1.5 Kinetics of a particle

What is Kinetics?

What is mass?

Newton's second law / Euler's first law

Earlier we said that $\bar{a} = \bar{D}(\bar{r}, \bar{v})$. Newton and Euler specified $\bar{D}(\bar{r}, \bar{v})$ such that the resultant external force acting on a particle $\bar{F}$ can be written

$$\boxed{\phantom{\bar{F} = \dot{\bar{G}}}} .$$

$\bar{G}$ is the linear momentum: $\bar{G} = m \cdot \bar{v}$. So the resultant force is the time rate change of linear momentum.

It is important to remember that $\bar{a}$ is the absolute accel.

In the Cartesian basis $(\bar{E}_x, \bar{E}_y, \bar{E}_z)$, the eq can be written:

1.5.1 Action + Reaction (Newton's Third Law) (Spivak)

1.5.2 The Four Steps

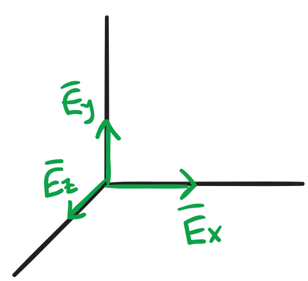
$\bar{F} = m\bar{a}$ can be used to analyze mechanical systems. We suggest these four steps.

1.6 A particle under the influence of gravity (an example)

Consider a particle of mass m launched from $\bar{r}_0$ with velocity $\bar{v}_0$ at time $t=0$. Include the gravitational force but not the drag force. Find $\bar{r}(t)$.

1.6.1 Kinematics

Cartesian basis:



1.6.2 Free-body diagram

1.6.3 $\bar{F} = m\bar{a}$

First, write the gravitational force eq.:

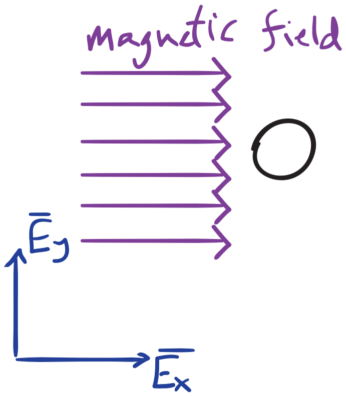
$\bar{F}$ is the resultant force, so it is the sum of forces.
In this case, there's only one force, so:

Now we can relate $\bar{F}$ and $\bar{a}$ with

1.6.4 Analysis (find $\bar{r}(t)$)

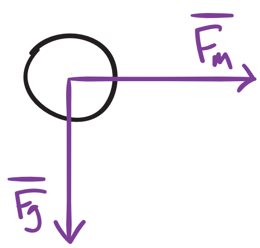
We have $\bar{a}$. Given our initial state $\bar{r}_0$ and $\bar{v}_0$, we want to know $\bar{r}(t)$.

1.7 A particle in magnetic + gravitational fields (example)



The ball is released from rest in a horizontal mag. field + vertical gravitational field.
If we measured the velocity as a function of time to be approx. $\bar{V}(t) = (t^2\,\bar{E}_x - 9.8t\,\bar{E}_y)\frac{m}{sec}$ what is $\bar{F}_m(t)$, the magnetic field force?

1.7.1 FBD



1.7.2 $\bar{F} = m\bar{a}$

Known force: $\bar{F}_g = -mg\,\bar{E}_y$

Unknown force: $\bar{F}_m$

Resultant force:

$$\bar{F} = \begin{array}{cc} F_m & \bar{E}_x \\ -\; mg & \bar{E}_y \end{array}$$

$\bar{F} = m\bar{a}$:

$$F_m \bar{E}_x - mg\,\bar{E}_y = m(a_x \bar{E}_x + a_y \bar{E}_y)$$

1.7.3 Analysis (find $F_m(t)$)

Solve for $\bar{a}$: $\bar{a} = \begin{bmatrix} \frac{1}{m} F_m \\ -g \end{bmatrix}$

From the measurement, we know:

$$\bar{a}(t) = \dot{\bar{v}}(t) = \begin{bmatrix} 2t \\ -9.8 \end{bmatrix} \qquad \therefore$$

$\boxed{F_m(t) = 2mt}$ and $g = -9.8\ m/sec^2$ (Earth)