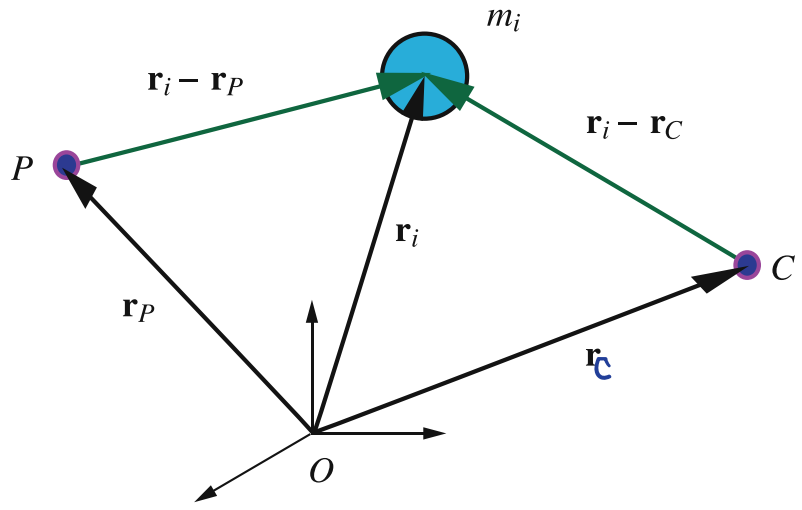
Chapter 7: Dynamics of a System of Particles

Topics:- linear momenta
angular momenta +
Kinetic energy for a system of particles
- center of mass
- Conservation of kinematical quantities

7.1 Preliminaries

Consider a system of n particles, each of mass m_i, with $i \in \mathbb{Z}$. The position vector of m_i is denoted $\bar{r}_i$. We use the following figure throughout this section. The following notation is used for each particle.



7.2 The Center of Mass, Momenta, & Kinetic Energy

7.2.1 The Center of Mass

The center of mass C of a system of particles is the point described by the position vector

where

The velocity of the center of mass is

From these expressions, we can write the identities

which we will use momentarily.

7.2.2 Linear Momentum

The linear momentum $\overline{G}$ of a system of particles is the sum of the linear momenta of the particles. The following demonstration is instructive.

This final expression is the definition of the linear momentum of a system of particles. We found it with the assumption that the linear momentum of the summed masses could be described as the sum of the masses multiplied by the velocity of a single point: the center of mass C. Therefore, our assumption was valid.

7.2.3 Angular Momentum

The angular momentum $\overline{H_P}$ of a system of particles about a point P is:

7.2.4 Kinetic Energy

The kinetic energy of a system of particles is defined to be

This can be rewritten as