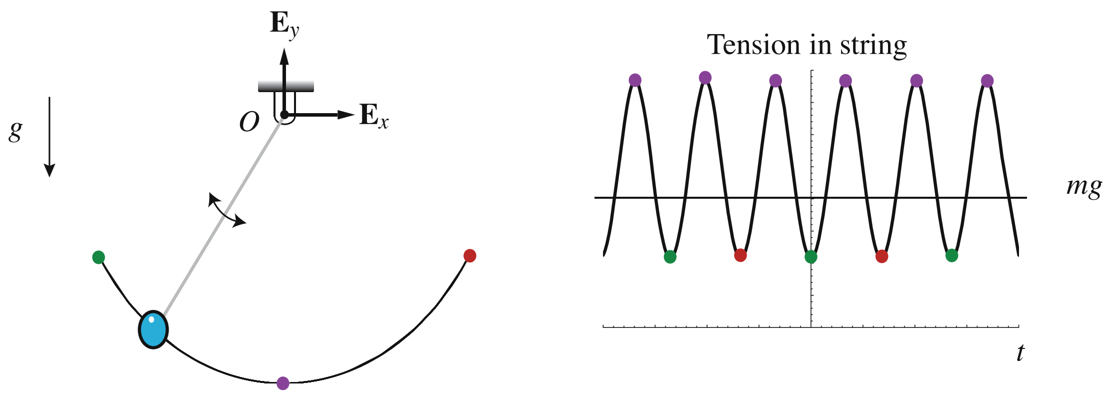
Chapter 2: Cylindrical Polar Coordinates

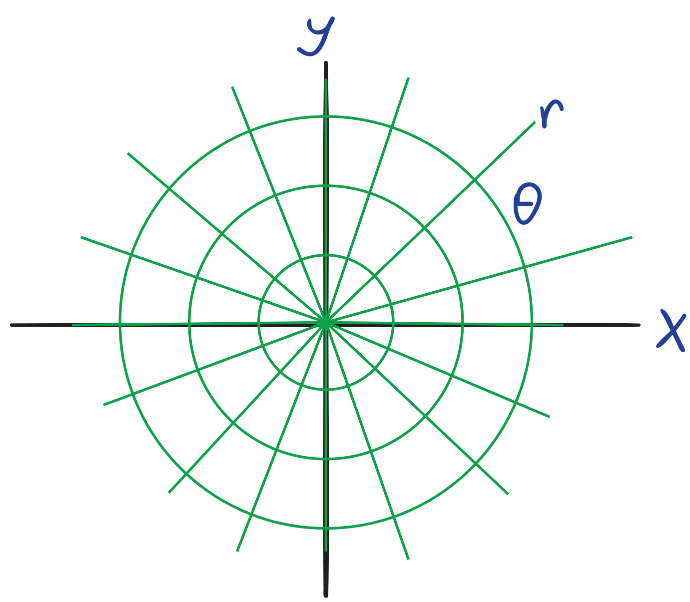
Topics:
- cylindrical polar coordinates
- basis $(\bar{e}_r, \bar{e}_\theta, \bar{E}_z)$
- Kinematics + Kinetics of particles w/ $(\bar{e}_r, \bar{e}_\theta, \bar{E}_z)$

2.1 The Cylindrical Polar Coordinate System

An example of a good system for polar coordinates:



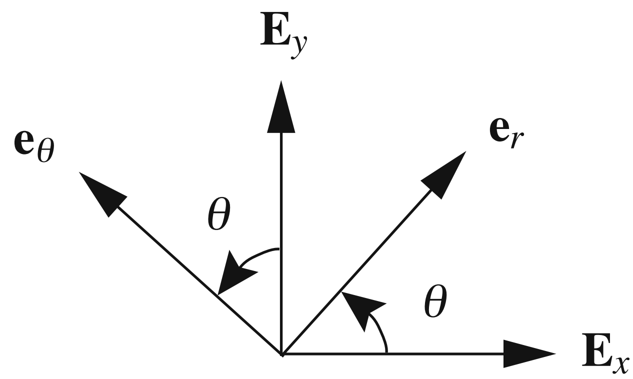
We will now define the cylindrical polar coordinate system $\{r, \theta, z\}$ in terms of the Cartesian system $\{x, y, z\}$.



Going the other way (assuming $\neg(X=0 \wedge y=0)$):

Now we can write our position vector $\bar{r}$ as

Define the basis vectors as:



(*)

Rewriting our position vector:

2.2 Velocity + Acceleration Vectors

Velocity

From (*) we know that $\boxed{} + \boxed{}$.

Combining this with the chain rule ($\dot{\bar{e}}_r = \dot{\theta}\, d\bar{e}_r/d\theta$),

$\boxed{}$

Acceleration

$\boxed{}$

2.3 Kinetics of a Particle

Writing $\bar{F} = m\bar{a}$ in cylindrical polar coordinates,

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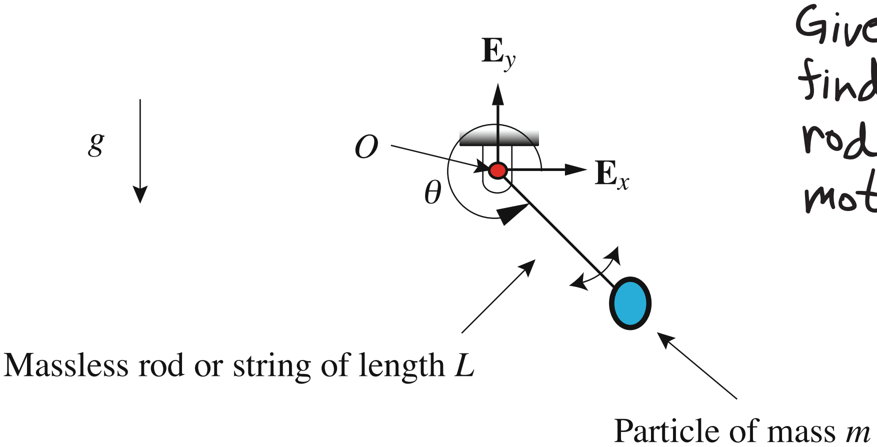
2.4 Planar Pendulum (example)



Given an initial state $\bar{r}_0, \bar{v}_0$, find the tension in the string/rod and the equations of motion.

2.4.1 Kinematics

Position:

Velocity:

Acceleration:

2.4.2 FBD + Forces

But we do know <u>something</u> about $\bar{T}$:

So

2.4.3 $\bar{F} = m\bar{a}$

$\bar{F} = m\bar{a}$ in the (r, θ, z)-basis:

2.4.4 Analysis

The $\bar{e}_\theta$ equation is an ODE from which we can find $\theta(t)$.

Example (Hibbeler 12-175)

A particle P moves along the spiral path $r = 10/\theta$ ft, where θ is in radians. If it maintains a constant speed of $v = v_0$, determine v_r and v_θ as functions of θ.