7.6 Conservation of Angular Momentum

In 7.2 we noted that the angular momentum of a system of particles about some point P is

Taking the time-derivative, it can be shown that

This is called the angular momentum theorem for a system of particles.

We define the resultant moment of the system relative to P as

We can use this to rewrite $\dot{H}_P$:

Two special cases of the angular momentum theorem:

If P is a fixed point O and $\bar{r}_0 = \bar{0}$:

If P is the center of mass C: $\bar{v} \times \bar{G} = \bar{0}$ and

To summarize an important point:
If P is fixed or C, the time rate of change of the angular momentum about P is the resultant moment about P.
If P is moving and not C, this is not true.

Finally, we consider when $\bar{H}_P$ is conserved in the direction of a vector $\bar{c}$:

This implies the necessary and sufficient condition for the conservation of $\bar{H}_P$ in the direction of $\bar{c}$:

7.8 Work, Energy, and Conservative Forces

Recalling the work-energy theorem for a single particle:

and defining the total kinetic energy of a system of particles as

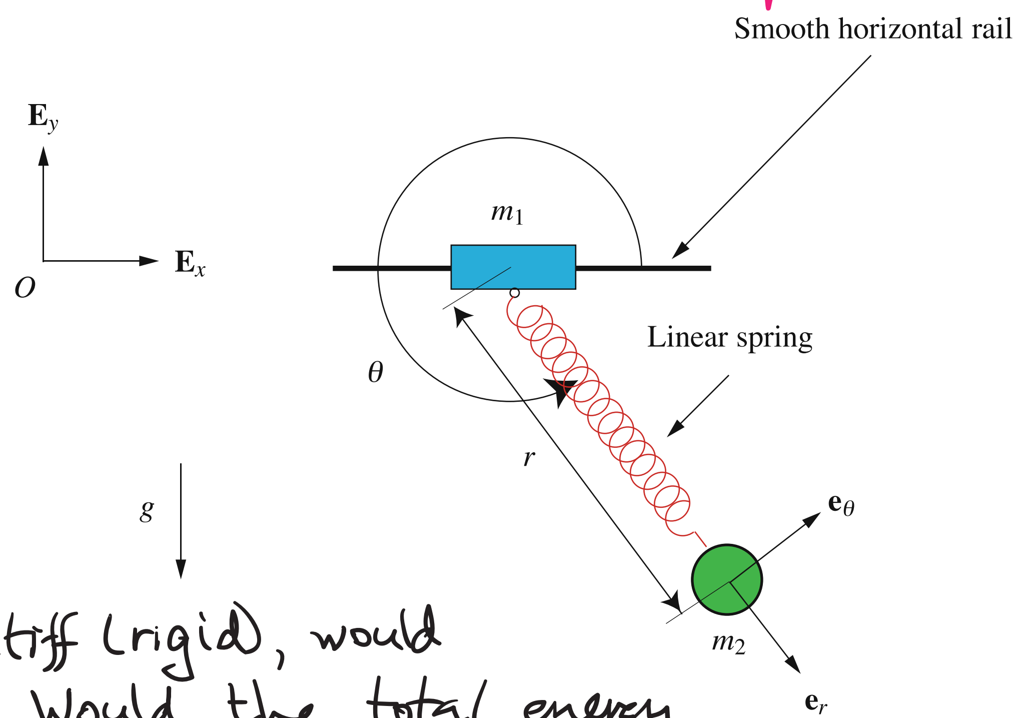
we can derive the work-energy theorem for a system of particles:

Similar to the development in chapter 5, we can rewrite this by separating conservative and non-conservative forces such that

where E is the total energy and $\bar{F}_{nci}$ is the resultant of the nonconservative forces on particle i.

The second form is usually more useful in solving problems.

7.8.1 The Cart + Pendulum Revisited (Example)



Part I

Is the energy of the system conserved?

Part II

If the spring was infinitly stiff (rigid), would it do work? Would the total energy of the system be conserved?

Part I solution

The work-energy theorem gives

The normal forces are perpendicular to the velocities, $\overline{N}_1 \cdot \overline{v}_1 = \overline{N}_2 \cdot \overline{v}_2 = 0$, so they do no work.

Furthermore, the spring powers combine to give

In summary,

$$\dot{T} = -\frac{d}{dt}\left(\frac{1}{2}k\left(\|\bar{r}_1-\bar{r}_2\|-L\right)^2 + m_1 g \bar{E}_y\cdot\bar{r}_1 + m_2 g \bar{E}_y\cdot\bar{r}_2\right).$$

Recognizing $U=\sum_i U_i$,

So the total energy of the system is conserved.

Part II solution

Kinematics: $\bar{r}_2-\bar{r}_1 = L\,\bar{e}_r \qquad \bar{v}_2-\bar{v}_1 = L\dot{\theta}\,\bar{e}_\theta$

Work-energy theorem:

$$\dot{T} = \left(S\bar{e}_r - m_1 g \bar{E}_y + \bar{N}_1\right)\cdot\bar{v}_1 + \left(-S\bar{e}_r - m_2 g \bar{E}_y + \bar{N}_2\right)\cdot\bar{v}_2$$

where $S\bar{e}_r$ is the tension force in the rod.