6.3 Collision of Particles

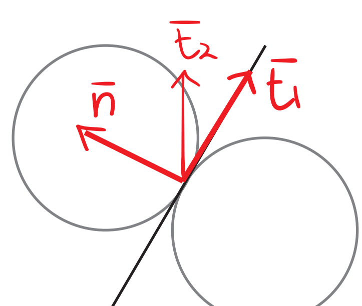
In this section we model the collision of particles. In order to lend the model some realism, we have to allow the particles to "deform," which is ad hoc.

6.3.1 The Model and the Impact Stages

We model two masses m_1 and m_2 as particles with position vectors locating the centers of mass.

Of interest are four distinct time intervals:

We use the basis $(\bar{n}, \bar{t}_1, \bar{t}_2)$:



6.3.2 Linear Impact During Impulses

Forces during II have subscript "d" and forces during III have subscript "r".

All other forces on m_1 + m_2 have resultant forces $\overline{R}_1$ + $\overline{R}_2$, respectively. We assume that the impulses during II + III are dominated by the inter-particle forces.

We can define the coefficient of restitution e as

So, if $e=1$, the compression and restitution impulses are equal. If $e=0$, there is no restitution impulse. The former is called a perfectly elastic collision, the latter a perfectly inelastic collision. In general, $0 \leq e \leq 1$, and e is determined experimentally.

Let "primed" velocities denote velocities after impact.
It can be shown that:

$$\boxed{} \; .$$

This is a very useful equation.

6.3.3 Linear Momenta

The integral form of the balance of linear momentum for each particle gives:

Using e and assuming that the effects of $\overline{R}_i$ are negligible during impact, and that $\overline{F}_{1d} = -\overline{F}_{2d}$ + $\overline{F}_{1r} = -\overline{F}_{2r}$, If we take the dot-product of these equations in the directions $\{\bar{n}, \bar{t}_1, \bar{t}_2\}$, we find that:

$$\boxed{\begin{aligned} &\bar{v}_1' \cdot \bar{t}_1 = \bar{v}_1 \cdot \bar{t}_1 \qquad \bar{v}_1' \cdot \bar{t}_2 = \bar{v}_1 \cdot \bar{t}_2 \\ &\bar{v}_2' \cdot \bar{t}_1 = \bar{v}_2 \cdot \bar{t}_1 \qquad \bar{v}_2' \cdot \bar{t}_2 = \bar{v}_2 \cdot \bar{t}_2 \\ &m_2 \bar{v}_2' \cdot \bar{n} + m_1 \bar{v}_1' \cdot \bar{n} = m_2 \bar{v}_2 \cdot \bar{n} + m_1 \bar{v}_1 \cdot \bar{n} \\ &m_2 \bar{v}_2' \cdot \bar{n} - m_2 \bar{v}_2 \cdot \bar{n} = -(1+e) \int_{t_0}^{t_1} \bar{F}_{1d}(\tau) \cdot \bar{n} \, d\tau . \end{aligned}}$$

I.e.:

in the $\bar{t}_1$ + $\bar{t}_2$-directions, each particle's momentum is separately conserved.

In the $\bar{n}$-direction, the system's momentum is conserved.

With the six boxed equations above, we can solve for the six components of unknown velocities $\bar{v}_1'$ + $\bar{v}_2'$, provided we know $\bar{v}_1$, $\bar{v}_2$, and the linear impulse of $\bar{F}_{1d}$ during the collision. This last one is often unknown, so we instead use e from experimental data.

6.3.4 Postimpact velocities

It is convenient to solve the above system of equations for the postimpact velocities, for which we often solve in typical problems.

$$\boxed{\begin{aligned}
\bar{V}_1' &= (\bar{V}_1 \cdot \bar{t}_1)\bar{t}_1 + (\bar{V}_1 \cdot \bar{t}_2)\bar{t}_2 + \frac{1}{m_1+m_2}\Big((m_1 - e m_2)\bar{V}_1 \cdot \bar{n} + (1+e) m_2 \bar{V}_2 \cdot \bar{n}\Big)\bar{n} \\
\bar{V}_2' &= (\bar{V}_2 \cdot \bar{t}_1)\bar{t}_1 + (\bar{V}_2 \cdot \bar{t}_2)\bar{t}_2 + \frac{1}{m_1+m_2}\Big((m_2 - e m_1)\bar{V}_2 \cdot \bar{n} + (1+e) m_1 \bar{V}_1 \cdot \bar{n}\Big)\bar{n}
\end{aligned}}$$