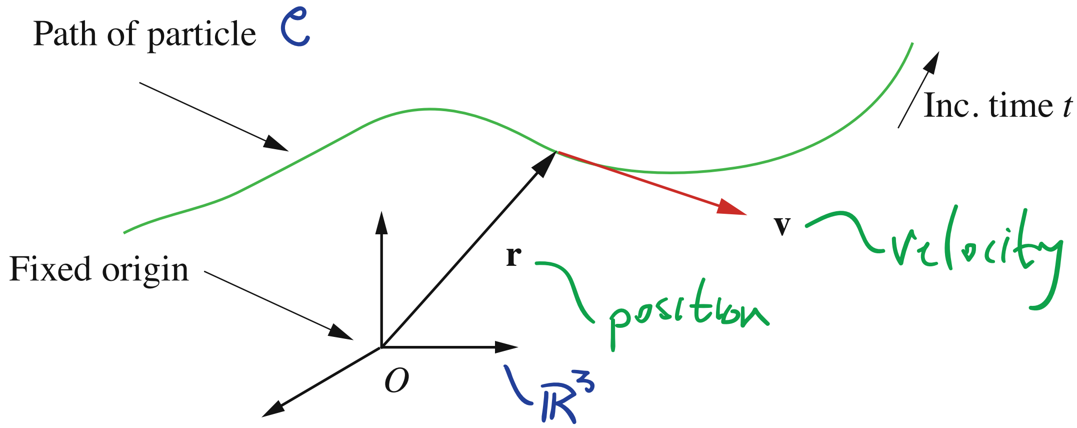
Forces:

1.2 Kinematics of a particle

What is kinematics?

*See Aristotle or Heidegger's "The Question Concerning Tech."

We will consider motion in a four-dimensional Euclidean Space $\mathbb{E}^4$ (3 spatial, 1 time).



Definitions

Position vector $\bar{r} \in \mathbb{R}^n$:

Absolute velocity vector-valued function $\bar{v}$: the time-rate of change of the location of a particle. It can be computed from $\bar{r}$ by:

Speed v: the magnitude of the velocity. It can be computed by:

Absolute acceleration vector-valued function $\bar{a}$: the time-rate of change of the velocity. It can be computed by:

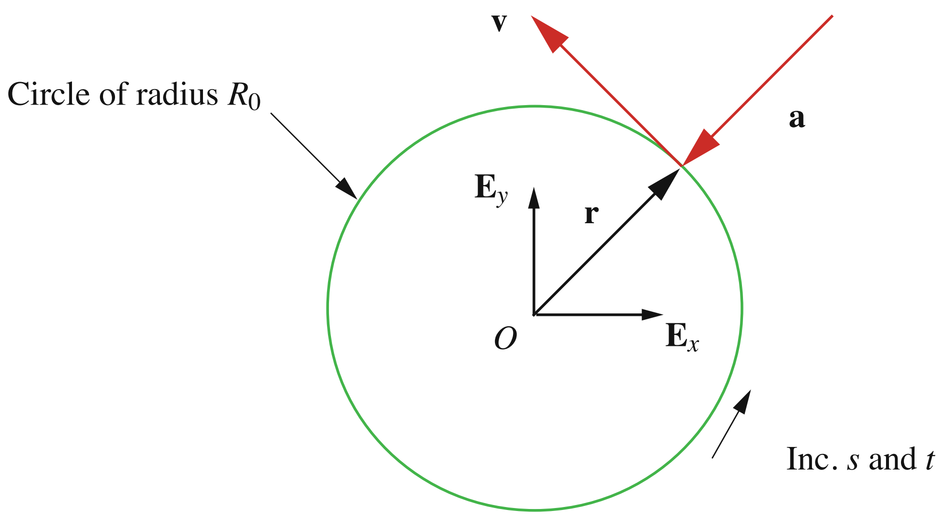
Distance along a path (arclength) s: the time derivative is the speed:

This can be integrated to find the distance travelled by the particle along its path $\mathcal{C}$ during the time interval $(t-t_0)$:

.

1.3 A Circular Motion (an example)

$$\bar{r} = \bar{r}(t) = R_0\left(\cos(\omega t)\bar{E}_x + \sin(\omega t)\bar{E}_y\right)$$



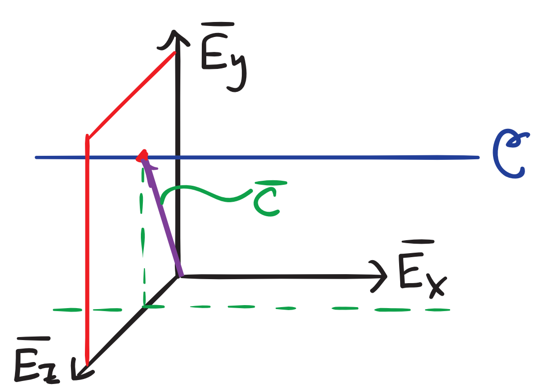
$\bar{E}_x, \bar{E}_y$:

Compute $\bar{v}(t)$:

Compute arclength:

.

1.4 Rectilinear Motions



$\bar{r} = \bar{r}(t) = x(t)\bar{E}_x + \bar{c}$

$\bar{v} = \bar{v}(t) = \dot{x}(t)\bar{E}_x = v(t)\bar{E}_x$

$\bar{a} = \bar{a}(t) = \ddot{x}(t)\bar{E}_x = a(t)\bar{E}_x$

1.4.1 Given $a = a(t)$

$$v(t) = v(t_0) + \int_{t_0}^{t} a(u)\,du$$

$$x(t) = x(t_0) + \int_{t_0}^{t} v(\tau)\,d\tau$$

1.4.2 Given $a = \hat{a}(v)$

Find $\hat{x}(v)$

We use the useful identity $\boxed{}$:

Find $\hat{t}(v)$

In order to find $\hat{t}(v)$, reason as follows:

$$\hat{a}(v) = \frac{dv}{d\hat{t}} \Longrightarrow d\hat{t} = \frac{dv}{\hat{a}(v)} \Longrightarrow \hat{t}(v) = \hat{t}(v_0) + \int_{v_0}^{v} \frac{1}{\hat{a}(u)}\,du.$$

1.4.3 Given $a = \hat{a}(x)$

Find $\hat{v}(x)$

Find $\hat{t}(x)$

$$\hat{v}(x) = \frac{d\hat{x}}{d\hat{t}} \Rightarrow d\hat{t} = \frac{d\hat{x}}{\hat{v}(x)} \Rightarrow \hat{t}(x) = \hat{t}(x_0) + \int_{x_0}^{x} \frac{1}{\hat{v}(u)}\,du$$