## 7.3 Kinetics of a System of Particles

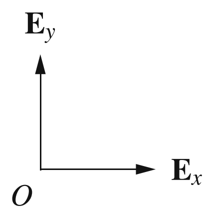
The resultant force $\overline{F}$ on a system of particles is

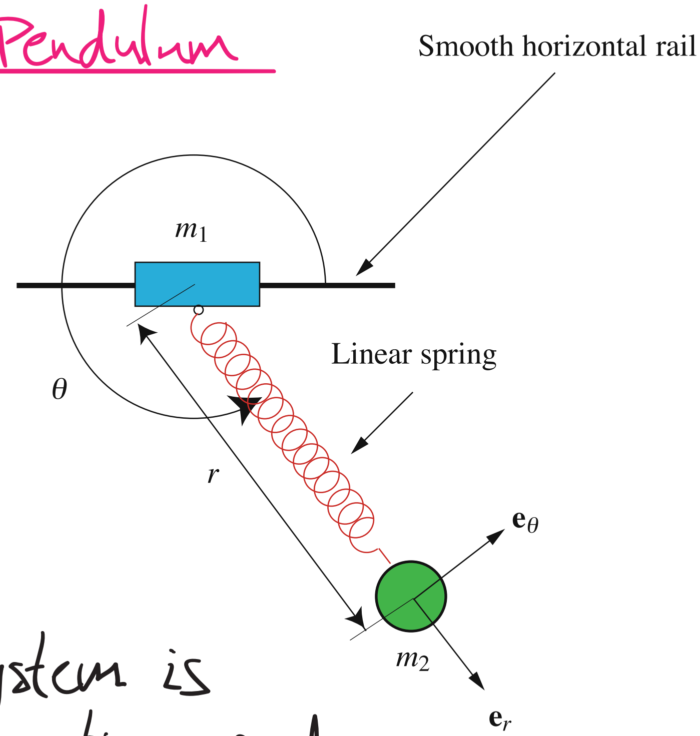
Newton's second Law (Euler's first law/balance of linear momentum) for each particle is

So $\overline{F} = \sum_{i=1}^{n} m_i \overline{a_i} = m \overline{a}$, by the definition of the center of mass.

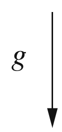
This is a useful fact. Solving the coupled equations of motion for every particle in a system is often very difficult.

# 7.5 The Cart + the Pendulum (Example)




The figure shows a two-particle system of masses $m_1$ + $m_2$.




Show that the linear momentum of the system is conserved in the $\mathbf{E}_x$-direction, and explore what this means for the motion of the particles.

## 7.5.1 Kinematics

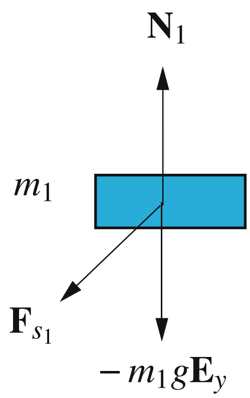
Position:

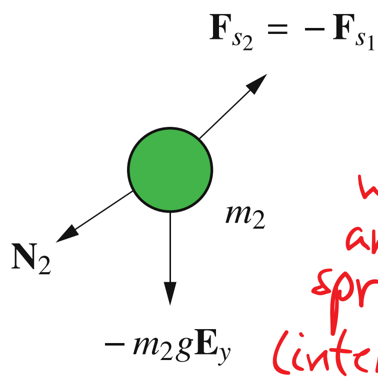
Velocity:

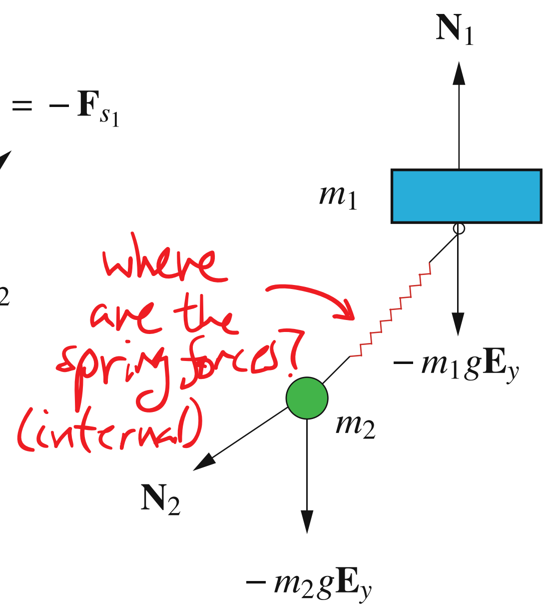
Acceleration:

## 7.5.2 Forces

FBDs:




Write what we know about the forces:

## 7.5.3 Balance Laws

The balances of linear momentum for the particles are:

With minimal rearranging of the six scalar equations, from which we want $r$, $x$, $\theta$, $N_{1y}$, $N_{1z}$, + $N_{2z}$ we find that

Given $r(t_0)$, $\theta(t_0)$, $x(t_0)$, $\dot{r}(t_0)$, $\dot{\theta}(t_0)$, + $\dot{x}(t_0)$, $r(t)$, $\theta(t)$, + $x(t)$ can be found by solving the coupled ODEs. Such a solution is beyond the scope of this class.

## 7.5.4 Analysis

Now let's consider the balance of linear momentum for the system:

Therefore $\bar{F} \cdot \bar{E}_x = 0$, and linear momentum is conserved in the x-direction.
Therefore, the x-component of the velocity of the center of mass is constant.

Writing out the $\bar{E}_x$-component of linear momentum,

If we write $G_0 = \bar{G} \cdot \bar{E}_x$ and solve for $\dot{x}$,

In the next lecture we will return to this analysis to show that energy is conserved.