Chapter 6: Momenta, Impulses, + Collisions

Topics:
- linear + angular momenta of a single particle
- conservation of momentum
- impact

6.1 Linear Momentum + Its Conservation

Consider a particle of mass m, position $\bar{r}$, and velocity $\bar{v}$. Recall the definition od linear momentum:

6.1.1 Linear Impulse + Linear Momentum

The integral form of the "balance of linear momentum" $\bar{F} = m\bar{a}$ is:

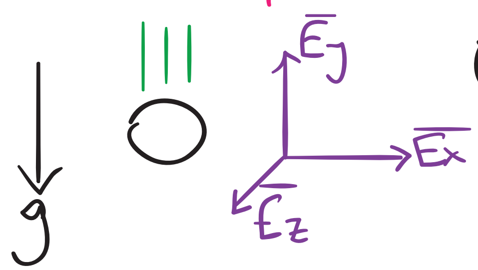
6.1.2 Conservation of Linear Momentum

Let $\bar{c}$ be a vector. $\bar{G}$ is conserved in the direction of $\bar{c}$ iff

This implies:

If $\bar{C}$ is a constant vector, $\bar{F} \cdot \bar{C} = 0$, i.e.

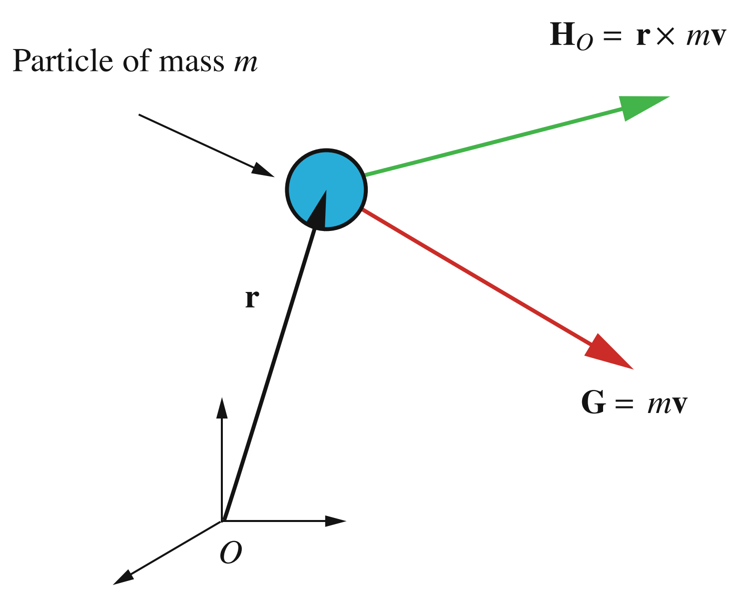
6.1.3 Example: Particle in a gravitational field



Q: In which directions is linear momentum conserved?

6.2 Angular Momentum + its Conservation

Let the angular momentum about the point O, $\bar{H}_0$, of a particle of mass m be defined as:



In Cartesian coordinates,

In cylindrical polar coordinates

When the motion is planar, this simplifies to:

6.2.1 Angular Momentum Theorem

How does the angular momentum evolve in time?

The final result we call the angular momentum theorem

$\boxed{}$.

6.2.2 Conservation of Angular Momentum

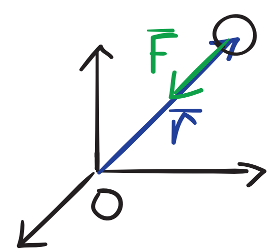
The angular momentum in the direction of a vector $\bar{c}$ is conserved iff

This implies

If $\bar{c}$ is constant, this reduces to

In this class, often we can choose $\bar{E}_z = \bar{c}$.

6.2.3 Central Force Problems

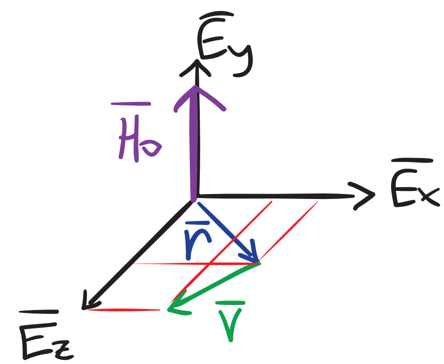


When the resultant force $\bar{F}$ is parallel to $\bar{r}$: $\dot{\bar{H}}_o = \bar{r} \times \bar{F} = \bar{0}$.

Therefore, $\bar{H}_o$ is conserved.

Let $\bar{H}_o = h\bar{h}$.

We can set-up these problems such that $\bar{E}_z = \bar{h}$, $\bar{r} = r\,\bar{e}_r$, + $\bar{v} = \dot{r}\bar{e}_r + r\dot{\theta}\,\bar{e}_\theta$ by choosing



$\bar{H}_o = h\bar{E}_z = \bar{r}(t_o) \times m\bar{v}(t_o)$ (w/ initial cond. $\bar{r}(t_o)$ + $\bar{v}(t_o)$)