3.4 Application to Particle Mechanics

We make two identifications:

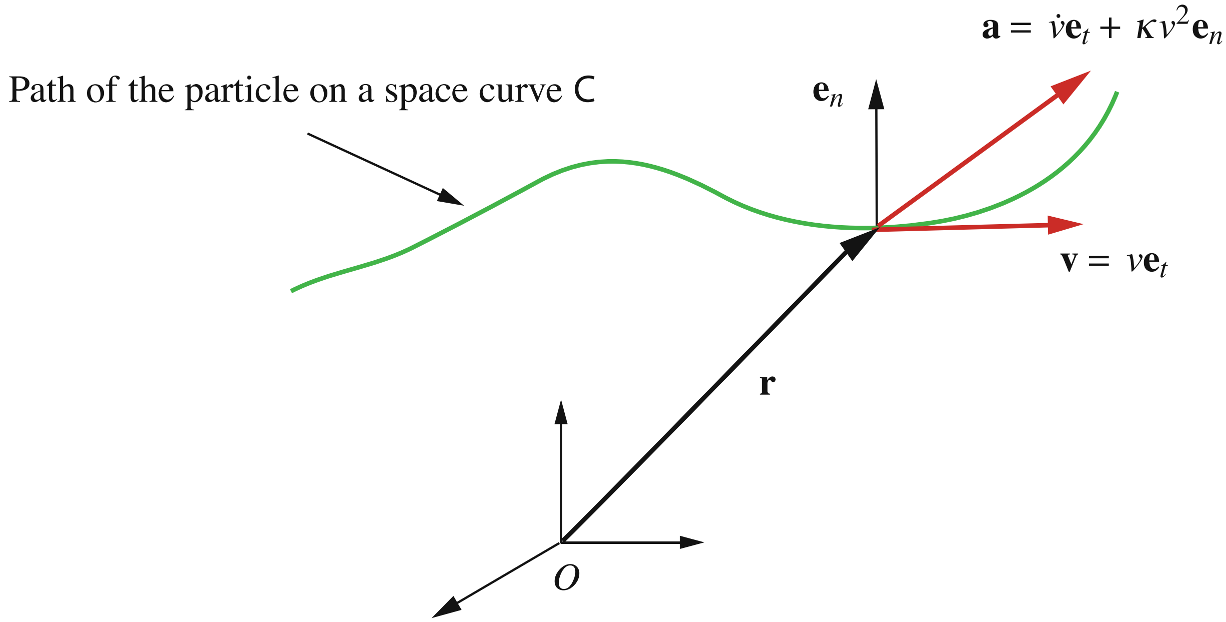
1. The space curve $\mathcal{C}$ is identified as the path of a particle.
2. The arclength parameter s is considered to be a function of time t.

Kinematics

Position:

Velocity:

Acceleration:



Kinetics

For a particle of mass m, $\bar{F} = m\bar{a}$.

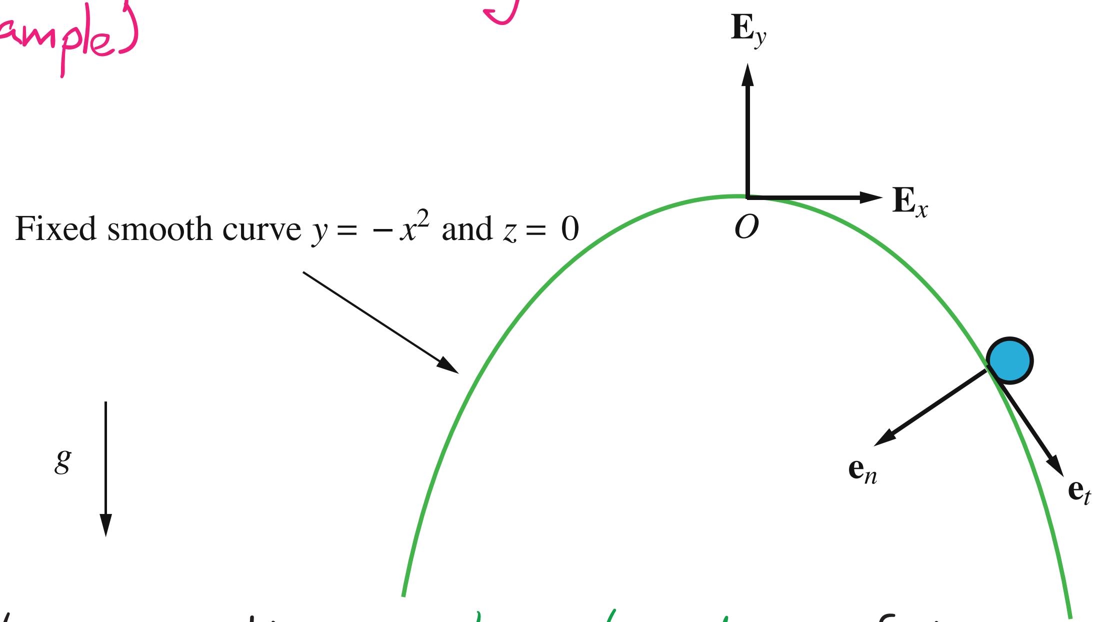
The resultant force $\bar{F}$ can be written as

So, in the Serret-Frenet basis,

$\boxed{}$.

Note that $F_b = 0$, so $\bar{F}$ is also entirely in the osculating plane.

3.5 A particle moving on a fixed curve under gravity (Example)



Determine the equation of motion of the particle and the force exerted "by the curve" to keep the particle on the curve.

3.5.1 Kinematics

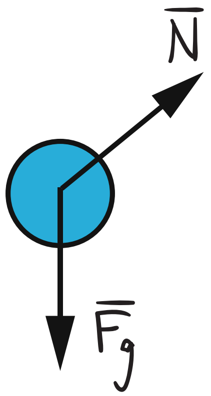
From 3.3.1 we know that the arclength is (with $x_0 = 0$)

We have $df/dx = d(-x^2)/dx = -2x$, so

We can also compute the curvature:

Acceleration:

3.5.2 Forces



We need to write $\bar{F}_g$ in the Serret-Frenet basis.
Recall:

Now we can write $\bar{F}$ in the Serret-Frenet basis:

3.5.3 $\bar{F} = m\bar{a}$

In the Serret-Frenet basis $\bar{F} = m\bar{a}$ is:

3.5.4 Analysis

First, note that:

and:

The $\bar{e}_t$-equation gives:

which is the equation of motion.

The $\bar{e}_n$-equation gives:

The $\overline{e_b}$ -equation gives:

So the force keeping the particle on the curve is:

$$\boxed{}\,.$$

So if we solve the equation of motion for $x(t)$, we will also know $\overline{N}(t)$.

Under what conditions will the particle deviate from the given spacecurve?